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Influenzfunktionale. Lineare Ankopplung

$$\mathcal{H} = H_S[q] + H_B[x] + H_{SB}[x,q] = H_S[q] + \sum_{\alpha=1}^{N} \left\{ \frac{p_\alpha^2}{2m_\alpha} + \frac{1}{2} m_\alpha \Omega_\alpha^2 x_\alpha^2 + f_\alpha[q] x_\alpha \right\}$$

$$\mathcal{F}[q_{t'}, q'_{t'}] = e^{-\phi[q_{t'}, q'_{t'}]}$$

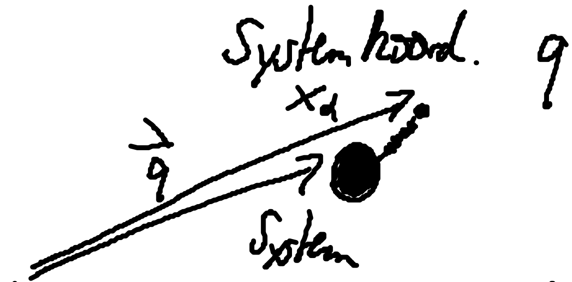
$$\Phi[q_{t'}, q'_{t'}] = \sum_{\alpha=1}^{N} \int_0^t dt' \int_0^{t'} ds \left\{ f_\alpha[q_{t'}] - f_\alpha[q'_{t'}] \right\}.$$

$$\cdot \left\{ S_\alpha(t'-s) f_\alpha[q_s] - S_\alpha^*(t'-s) f_\alpha[q'_s] \right\}$$

$$S_\alpha(\tau) = \frac{1}{2 m_\alpha \Omega_\alpha} \left( \coth \frac{\beta \Omega_\alpha}{2} \cos \Omega_\alpha \tau - i \sin \Omega_\alpha \tau \right)$$

Lineare Ankopplung an $q$

$$H_B[x] + H_{SB}[xq] = \sum_{\alpha=1}^{N} \left[ \frac{p_\alpha^2}{2m_\alpha} + \frac{1}{2} m_\alpha \Omega_\alpha^2 \left( X_\alpha - \frac{c_\alpha}{m_\alpha \Omega_\alpha^2} q \right)^2 \right]$$




$$= \sum_{\alpha=1}^{N} \left\{ \frac{p_\alpha^2}{2m_\alpha} + \frac{1}{2} m_\alpha \Omega_\alpha^2 x_\alpha^2 - c_\alpha q X_\alpha + \frac{1}{2} \frac{c_\alpha^2}{m_\alpha \Omega_\alpha^2} \cdot q^2 \right\}$$

$$\phi[q_{t'}, q'_{t'}] = \int_0^t dt' \int_0^{t'} ds \, \{ q_{t'} - q'_{t'} \} \{ L(t'-s) q_s - L^*(t'-s) q'_s \}$$

$$+ i \frac{\mu}{2} \int_0^t dt' \{ q_{t'}^2 - (q'_{t'})^2 \}$$

$$L(\tau) \equiv \frac{1}{\pi} \int_0^\infty d\omega \, J(\omega) \left\{ \coth \frac{\beta \omega}{2} \cos \omega \tau - i \sin \omega \tau \right)$$

Spektraldichte des Bades

$$J(\omega) \equiv \frac{\pi}{2}\sum_{\alpha=1}^{N}\frac{c_\alpha^2}{M_\alpha\Omega_\alpha}\,\delta(\omega-\Omega_\alpha)$$

Jetzt Modelle für $J(\omega)$, z.B.

- $J(\omega) = 2\alpha\,\omega\, e^{-\omega/\omega_c}\,\theta(\omega)$

Ohmsche Spektralfkt. mit exponentiellen Cutoff und Kopplungsstärke $\alpha$

$$\mu \equiv \frac{1}{2}\sum_{\alpha=1}^{N}\frac{c_\alpha^2}{M_\alpha\Omega_\alpha^2} = \frac{2}{\pi}\int_0^\infty d\omega\,\frac{J(\omega)}{\omega}.$$

Zusätzliches Systempotential

$$V_{counter}[q] = \frac{1}{2}\mu q^2$$

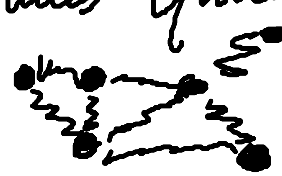
gibt

$$\exp\left(i\frac{\mu}{2}\int_0^t dt'\left\{q_{t'}^2 - q_{t'}'^2\right\}\right).$$

## Propagator für den gedämpften harm. Oszillator

– H. Grabert, P. Schramm, G.-L. Ingold, Phys. Rep. 168, 115 (1988).

exakte Lösung für System = harm. Oszillator weil alles quadratisch ist!



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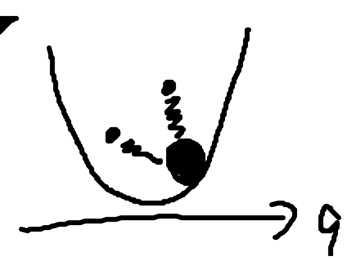


— Vergleich mit Mastergleichung: R. Karrlein, H. Grabert, Phys. Rev. E 55, 153 (1997).

## Influenzfunktionale für allgemeine Bäder

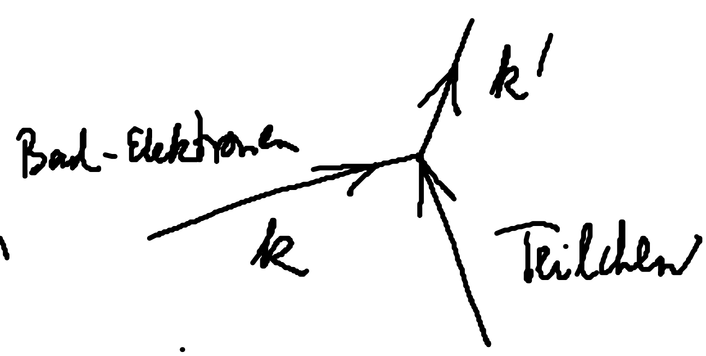
$$\mathcal{F}[q(t'), q'(t')] = \mathrm{Tr}_B\left(\rho_B\, U_B^+[q']\, U_B[q]\right)$$

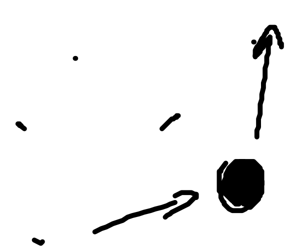
$$\left.\begin{aligned} H_B(t) &= \mathcal{H}_0 + V(t) \\ H'_B(t) &= \mathcal{H}_0 + V'(t) \end{aligned}\right\}$$ beziehen sich auf verschiedene Pfade $q$ und $q'$

Beispiele: $\mathcal{H}_0 = \sum_k \varepsilon_k c_k^+ c_k$ freie Fermionen

$$V(t) \equiv V[q_t] = \sum_{kk'} M_{kk'}\, c_{k'}^+ c_k\, e^{i(k-k')q_t}$$

Störoperator in 2. Quantisierung. Einteilchenoperator



(SG)

$$i\frac{\partial}{\partial t}\tilde{U}_B = \tilde{V}(t)\,\tilde{U}_B$$

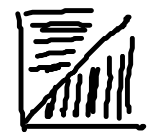
Jetzt WW-Bild

im WW Bild

$$U_B[q] = e^{-i\hat{H}_0 t}\Big\{1 + i\int_0^t dt'\,\tilde{V}(t') - \int_0^t dt\int_0^{t'} dt'\,ds\,\tilde{V}(t')\tilde{V}(s) + \dots\Big\}$$

$$U_B^+[q'] = \Big\{1 - i\int_0^t dt'\,\tilde{V}'(t') - \int_0^t dt\int_0^t ds\,\tilde{V}'(s)\tilde{V}'(t') \dots\Big\} \times e^{+iH_0 t}$$

$$U_B^+[q']U_B[q] = 1 - i\int_0^t dt'\,\{\tilde{V}'(t') - \tilde{V}(t')\} +$$



$$+ \int_0^t dt'\,\tilde{V}'(t')\int_0^t ds\,\tilde{V}(s) - \int_0^t dt'\int_0^{t'} ds\,\{\tilde{V}'(s)\tilde{V}'(t') + \tilde{V}(t')\tilde{V}(s)\} + \dots$$

…

…

ÜA

$$= 1 - i\int_0^t dt'\,\{\tilde{V}'(t') - \tilde{V}(t')\} +$$

$$+ \int_0^t dt'\int_0^{t'} ds\,\{[\tilde{V}'(t') - \tilde{V}(t')]\tilde{V}(s)\} +$$

$$- \int_0^t dt'\int_0^{t'} ds\,\{\tilde{V}'(s)[\tilde{V}'(t') - \tilde{V}(t')]\} + \dots$$

Jetzt $\hat{V}(t) \equiv \sum_{\alpha\beta} g_{\alpha\beta}(t)\hat{X}_{\alpha\beta}$ ; $\hat{V}'(t) = \sum_{\alpha\beta} g'_{\alpha\beta}(t)\hat{X}_{\alpha\beta}$

$t$

$$\mathcal{F}[q_{t'}, q'_{t'}] = 1 - i \sum_{\alpha\beta} \int_0 dt' \{ g'_{\alpha\beta}(t') - g_{\alpha\beta}(t') \} \cdot \langle \tilde{X}_{\alpha\beta}(t') \rangle_0$$

lineare Response

$$+ \sum_{\alpha\beta\gamma\delta} \int_0^t dt' \int_0^{t'} ds \{ g'_{\alpha\beta}(t') - g_{\alpha\beta}(t') \} \times$$

$$\times \left[ g_{\gamma\delta}(s) \langle \tilde{X}_{\alpha\beta}(t') \tilde{X}_{\gamma\delta}(s) \rangle_0 - g'_{\gamma\delta}(s) \langle \tilde{X}_{\gamma\delta}(s) \tilde{X}_{\alpha\beta}(t') \rangle_0 \right]$$

quadr. Response

$$+ \dots$$

z.B. $X_{\alpha\beta} = c^+_k c_{k'}$

Einführen: Korrelations-Tensor 4. Stufe

$$L_{\alpha\beta\gamma\delta}(t', s) = \langle \tilde{X}_{\alpha\beta}(t') \tilde{X}_{\gamma\delta}(s) \rangle_0$$

$$\Rightarrow \mathcal{F}[q, q'] = 1 - i \sum_{\alpha\beta} \int_0^t dt' \{ g'_{\alpha\beta}(t') - g_{\alpha\beta}(t') \} \langle \tilde{X}_{\alpha\beta}(t') \rangle_0$$

$$+ \sum_{\alpha\beta\gamma\delta} \int_0^t dt' \int_0^{t'} ds \{ g'_{\alpha\beta}(t') - g_{\alpha\beta}(t') \} \left[ g_{\gamma\delta}(s) L_{\alpha\beta\gamma\delta}(t', s) - g'_{\gamma\delta}(s) L_{\gamma\delta\alpha\beta}(s, t') \right] + \dots$$

Nächster Schritt: Re-Exponentiation

$$1 + x + \frac{1}{2}x^2 + \ldots \approx e^x$$

Eine Vereinfachung: $\langle \tilde{X}_{\alpha\beta}(t') \rangle_0 \equiv 0$

$$\Rightarrow \quad V[q_{t'}] = \sum_{\alpha\beta} g_{\alpha\beta} X_{\alpha\beta}$$

$$\mathcal{F}^{\text{pert.}}[q(t'), q'(t')] = e^{-\Phi_{t\,t'}^{\text{pert.}}[q(t'), q'(t')]}$$

$$\Phi^{\text{pert}}[q(t'), q'(t')] = \sum_{\alpha\beta\gamma\delta} \int_0^t dt' \int_0^{t'} ds \left\{ g_{\alpha\beta}[q_{t'}] - g_{\alpha\beta}[q'_{t'}] \right\}$$

$$\times \left[ g_{\gamma\delta}[q_s] L_{\alpha\beta\gamma\delta}(t', s) - g_{\gamma\delta}[q'_s] L_{\gamma\delta\alpha\beta}(s, t') \right]$$

Das gilt jetzt für <u>beliebige</u> Bäder.

- Falls $\langle \tilde{X}_{\alpha\beta} \rangle \neq 0$, dann

  $L$ definieren mit $\left\langle \left( \tilde{X}_{\alpha\beta}(t) - \langle X_{\alpha\beta} \rangle \right) \times \left( \tilde{X}_{\gamma\delta}(s) - \langle X_{\gamma\delta} \rangle \right) \right\rangle$.

- Reproduzieren lineare Ankopplung $\hat{V} = f[q] \cdot X$ mit Oszillator-Bad:

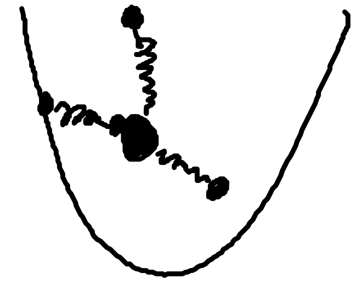
$$\Phi[q_{t'}, q'_{t'}] = \int_0^t dt' \int_0^{t'} ds \left\{ f[q_{t'}] - f[q'_{t'}] \right\} \times$$

$$\times \left\{ L(t', s) f[q_s] - L(s, t') f[q'_s] \right\}$$

$$L(t',s) = \langle x(t')x(s)\rangle_0 = \langle x(t'-s)x\rangle_0$$

Bemerkung: Zur Spektraldichte

$$J(\omega) = \frac{\pi}{2}\sum_{\alpha=1}^{N} \frac{c_\alpha^2}{M_\alpha \Omega_\alpha}\,\delta(\omega-\Omega_\alpha)$$



N: Anzahl der Bad-Oszillatoren

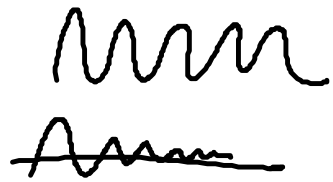
z.B. Phononen, $\Omega_\alpha = c_s|k|$, $\alpha = \vec{k}$

Wir brauchen thermodynamischen Limes $N \Longrightarrow \infty$, um sinnvoll Dissipation beschreiben zu können.

## Semiklassik für gedämpfte Bewegung

$$\ddot{x} + \boxed{\gamma \dot{x}} + \omega^2 x = 0$$

Wie verstehen?



Caldeira-Leggett (1982)

$$H_S = \frac{p^2}{2M} + V(q)$$

mikroskopisch: $\rho(x,y;t) = \langle \underbrace{x+\frac{y}{2}}_{q} | \rho(t) | \underbrace{x-\frac{y}{2}}_{q'} \rangle$

$x = \frac{1}{2}(q+q')$ Schwerpunktskoord.

$y = q-q'$ Relativkoordinate

$\infty$

$$f(x,p,t) \equiv \int_{-\infty} \frac{dy}{2\pi} \rho(x,y,t) e^{-ipy} \qquad (\hbar = 1)$$

Wigner-Verteilungsfunktion