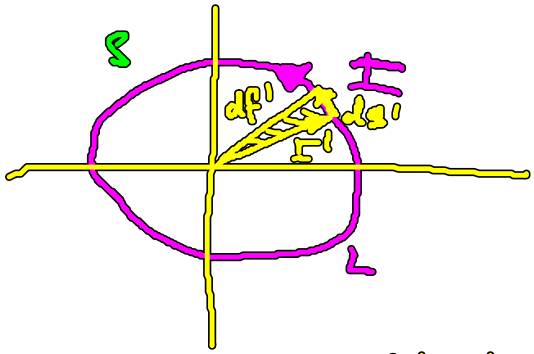
Beispiel für einen magnetischen Dipol:



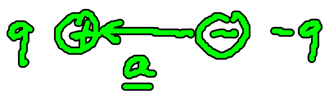
$$d\underline{f}' = \frac{1}{2}\,\underline{r}' \times d\underline{s}'$$

$$d^3r'\,\underline{j}(\underline{r}') = d\underline{s}'\,I$$

$$\underline{m} = \frac{1}{2}\oint_L d^3r'\,\underline{r}' \times \underline{j}(\underline{r}') = \frac{I}{2}\oint_L \underline{r}' \times d\underline{s}' = I\int_F d\underline{f}' = I F \underline{n}$$

Ringstrom $\longrightarrow$ magn. Dipolmoment $\underline{m} = I F \underline{n}$

(analog: 2 Punktlad. $\longrightarrow$ el. Dipolmoment $\underline{p} = q\underline{a}$)



2. Beispiel: Bewegte Ladungen

N Teilchen mit Massen m_i u. Ladungen q_i, spezif. Ladung $\frac{q_i}{m_i} = \frac{q}{m} = \text{const.}$

$$\rho(\underline{r}) = \sum_i q_i\,\delta(\underline{r} - \underline{r}_i) \qquad \underline{r}_i = \underline{r}_i(t)$$

$$\underline{j}(\underline{r}) = \sum_i q_i\,\underline{v}_i\,\delta(\underline{r} - \underline{r}_i) \qquad \underline{v}_i = \frac{d\underline{r}_i}{dt}$$

magn. Dipolmoment:

$$\underline{m} = \frac{1}{2}\int d^3r'\,\underline{r}' \times \underline{j}(\underline{r}') = \frac{1}{2}\sum_i q_i \int d^3r'\,\underline{r}' \times \underline{v}_i\,\delta(\underline{r}' - \underline{r}_i)$$

$$= \frac{1}{2}\sum_i q_i \underline{r}_i \times \underline{v}_i = \frac{1}{2}\sum_i \underbrace{\frac{q_i}{m_i}}_{q/m} m_i \underline{r}_i \times \underline{v}_i = \frac{q}{2m}\underline{L}$$

$\boxed{\underline{m} = \frac{q}{2m}\underline{L}}$ gilt auch für starre Körper!

(Spin eines Elektrons: $\underline{S}$, $\underline{m} = g\frac{e}{2m}\underline{S}$ mit $g \approx 2$)

quantenmech. Effekt

$$\underline{A} = \frac{\mu_0}{4\pi r^3}\underline{m} \times \underline{r}$$

Kraft auf Stromverteilung $\underline{j}(\underline{r}') = \rho(\underline{r}')\underline{v}(\underline{r}')$ im Feld einer magn. Induktion $\underline{B}(\underline{r}')$ (extern):

$$\underline{F} = \int d^3r' \, \underline{j}(\underline{r}') \times \underline{B}(\underline{r}') \quad \text{Lorentz-Kraft}$$

Taylorentw.:

$$\underline{B}(\underline{r}') = \underline{B}(\underline{r}) + [(\underline{r}'-\underline{r})\cdot\underline{\nabla}_r]\underline{B}(\underline{r}) + \dots$$

$$\underline{F} = \underbrace{\left[\int d^3r' \, \underline{j}(\underline{r}')\right]}_{0 \text{ (stationär)}} \times \underline{B}(\underline{r}) + \int d^3r' \, \underline{j}(\underline{r}') \times [(\underline{r}'-\underline{r})\cdot\underline{\nabla}_r]\underline{B}(\underline{r}) + \dots$$

$$= \int d^3r' \, \underline{j}(\underline{r}') \times \underbrace{[(\underline{r}'\cdot\underline{\nabla}_r)\underline{B}(\underline{r})]}_{\underline{\nabla}_r(\underline{r}'\cdot\underline{B}(\underline{r})) - \underline{r}' \times \underbrace{(\underline{\nabla}_r \times \underline{B}(\underline{r}))}_{0 \text{ externes Feld!}}} - \underbrace{\int d^3r' \, \underline{j}(\underline{r}')}_{0 \text{ (stat.)}} \times [(\underline{r}\cdot\underline{\nabla}_r)\underline{B}(\underline{r})] + \dots$$

$$= \underbrace{\int d^3r' \, \underline{j}(\underline{r}') \times \underline{\nabla}_r(\underline{r}'\cdot\underline{B}(\underline{r}))}_{-\underline{\nabla}_r \times [(\underline{r}'\cdot\underline{B})\underline{j}(\underline{r}')]}$$

$$= -\underline{\nabla}_r \times (\underline{m} \times \underline{B}(\underline{r})) \qquad \underline{m} = \frac{1}{2}\int d^3r \, \underline{r} \times \underline{j}$$

$$\underset{\nabla}{\underline{a}} \times (\underset{m}{\underline{b}} \times \underset{B}{\underline{c}}) = \underline{b}(\underline{a}\underline{c}) - \underline{c}(\underline{a}\underline{b}) \qquad \underline{m}(\nabla B) \quad \underline{B}(\nabla m)$$

$$= (\underline{m}\cdot\underline{\nabla}_r)\underline{B}(\underline{r})$$

$\Rightarrow \quad V = -\underline{m}\,\underline{B}(\underline{r})$ pot. Energie eines magn. Dipols im Magnetfeld

3. Maxwell-Gleichungen

Ziel: Dynamik der Felder

Methode: Erweiterung der elektrostat. u. magnetostat. Feldgln., so dass allg. Invarianz-Prinzipien erfüllt sind.

3.1 TCP-Invarianz

Zeitumkehr T: $t \to t' = -t$

Ladungsumkehr C: $Q \to Q' = -Q$

Paritätumkehr P: $\underline{r} \to \underline{r}' = -\underline{r}$

(i) Zeitumkehr-Trafo

$$T_g := \{T\text{-invar. Obs. } A : TA = A\} \quad \text{gerade unter } T$$
$$= \{\underline{r}, d\underline{r}, \underline{a} = \frac{d^2\underline{r}}{dt^2}, m, q, \rho, \underline{F} = m\underline{a}, \underline{E} = \frac{\underline{F}}{q}, \phi, \dots\}$$

$$T_u := \{A : TA = -A\} \quad \text{ungerade unter } T$$
$$= \{\underline{v} = \frac{d\underline{r}}{dt}, \underline{j} = \rho\underline{v}, \underline{B}, \underline{A}, \dots\}$$

$$\underset{g}{\underline{F}} = \underset{g}{q}\,\underset{u}{\underline{v}} \times \underset{u}{\underline{B}} \qquad \underset{u}{\underline{B}} = \underset{g}{\underline{\nabla}} \times \underset{u}{\underline{A}}$$

$\Rightarrow$ T-Invarianz der stat. Grundgln.:

$$T: \{\underline{\nabla}_r \times \underline{E} = 0\} \longrightarrow \{\underline{\nabla} \times \underline{E} = 0\}$$
$$T: \{\epsilon_0 \underline{\nabla}_r \cdot \underline{E} = \rho\} \longrightarrow \{\epsilon_0 \underline{\nabla}_r \cdot \underline{E} = \rho\}$$
$$T: \{\underline{\nabla} \cdot \underline{B} = 0\} \longrightarrow \{-\underline{\nabla}_r \cdot \underline{B} = 0\}$$

$$T: \{\underline{\nabla}\times\underline{B} = \mu_0 \underline{j}\} \longrightarrow \{-\underline{\nabla}\times\underline{B} = -\mu_0 \underline{j}'\}$$

Kontinuitätsgl.:

$$T: \{\frac{\partial\varrho}{\partial t} + \underline{\nabla}\cdot\underline{j}' = 0\} \rightarrow \{-\frac{\partial\varrho}{\partial t} - \underline{\nabla}\cdot\underline{j}' = 0\}$$

Diese Gln. sind forminvariant

(ii) Ladungsumkehr (-konjugation)

$C_g = \{\underline{F}, m, \underline{r}, \underline{v}, \underline{a}, \ldots\}$ gerade

$C_u = \{\underline{E} = \frac{1}{q}\underline{F}, \underline{B}, \varrho, \underline{j}, \ldots\}$ ungerade

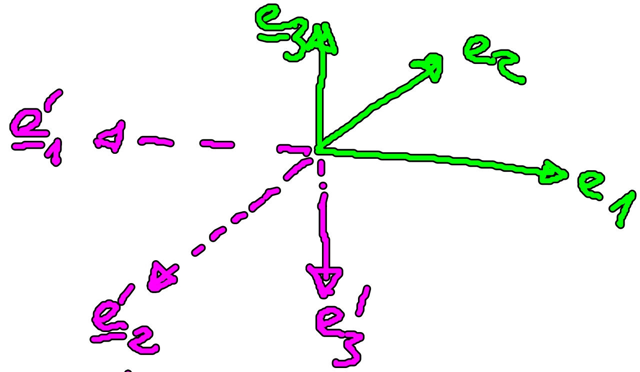
$$\underset{g}{\underline{F}} = \underset{ug}{q\underline{v}} \times \underset{u}{\underline{B}}$$

$\Rightarrow$ C-Invarianz der stat. Gln:

$$C: \{\underline{\nabla}\times\underline{E} = 0\} \longrightarrow \{-\underline{\nabla}\times\underline{E} = 0\}$$
$$C: \{\epsilon_0\underline{\nabla}\cdot\underline{E} = \varrho\} \longrightarrow \{-\epsilon_0\underline{\nabla}\cdot\underline{E} = -\varrho\}$$
$$C: \{\underline{\nabla}\cdot\underline{B} = 0\} \longrightarrow \{-\underline{\nabla}\cdot\underline{B} = 0\}$$
$$C: \{\underline{\nabla}\times\underline{B} = \mu_0\underline{j}\} \longrightarrow \{-\underline{\nabla}\times\underline{B} = -\mu_0\underline{j}'\}$$
$$C: \{\frac{\partial\varrho}{\partial t} + \underline{\nabla}\cdot\underline{j} = 0\} \longrightarrow \{-\frac{\partial\varrho}{\partial t} - \underline{\nabla}\cdot\underline{j} = 0\}$$

(iii) Paritätsumkehr (Räuml. Spiegelung)

Vertauschung „rechts $\leftrightarrow$ links"



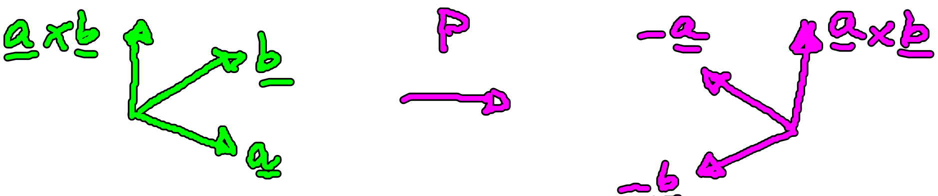
Rechtssystem

Linkssystem

$P\underline{r} = -\underline{r}$ polarer Vektor

aber

$$P(\underline{a}\times\underline{b}) = (-\underline{a})\times(-\underline{b}) = \underline{a}\times\underline{b}$$



P-invariant (axialer Vektor, Pseudovektor)

Seien $\underline{a}, \underline{b}$ polar $\Rightarrow$ $\underline{a}\times\underline{\omega}$ polar

$\underline{\omega}, \underline{\sigma}$ axial $\underline{a}\times\underline{b}$, $\underline{\omega}\times\underline{\sigma}$ axial

$\underline{a}\cdot\underline{b}$ skalar $(P(\underline{a}\underline{b}) = \underline{a}\underline{b})$

$\underline{a}\cdot\underline{\omega}$ pseudoskalar $(P(\underline{a}\underline{\omega}) = -\underline{a}\underline{\omega})$

$P_q = \{$ Skalare m, q; axiale Vektoren $\underline{B}$ $\}$ gerade

$$\underline{F} = q\underline{v}\times\underline{B}$$

$P_u = \{$ polare Vektoren $\underline{r}$, $d\underline{r}$, $\underline{v}$, $\underline{a}$, $\underline{F}$, $\underline{E} = \frac{1}{q}\underline{F}$, $\underline{j} = \rho\underline{v}$, $\underline{A}$,
Pseudoskalar $\underline{\nabla}\cdot\underline{B}$ $\}$

$\underset{g}{\underline{B}} = \underset{u}{\underline{\nabla}} \times \underset{u}{\underline{A}}$

P-Invarianz der El./Magn. statik

$P: \{\underline{\nabla}\times\underline{E} = 0\} \rightarrow \{\underline{\nabla}\times\underline{E} = 0\}$

$P: \{\epsilon_0 \underline{\nabla}\cdot\underline{E} = \rho\} \rightarrow \{\epsilon_0 \underline{\nabla}\cdot\underline{E} = \rho\}$

$P: \{\underline{\nabla}\cdot\underline{B} = 0\} \rightarrow \{-\underline{\nabla}\cdot\underline{B} = 0\}$

$P: \{\underline{\nabla}\times\underline{B} = \mu_0\underline{j}\} \rightarrow \{-\underline{\nabla}\times\underline{B} = -\mu_0\underline{j}'\}$

$P: \{\frac{\partial}{\partial t}\rho + \underline{\nabla}\cdot\underline{j}\} \rightarrow \{\frac{\partial}{\partial t}\rho + \underline{\nabla}\cdot\underline{j}' = 0\}$

NB: gäbe es magn. Ladungen, dann wären sie Pseudoskalare!

NB: Schwache WW verletzt Paritätserh.

3.2 Maxwell-Gln. im Vakuum

Forderungen an dynam. Gln. für zeitabh. Felder $\underline{E}(\underline{r},t)$ u. $\underline{B}(\underline{r},t)$:

(1) Stat. Grenzfall

$$\underline{\nabla}\times\underline{E} = 0$$
$$\epsilon_0\underline{\nabla}\cdot\underline{E} = \rho$$
$$\underline{\nabla}\cdot\underline{B} = 0$$

$$\nabla \times \underline{B} = \mu \underline{j}$$

(2) linear in $\underline{E}$, $\underline{B}$ (Superpositionsprinzip)

<u>1. Ordnung</u> in t (Kausalitätsprinzip: zur Zeit $t=0$ soll Zustand für $t>0$ vollst. festlegen)

(3) <u>TCP-Invarianz</u>

(4) <u>Ladungserhaltung</u>

(5) <u>Lorentzkraft</u>