$$\hat{A} = \sum_{\lambda,\mu}^{1\dots\infty} a_\lambda^+ a_\mu A_{\lambda\mu} \quad \text{mit} \quad A_{\lambda\mu} = \langle \lambda | A(\underline{x}) | \mu \rangle$$

$$[b,b^+] = bb^+ - b^+b = 1$$

$$\Delta x^2 = \langle n | (\hat{x} - \langle x \rangle)^2 | n \rangle = \langle n | \hat{x}^2 | n \rangle$$

$$= \frac{\hbar}{2m\omega} \langle n | (b^+ + b)^2 | n \rangle = \frac{\hbar}{2m\omega} \langle n | b^+b + bb^+ | n \rangle$$

$$= \frac{\hbar}{2m\omega} \langle n | 2b^+b + 1 | n \rangle = \frac{\hbar}{m\omega} \left(n + \frac{1}{2}\right)$$

$$H = \underbrace{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{m}{2}\omega^2 x^2}_{H_0} + \underbrace{C x^3 + D x^4}_{H_1}$$

$$E_n^{(1)} = \langle n | H_1 | n \rangle = \langle n | C\hat{x}^3 + D\hat{x}^4 | n \rangle = D \langle n | \hat{x}^4 | n \rangle$$

$$\hat{x}^4 = \left(\frac{\hbar}{2m\omega}\right)^2 (b^+ + b)^4 \Rightarrow (b^+ + b)^4 = (b^{+2} + b^+b + bb^+ + b^2)^2$$

$$E_n^{(1)} = D\left(\frac{\hbar}{2m\omega}\right)^2 \langle n | b^{+2}b^2 + (b^+b)^2 + b^+bbb^+ + (bb^+)^2 + bb^+b^+b + b^2b^{+2} | n \rangle$$

$$(1) \quad b^+b^+bb = b^+bb^+b - b^+b = (b^+b)^2 - b^+b \quad \dots \qquad [b,b^+] = 1$$

$$E_n^{(1)} = D\left(\frac{\hbar}{2m\omega}\right)^2 \langle n | 6(b^+b)^2 + 6(b^+b) + 3 | n \rangle$$

$$= D\left(\frac{\hbar}{2m\omega}\right)^2 (6n^2 + 6n + 3)$$

$$= \int_{-\infty}^{+\infty} \phi_n^*(x) (D x^4) \phi_n(x)\, dx \; ; \quad \phi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{n! 2^n}} \exp\left\{-\frac{\xi^2}{2}\right\} H_n(\xi)$$

$$\text{mit } \xi = \sqrt{\frac{m\omega}{\hbar}}\, x$$

$$\hat{V} = \int \hat{n}(\vec{r})\, v(\vec{r})\, d^3r$$

$$\hat{H}|g\rangle = E_g|g\rangle \Rightarrow E_g = \langle g|\hat{H}|g\rangle$$

$$\langle g|g\rangle = 1 \qquad \hat{H} = \hat{T} + \hat{V} + \hat{V}_{ee}$$

$$E_g = \langle g|\hat{T}|g\rangle + \langle g|\hat{V}|g\rangle + \langle g|\hat{V}_{ee}|g\rangle$$

$$\langle g|\hat{V}|g\rangle = \int \langle g|\hat{n}(\vec{r})|g\rangle\, v(\vec{r})\, d^3r = \int n_g(\vec{r})\, v(\vec{r})\, d^3r$$