$$\frac{\delta}{\delta \varphi(x)} \int \varphi(x') f(x') dx' = f(x)$$

$$= \frac{d}{d\alpha} \int (\varphi + \alpha \eta) f(x') dx' \Big|_{\alpha=0} = \int \eta(x') f(x') dx' = f(x)$$

$$\frac{\delta}{\delta \varphi(x)} \int f(\varphi(x'), \varphi'(x'), x') dx' = \frac{\partial f}{\partial \varphi(x)} - \frac{d}{dx} \frac{\partial f}{\partial \varphi'(x)}$$

$$\frac{d}{d\alpha} \int f(\varphi(x') + \alpha \eta(x'), \varphi'(x') + \alpha \eta'(x'), x') dx' \Big|_{\alpha=0} =$$

$$\int \left[\frac{\partial f}{\partial \varphi} \eta(x') + \frac{\partial f}{\partial \varphi'} \eta'(x') \right] dx' = \int \left[\frac{\partial f}{\partial \varphi} \eta - \frac{d}{dx'} \frac{\partial f}{\partial \varphi'} \eta \right] dx'$$

$$= \int \left[\frac{\partial f}{\partial \varphi} - \frac{d}{dx'} \frac{\partial f}{\partial \varphi'} \right] \eta(x') dx'$$