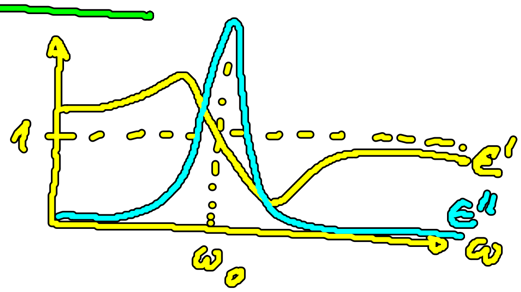
Beziehungen zwischen $\varepsilon'(\omega)$ und $\varepsilon''(\omega)$:

(Kramers-Kronig-Relationen)



- allg. gültiger Zus.hang zwischen Dispersion ($n(\omega)$) und Absorption ($\kappa(\omega)$); $\tilde{n}^2 = \varepsilon$;
  erlaubt z.B. Berechnung der Dispersionsbeziehung aus dem Absorptionsspektrum und umgekehrt!
- folgt aus dem Kausalitätsprinzip!

Beweis (Methode: Funktionentheorie)

Für eine kausale Funktion $\chi(t)$ gilt:

$$\chi(t) = \Theta(t)\,\chi(t) \quad \text{mit } \Theta(t) = \begin{cases} 0 & \text{für } t<0 \\ 1 & \text{für } t \geq 0 \end{cases}$$

Heaviside-Fkt.

Fourier-Trafo:

$$\hat{\chi}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\omega'\, \hat{\Theta}(\omega-\omega')\,\hat{\chi}(\omega')$$

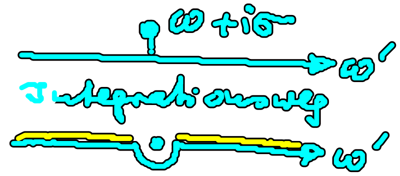
$$\hat{\Theta}(\omega) := \lim_{\sigma \to 0^+} \int_0^{\infty} dt\, e^{i\omega t - \sigma t}$$

(konvergenzerzeugender Faktor $e^{-\sigma t}$)

$$= -\lim_{\sigma \to 0^+} \frac{1}{\sqrt{2\pi}} \frac{1}{i\omega - \sigma}$$

$$\Rightarrow \hat{\chi}(\omega) = \lim_{\sigma \to 0^+} \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega' \frac{1}{\omega' - \omega - i\sigma}\,\hat{\chi}(\omega')$$

Integrand hat Pol bei $\omega' = \omega + i\sigma$



äquiv. Int. weg

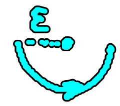
Zerlegung:

$$\int_{-\infty}^{\infty} d\omega' \, \frac{\hat{\chi}(\omega')}{\omega' - \omega} = \underbrace{\lim_{\varepsilon \to 0^+} \left[\int_{-\infty}^{\omega-\varepsilon} + \int_{\omega+\varepsilon}^{\infty} \right] d\omega' \, \frac{\hat{\chi}(\omega')}{\omega' - \omega}}_{\mathcal{P}\int_{-\infty}^{\infty} d\omega' \frac{\chi'}{\omega'-\omega} \text{ „Hauptwert" (principal value)}} + \underbrace{\int_{\curvearrowright} d\omega' \, \frac{\chi'(\omega')}{\omega' - \omega}}_{\text{halbes Residuum}}$$

$$\int_{\curvearrowright} d\zeta \, \frac{f(\zeta)}{\zeta} = f(0) \int \frac{d\zeta}{\zeta}$$

$$\zeta = \varepsilon e^{i\varphi} \qquad = f(0)\, i \int_0^{\pi} d\varphi = i\pi f(0)$$

$$d\zeta = i\zeta d\varphi$$



$$\Longrightarrow \hat{\chi}(\omega) = \frac{1}{2\pi i} \mathcal{P}\int_{-\infty}^{\infty} d\omega' \, \frac{\hat{\chi}(\omega')}{\omega' - \omega} + \frac{1}{2} \hat{\chi}(\omega)$$

$$\Longrightarrow \boxed{\hat{\chi}(\omega) = \frac{1}{\pi i} \mathcal{P}\int_{-\infty}^{\infty} d\omega' \, \frac{\hat{\chi}(\omega')}{\omega' - \omega}}$$

Zerlegung in Re und Im mit $\mathrm{Re}\,\hat{\chi}(\omega) = \varepsilon'(\omega) - 1$

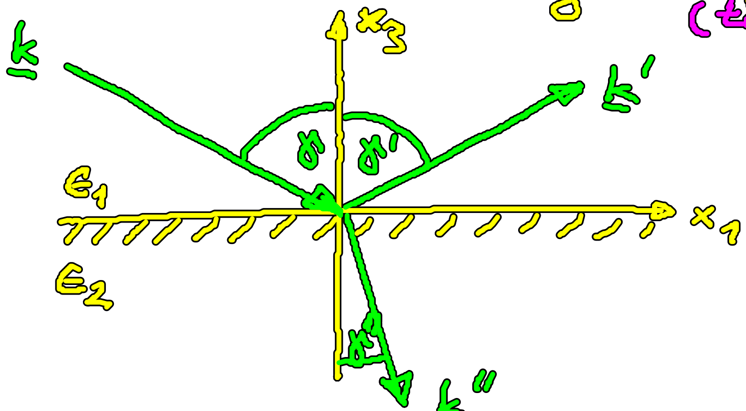
$\mathrm{Im}\,\hat{\chi}(\omega) = \varepsilon''(\omega)$

$$\boxed{\begin{aligned} \varepsilon'(\omega) - 1 &= \frac{1}{\pi} \mathcal{P}\int_{-\infty}^{\infty} d\omega' \, \frac{\varepsilon''(\omega')}{\omega' - \omega} \\ \varepsilon''(\omega) &= -\frac{1}{\pi} \mathcal{P}\int_{-\infty}^{\infty} d\omega' \, \frac{\varepsilon'(\omega') - 1}{\omega' - \omega} \end{aligned}}$$

Kramers-Kronig-Relationen

5.7 Brechung und Reflexion

Wellenausbreitung in geschichtete Medien: (transparent $\Rightarrow \varepsilon_i \in \mathbb{R}$)



Einfallende Welle $\underline{E} = \underline{E}_0\, e^{i(\underline{k}\cdot\underline{r}-\omega t)}$

Reflektierte Welle $\underline{E}' = \underline{E}_0'\, e^{i(\underline{k}'\cdot\underline{r}-\omega' t)}$

Transmittierte Welle $\underline{E}'' = \underline{E}_0''\, e^{i(\underline{k}''\cdot\underline{r}-\omega'' t)}$

Grenzbed. für $\underline{E}$ (§5.4)

$$E_1 + E_1'\Big|_{x_3=0} = E_1''\Big|_{x_3=0}$$ (Tang. komp. stetig)

$$\underline{r}=0:\ E_{01}e^{-i\omega t} + E_{01}'e^{-i\omega' t} = E_{01}''e^{-i\omega'' t}$$

$$\Rightarrow \boxed{\begin{matrix}\omega = \omega' = \omega'' \\ E_{01}+E_{01}' = E_{01}''\end{matrix}}$$

$$t=0:\ E_{01}e^{ik_1x_1} + E_{01}'e^{ik_1'x_1} = E_{01}''e^{ik_1''x_1}$$

$$\Rightarrow \boxed{k_1 = k_1' = k_1''}$$

$$\Rightarrow \underbrace{|\underline{k}|}_{\omega/c_1}\sin\gamma = \underbrace{|\underline{k}'|}_{\omega/c_1}\sin\gamma' = \underbrace{|\underline{k}_1''|}_{\omega/c_2}\sin\gamma''$$

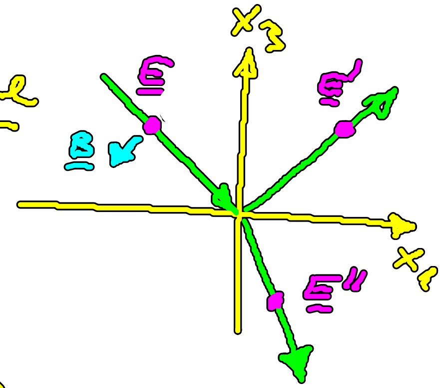
$$\Rightarrow \boxed{\begin{matrix}\sin\gamma = \sin\gamma' \\ \dfrac{\sin\gamma''}{\sin\gamma} = \dfrac{c_2}{c_1} = \dfrac{n_1}{n_2}\end{matrix}}$$

Reflexionsgesetz

Brechungsgesetz (Snellius)

Bestimmung der Amplituden:

(a) Polarisation von $\underline{E} \perp$ Einfallsebene



$$E_{01} = E_{01}' = E_{01}'' = 0$$

$$E_{03} = E_{03}' = E_{03}'' = 0$$

$$(1) \quad \boxed{E_{02} + E_{02}' = E_{02}''} \quad \text{(Tang. komp.)}$$

Mit $\underline{B}_0 = \frac{c}{\omega}(\underline{k} \times \underline{E}_0) = \frac{c}{\omega} E_{02} \begin{pmatrix} -k_3 \\ 0 \\ k_1 \end{pmatrix}$ folgt Tang. komp. von $\underline{B}$: $B_{01} + B_{01}' = B_{01}''$

$$\Longrightarrow k_3 E_{02} + k_3' E_{02}' = k_3'' E_{02}''$$

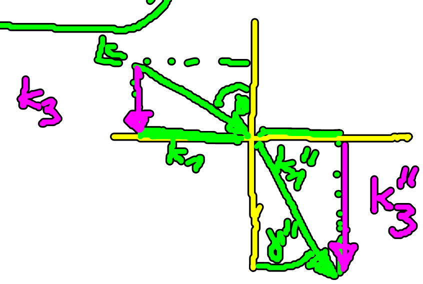
Reflex. gesetz $\Longrightarrow k_3 = -k_3' \Longrightarrow \boxed{k_3(E_{02} - E_{02}') = k_3'' E_{02}''}$ (2)

$$(1) \text{ in } (2) \Longrightarrow k_3(E_{02} - E_{02}') = k_3''(E_{02} + E_{02}')$$

$$\Longrightarrow \boxed{\frac{E_{02}'}{E_{02}} = \frac{k_3 - k_3''}{k_3 + k_3''}, \quad \frac{E_{02}''}{E_{02}} = \frac{2k_3}{k_3 + k_3''}}$$

$$k_3'' = |\underline{k}''| \cos\gamma'' = |k| \underbrace{\left(\frac{n_2}{n_1}\right)}_{\frac{\sin\gamma}{\sin\gamma''}} \cos\gamma''$$

$$k_3 = |\underline{k}| \cos\gamma$$



$$\Longrightarrow \boxed{\begin{aligned} \frac{E_{02}'}{E_{02}} &= \frac{\cos\gamma \sin\gamma'' - \sin\gamma \cos\gamma''}{\cos\gamma \sin\gamma'' + \sin\gamma \cos\gamma''} = \frac{\sin(\gamma'' - \gamma)}{\sin(\gamma'' + \gamma)} \\ \frac{E_{02}''}{E_{02}} &= \frac{2 \sin\gamma'' \cos\gamma}{\sin(\gamma'' + \gamma)} \end{aligned}}$$

Intensitätsverhältnisse:

Zeitmittel des Poyntingvektors $\langle S\rangle = \frac{1}{T}\int_0^T dt\,(\underline{E}\times\underline{H}) \sim |\underline{E}_0|^2$

<u>Reflexionsvermögen</u>:

$$R_\perp = \left|\frac{E'_{02}}{E_{02}}\right|^2 = \frac{\sin^2(\gamma''-\gamma)}{\sin^2(\gamma''+\gamma)}$$

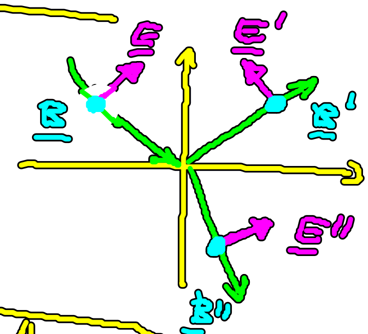
<u>Transmissionsvermögen</u>

$$T_\perp = \left|\frac{E''_{02}}{E_{02}}\right|^2 = \frac{4\sin^2\gamma\cos^2\gamma}{\sin^2(\gamma''+\gamma)} = 1-R_\perp$$

$\perp$ polarisiert

(b) <u>Polarisation von $\underline{E}$ $\parallel$ Einfallsebene</u>

$\underline{B}$ $\perp$ Einfallsebene



$\Rightarrow$ analoge Argumentation für B_{02}, B'_{02}, B''_{02} wie in (a)

$$\Rightarrow \boxed{\frac{E'_\parallel}{E_\parallel} = \frac{\tan(\gamma-\gamma'')}{\tan(\gamma+\gamma'')}, \quad \frac{E''_\parallel}{E_\parallel} = \frac{2\sin\gamma''\cos\gamma}{\sin(\gamma+\gamma'')\cos(\gamma''-\gamma)} \qquad R_\parallel = 1 - T_\parallel = \frac{\tan^2(\gamma''-\gamma)}{\tan^2(\gamma''+\gamma)}}$$

<u>Bem.</u>: (i) Bei Reflexion u. Brechung wird i.a. Polaris.richt. gedreht.

Speziell für $\gamma''+\gamma = \frac{\pi}{2}$: $\tan(\gamma''+\gamma)\to\infty \Rightarrow R_\parallel = 0$

→ reflektierte Welle vollständig polarisiert ⊥ Einfallsebene (γ = Brewster-Winkel γ_B

$$\tan \gamma_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \frac{\sin\gamma}{\cos\gamma} = \frac{\sin\gamma}{\sin\left(\frac{\pi}{2}-\gamma\right)} = \frac{\sin\gamma}{\sin\gamma''} = \frac{n_2}{n_1}$$

(ii) Totalreflexion

Sei $\epsilon_2 < \epsilon_1$ → für $\gamma = \gamma_G$ Grenzwinkel der Totalreflexion

$$\sin\gamma_G = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

für $\gamma > \gamma_G$: $k_3'' = i/d$

evaneszente Welle $\underline{E}'' = \underline{E}_0'' \, e^{-|x_3|/d} \, e^{i(k_1'' x_1 - \omega t)}$