

13.6.2007

Impulsfunktionale. Lineare Ankopplung

$$H = H_S[q] + H_B[x] + H_{SB}[x, q] = H_S[q] + \sum_{\alpha=1}^N \left\{ \frac{p_{\alpha}^2}{2m_{\alpha}} + \frac{1}{2} m_{\alpha} \Omega_{\alpha}^2 x_{\alpha}^2 + f_{\alpha}[q] x_{\alpha} \right\}$$

$$F[q_{t'}, q_t] = e^{-\frac{i}{\hbar} \Phi[q_{t'}, q_t]}$$

$$\Phi[q_{t'}, q_t] = \sum_{\alpha=1}^N \int_0^{t'} dt' \int_0^t ds \left\{ f_{\alpha}[q_{t'}] - f_{\alpha}[q_t] \right\}$$

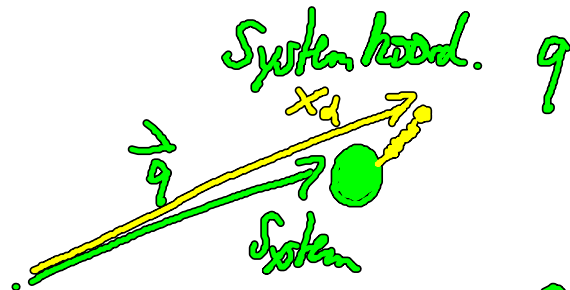
$$\cdot \{ S_2(t'-s) \rho_2[q_s] - S_2^*(t'-s) \rho_2[q'_s] \}$$

$$S_2(\tau) = \frac{1}{2h_2 \Omega_2} \left( \coth \frac{\beta \Omega_2}{2} \cos \Omega_2 \tau - i \sin \Omega_2 \tau \right)$$

Lineare Ankopplung an  $q$

$$H_B[x] + H_{SB}[xq] = \sum_{\alpha=1}^N \left[ \frac{p_\alpha^2}{2h_\alpha} + \frac{1}{2} h_\alpha \Omega_\alpha^2 \left( x_\alpha - \frac{c_\alpha}{h_\alpha \Omega_\alpha^2} q \right)^2 \right]$$

.....



$$= \sum_{\alpha=1}^N \left\{ \frac{p_\alpha^2}{2h_\alpha} + \frac{1}{2} h_\alpha \Omega_\alpha^2 x_\alpha^2 - c_\alpha q x_\alpha + \frac{1}{2} \frac{c_\alpha^2}{h_\alpha \Omega_\alpha^2} \cdot q^2 \right\}$$

$$\phi[q_t, q'_t] = \int_0^t dt' \int_0^{t'} ds \{ q_{t'} - q'_s \} \{ L(t'-s) q_s - L^*(t'-s) q'_s \} \\ + i \frac{k}{2} \int_0^t dt' \{ q_{t'}^2 - (q'_{t'})^2 \}$$

$$L(\tau) = \frac{1}{\pi} \int_0^\infty d\omega \mathcal{F}(\omega) \left\{ \coth \frac{\beta \omega}{2} \cos \omega \tau - i \sin \omega \tau \right\}$$

Spektraldichte des  $N$  Bosons  $z$

$$F(\omega) = \frac{\pi}{2} \sum_{d=1}^N \frac{c_d^2}{M_d \Omega_d} \delta(\omega - \Omega_d)$$

Folgt Modelle für  $F(\omega)$ , z.B.

- $F(\omega) = 2\alpha \omega e^{-\omega/\omega_c} \theta(\omega)$

Ohmsche Spektralfkt.  
mit exponentiellem cutoff  
und Kopplungskonstante  $\alpha$

$$\mu = \frac{1}{2} \sum_{d=1}^N \frac{c_d^2}{M_d \Omega_d^2} = \frac{2}{\pi} \int_0^{\infty} d\omega \frac{F(\omega)}{\omega}$$

Zusätzliches Systempotential

$$V_{\text{couple}}[q] = \frac{1}{2} \mu q^2$$

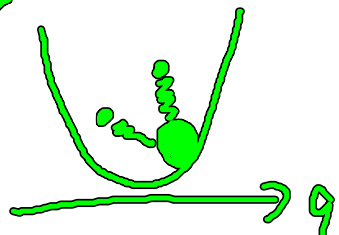
gilt

$$\exp\left(i \frac{\mu}{2} \int_0^t dt' \{q_{t'}^2 - q_{t'}^2\}\right)$$

Propagator für den gedämpften harm. Oszillator

- H. Grabert, P. Schramm, G.-L. Ingold  
Phys. Rep. 168, 115 (1988).

exakte Lösung für System = harm. Oszillator  
weil alles quadratisch ist!



- Vergleich mit Mastergleichung: R. Karleini, H. Fehst  
 Phys. Rev. E 55, 153 (1997).

## Influenzfunktionale für allgemeine Bäder

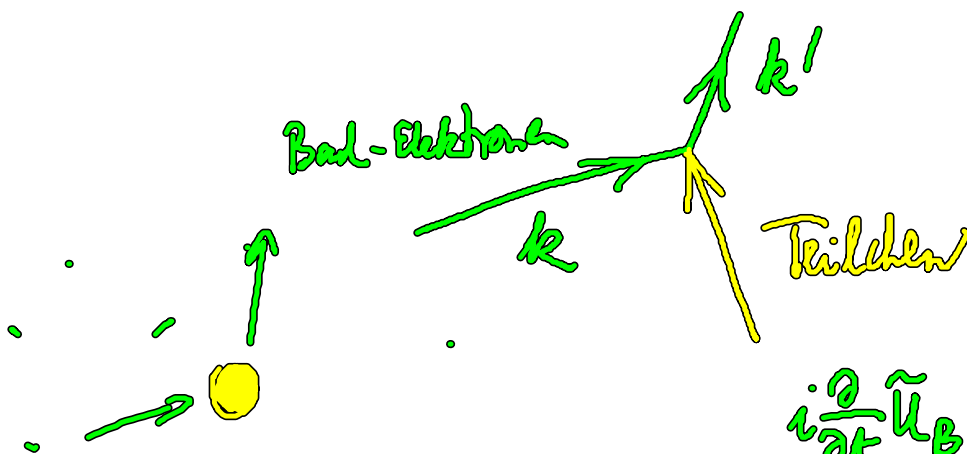
$$\mathcal{F}[q(t), q'(t')] = \text{Tr}_B \left( \rho_B U_B^\dagger[q'] U_B[q] \right)$$

$$\left. \begin{aligned} H_B(t) &= H_0 + V(t) \\ H'_B(t) &= H_0 + V'(t) \end{aligned} \right\} \text{beziehen sich auf} \\ \text{verschiedenen Platte} \\ q \text{ und } q'$$

Beispiele:  $H_0 = \sum_k \epsilon_k c_k^\dagger c_k$  freie Fermionen

$$V(t) = V[q_t] = \sum_{kk'} M_{kk'} c_{k'}^\dagger c_k e^{i(k-k')q_t}$$

Störoperator  
 in 2. Quantisierung.  
 Erzeugendoperator



$$(SG) \\ i \frac{\partial}{\partial t} \tilde{U}_B = \tilde{V}(t) \tilde{U}_B$$

Zeit WW-Bild im WW Bild

$$u_B[q] = e^{-i\hat{H}_0 t} \left\{ 1 + i \int_0^t dt' \tilde{v}(t') - \int_0^t dt' \int_0^{t'} ds \tilde{v}(t') \tilde{v}(s) + \dots \right\}$$

$$u_B^\dagger[q'] = \left\{ 1 - i \int_0^t dt' \tilde{v}'(t') - \int_0^t dt' \int_0^{t'} ds \tilde{v}'(s) \tilde{v}'(t') + \dots \right\}$$

$$u_B^\dagger[q'] u_B[q] = 1 - i \int_0^t dt' \{ \tilde{v}'(t') - \tilde{v}(t') \} + \int_0^t dt' \tilde{v}'(t') \int_0^{t'} ds \tilde{v}(s) - \int_0^t dt' \int_0^{t'} ds \{ \tilde{v}'(s) \tilde{v}'(t') + \tilde{v}(t') \tilde{v}(s) \} + \dots$$

$$\dots \hat{U}_A = 1 - i \int_0^t dt' \{ \tilde{v}'(t') - \tilde{v}(t') \} + \int_0^t dt' \int_0^{t'} ds \{ [\tilde{v}'(t') - \tilde{v}(t')] \tilde{v}(s) \} + \int_0^t dt' \int_0^{t'} ds \{ \tilde{v}'(s) [\tilde{v}'(t') - \tilde{v}(t')] \} + \dots$$

$$\text{Zeit } \hat{v}(t) = \sum_{\alpha\beta} g_{\alpha\beta}(t) \hat{X}_{\alpha\beta}; \quad \hat{v}'(t) = \sum_{\alpha\beta} g'_{\alpha\beta}(t) \hat{X}_{\alpha\beta}$$

$$\mathcal{F}[q, q'] = 1 - i \sum_{\alpha\beta} \int_0^t dt' \{g'_{\alpha\beta}(t') - g_{\alpha\beta}(t')\} \cdot \langle \tilde{X}_{\alpha\beta}(t') \rangle_0$$

linear  
Response

$$+ \sum_{\alpha\beta\gamma\delta} \int_0^t dt' \int_0^{t'} dt'' \{g'_{\alpha\beta}(t') - g_{\alpha\beta}(t')\} \times$$

$$\times [g_{\gamma\delta}(s) \langle \tilde{X}_{\alpha\beta}(t') \tilde{X}_{\gamma\delta}(s) \rangle_0 - g'_{\gamma\delta}(s) \langle \tilde{X}_{\gamma\delta}(s) \tilde{X}_{\alpha\beta}(t') \rangle_0]$$

quadr.  
Response

+ ...

z.B.  $X_{\alpha\beta} = C_k^T C_k$

Einführen: Korrelations-Tensor 4. Stufe

$$L_{\alpha\beta\gamma\delta}(t', s)_t = \langle \tilde{X}_{\alpha\beta}(t') \tilde{X}_{\gamma\delta}(s) \rangle_0$$

$$\Rightarrow \mathcal{F}[q, q'] = 1 - i \sum_{\alpha\beta} \int_0^t dt' \{g'_{\alpha\beta}(t') - g_{\alpha\beta}(t')\} \langle \tilde{X}_{\alpha\beta}(t') \rangle_0$$

$$+ \sum_{\alpha\beta\gamma\delta} \int_0^t dt' \int_0^{t'} dt'' \{g'_{\alpha\beta}(t') - g_{\alpha\beta}(t')\} [g_{\gamma\delta}(s) L_{\alpha\beta\gamma\delta}(t', s) - g'_{\gamma\delta}(s) L_{\gamma\delta\alpha\beta}(s, t')] + \dots$$

Nächster Schritt: Re-Exponentiation

$$1 + x + \frac{1}{2} x^2 + \dots \approx e^x$$

Eine Vereinfachung:  $\langle \tilde{X}_{\alpha\beta}(t) \rangle_0 = 0$

$$\Rightarrow V[q_{\alpha}] = \sum_{\alpha\beta} g_{\alpha\beta} X_{\alpha\beta}$$

$$\mathcal{F}^{\text{pert.}} [q(t'), q'(t')] = e^{-\frac{1}{\hbar} \mathcal{I}^{\text{pert.}} [q(t'), q'(t')]}$$

$$\mathcal{I}^{\text{pert.}} [q(t'), q'(t')] = \sum_{\alpha\beta\gamma\delta} \int_0^t dt' \int_0^{t'} ds \{ g_{\alpha\beta} [q_{\alpha}] - g_{\alpha\beta} [q'_{\alpha}] \}$$

$$\times [ g_{\gamma\delta} [q_{\delta}] L_{\alpha\beta\gamma\delta}(t', s) - g_{\gamma\delta} [q'_{\delta}] L_{\gamma\delta\alpha\beta}(s, t') ]$$

Das gilt jetzt für beliebige Bänder.

- Falls  $\langle \tilde{X}_{\alpha\beta} \rangle \neq 0$ , dann

$$L \text{ definieren mit } \langle (\tilde{X}_{\alpha\beta}(t) - \langle X_{\alpha\beta} \rangle) \times (\tilde{X}_{\gamma\delta}(s) - \langle X_{\gamma\delta} \rangle) \rangle.$$

- Reproduzieren lineare Ankodung  $\hat{V} = f[q] \cdot X$   
mit Oszillator-Beh:

$$\Phi [q_{\alpha}, q'_{\alpha}] = \int_0^t dt' \int_0^{t'} ds \{ f [q_{\alpha}] - f [q'_{\alpha}] \} \times$$

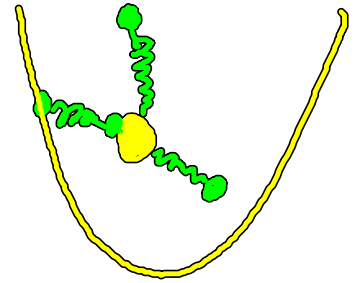
$$\times \{ L(t|s) f [q_{\delta}] - L(s|t') f [q'_{\delta}] \}$$

$$L(t's) = \langle x(t)/x(s) \rangle_0 = \langle x(t-s)/x \rangle_0$$

Bemerkung: zur Spektraldichte

$$J(\omega) = \frac{\pi}{2} \sum_{\alpha=1}^N \frac{c_{\alpha}^2}{M_{\alpha} \Omega_{\alpha}} \delta(\omega - \Omega_{\alpha})$$

$N$ : Anzahl der Bad-Oszillatoren



z.B. Phononen,  $\Omega_{\alpha} = \frac{c_s |k|}{d = k}$

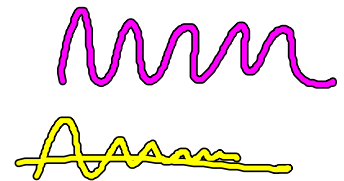
Wir brauchen thermodynamischer Limes  $N \rightarrow \infty$ ,  
um sinnvoll Dissipation beschreiben zu  
können.

## Stochastik für gedämpfte Bewegung

$$\ddot{x} + \gamma \dot{x} + \omega^2 x = 0$$

Wie verhalten?

Caldiera-Leggett (1982)



$$H_S = \frac{p^2}{2M} + V(q)$$

mikroskopisch:

$$P(x, y, t) = \langle \underbrace{x + \frac{y}{2}}_{q} | \rho(t) | \underbrace{x - \frac{y}{2}}_{q'} \rangle$$

$x = \frac{1}{2}(q + q')$  Schwerpunktbed.  $q$

$y = q - q'$  Relativkoordinaten

$\infty$



$$f(x, p, t) \equiv \int_{-\infty}^{\infty} \frac{dy}{2\pi} \rho(x|y, t) e^{-ipy} \quad (\hbar=1)$$

Wigner-Verteilungsfunktion