

$$\hat{A} = \sum_{\lambda, \mu} a_{\lambda}^{\dagger} a_{\mu} A_{\lambda\mu} \text{ mit } A_{\lambda\mu} = \langle \lambda | A(\underline{x}) | \mu \rangle$$

$$\begin{aligned} \Delta x^2 &= \langle n | (\hat{x} - \underline{x})^2 | n \rangle = \langle n | \hat{x}^2 | n \rangle \quad [b, b^{\dagger}] = bb^{\dagger} - b^{\dagger}b = 1 \\ &= \frac{\hbar}{2m\omega} \langle n | (b^{\dagger} + b)^2 | n \rangle = \frac{\hbar}{2m\omega} \langle n | b^{\dagger}b + bb^{\dagger} | n \rangle \\ &= \frac{\hbar}{2m\omega} \langle n | 2b^{\dagger}b + 1 | n \rangle = \frac{\hbar}{m\omega} \left(n + \frac{1}{2} \right) \end{aligned}$$

$$H = \underbrace{-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \frac{m}{2} \omega^2 x^2}_{H_0} + \underbrace{Cx^3 + Dx^4}_{H_1}$$

$$E_n^{(1)} = \langle n | H_1 | n \rangle = \langle n | C\hat{x}^3 + D\hat{x}^4 | n \rangle = D \langle n | \hat{x}^4 | n \rangle$$

$$\hat{x}^4 = \left(\frac{\hbar}{2m\omega} \right)^2 (b^{\dagger} + b)^4 \Rightarrow (b^{\dagger} + b)^4 = (b^{\dagger 2} + b^{\dagger}b + bb^{\dagger} + b^2)^2$$

$$E_n^{(1)} = D \left(\frac{\hbar}{2m\omega} \right)^2 \langle n | (b^{\dagger 2}b^2 + (b^{\dagger}b)^2 + b^{\dagger}b b b^{\dagger} + (bb^{\dagger})^2 + bb^{\dagger}b^{\dagger}b + b^2b^{\dagger 2} | n \rangle$$

$$(1) \quad b^{\dagger}b^{\dagger}bb = b^{\dagger}b b^{\dagger}b - b^{\dagger}b = (b^{\dagger}b)^2 - b^{\dagger}b \quad \dots \quad [b, b^{\dagger}] = 1$$

$$E_n^{(1)} = D \left(\frac{\hbar}{2m\omega} \right)^2 \langle n | 6(b^{\dagger}b)^2 + 6(b^{\dagger}b) + 3 | n \rangle$$

$$= D \left(\frac{\hbar}{2m\omega} \right)^2 (6n^2 + 6n + 3)$$

$$= \int_{-\infty}^{\infty} \phi_n^*(x) (Dx^4) \phi_n(x) dx \quad ; \quad \phi_n(x) = \left(\frac{m\omega}{\pi\hbar} \right)^{1/4} \frac{1}{\sqrt{n!2^n}} \exp\left\{ -\frac{m\omega}{2\hbar} x^2 \right\} H_n(\xi)$$

mit $\xi = \sqrt{\frac{m\omega}{\hbar}} x$

$$\hat{V} = \int \hat{n}(\vec{r}) v(\vec{r}) d^3r$$

$$\left. \begin{aligned} H|g\rangle &= E_g|g\rangle \Rightarrow E_g = \langle g|\hat{H}|g\rangle \\ \langle g|g\rangle &= 1 \quad \hat{H} = \hat{T} + \hat{V} + \hat{V}_{ee} \\ E_g &= \langle g|\hat{T}|g\rangle + \langle g|\hat{V}|g\rangle + \langle g|\hat{V}_{ee}|g\rangle \\ \langle g|\hat{V}|g\rangle &= \int \langle g|\hat{n}(\vec{r})|g\rangle v(\vec{r}) d^3r = \int n_g(\vec{r})v(\vec{r}) d^3r \end{aligned} \right\}$$