3.7.2008

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$\boxed{\text{Gradient, Rotation, Divergenz}}$

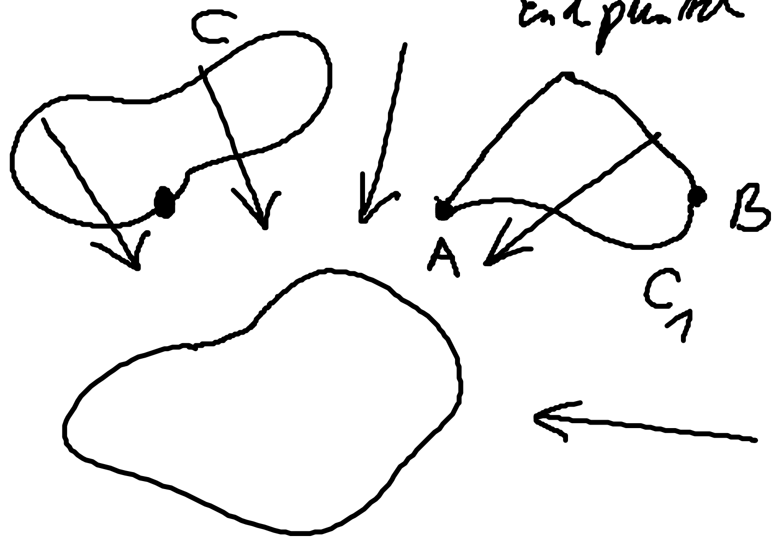
$$\underline{F} = -\underline{\nabla}\phi$$

$\underline{\nabla}$: grad

ϕ : Potential

Konservative Kräfte :

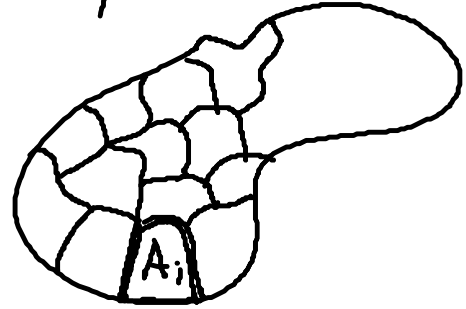
$\int \underline{F}\, ds$ nur vom Anfangs- und Endpunkt abhängig



$$\oint_C \underline{F}\, d\underline{s} \equiv W[C] \quad \text{Wirbelstärke}$$

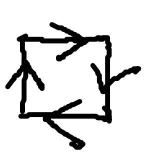
Für konservative Kräfte ist $W[C] = 0$.

Rotation $\operatorname{rot} \underline{F}$ als lokales Maß für die Wirbelstärke



$$(\operatorname{rot} \underline{F})\, \underline{n} \equiv \lim_{A_i \to 0} \frac{\oint_C \underline{F}\, d\underline{s}}{A_i}$$

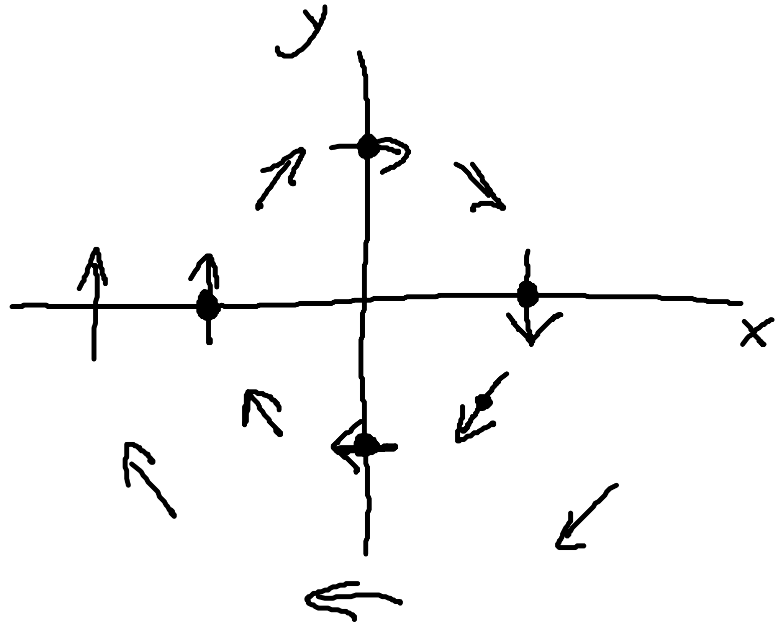
Ausrechnen für kart. KO:



$$\Rightarrow \quad \operatorname{rot} \underline{F} \equiv \underline{\nabla} \times \underline{F} = \begin{vmatrix} \underline{e}_x & \underline{e}_y & \underline{e}_z \\ \partial_x & \partial_y & \partial_z \\ F_x & F_y & F_z \end{vmatrix},$$

wobei $\underline{F} = (F_x, F_y, F_z)$

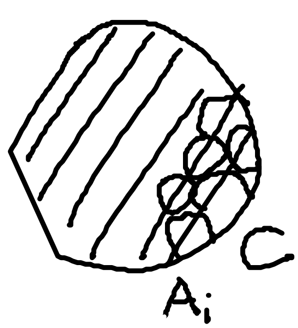
Beispiel: $\underline{f}(x,y,z) = (y, -x, 0)$



$$\text{rot } f = \begin{vmatrix} \cdot & \cdot & \cdot \\ \partial_x & \partial_y & \partial_z \\ y & -x, & 0 \end{vmatrix} = (0, 0, -2)$$

Integralsatz von Stokes

$$W[C] \equiv \oint_C \underline{F}(\underline{x})\,d\underline{x} = \sum_i A_i \underbrace{\frac{W_i}{A_i}}_{\downarrow\ \text{rot } F}$$



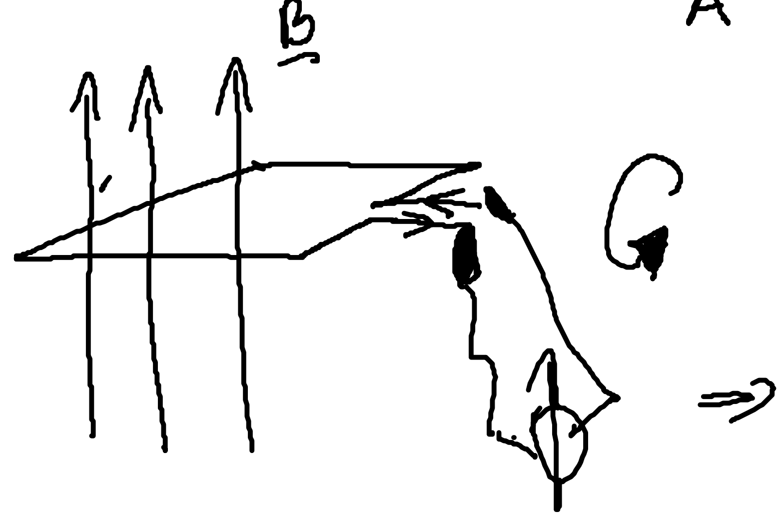
$$\rightarrow \int_A \text{rot } \underline{F}\, dA$$

$$\oint_C F\,dx = \int_A \text{rot } F\, dA$$

Induktionsgesetz:

$$-\frac{\partial}{\partial t}\int_A \underline{B}\, d\underline{A} = \underbrace{\oint_C \underline{E}\, d\underline{s}}$$

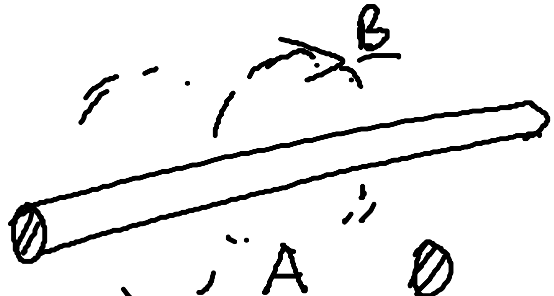
$$= \int_A \text{rot}\, \underline{E}\, d\underline{A}$$



$$-\frac{\partial}{\partial t}\underline{B} = \text{rot}\, \underline{E}$$

1. Maxwell'sche Gleichung (Induktionsgesetz)

Magnetostatik:



$\underline{J}(\underline{x}) \rightarrow$

Stromdichte

$$\underbrace{\oint_C \underline{B}\, d\underline{s}} = \mu_0 \int_A \underline{J}\, d\underline{A}$$

$$\int_A \operatorname{rot} \underline{B} \, dA$$

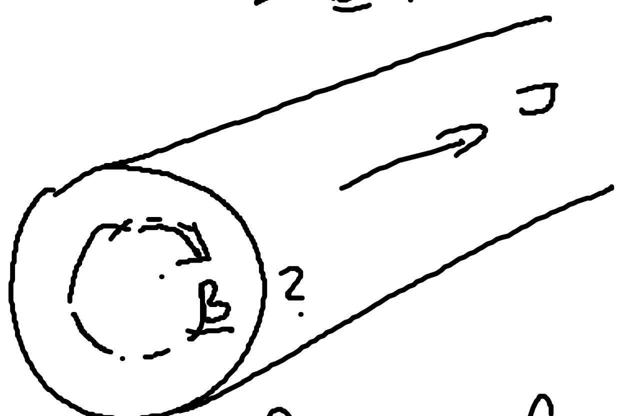
$$\operatorname{rot} \underline{B} = \mu_0 \underline{J} + \frac{1}{c^2} \frac{\partial}{\partial t} \underline{E}$$

3. Maxwellsche Gleichung.

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{Vs}{Am}$$ magnetische Feldkonstante.

$\to \underline{B}$?

AUFGABE:



$\operatorname{rot} \underline{B} = \mu_0 \underline{J}$

$$\oint B \, ds = \mu_0 \int \underline{J} \, d\underline{A}$$

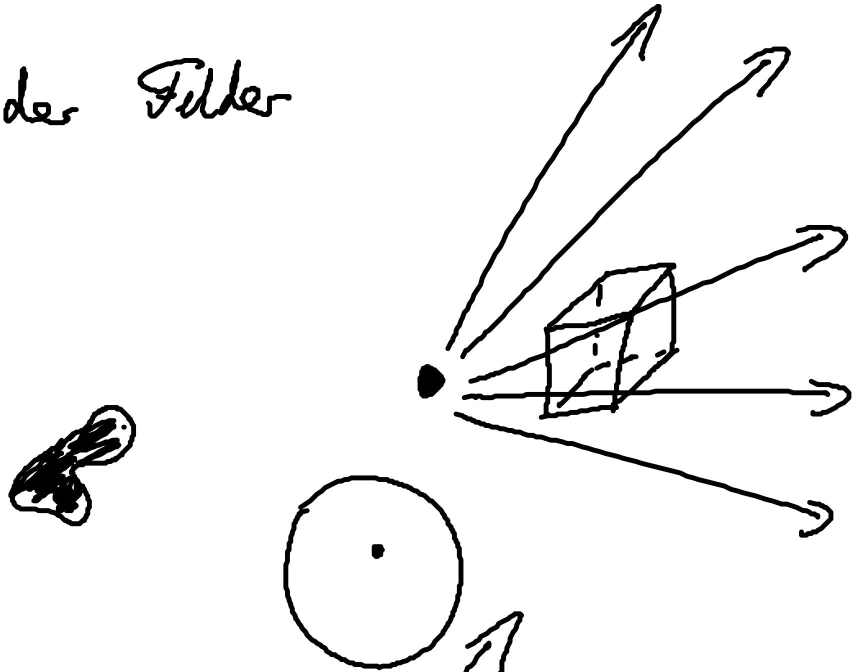
Berkeley Physik-Kurs Band II
(E.M. Purcell)

bestes Physikbuch.

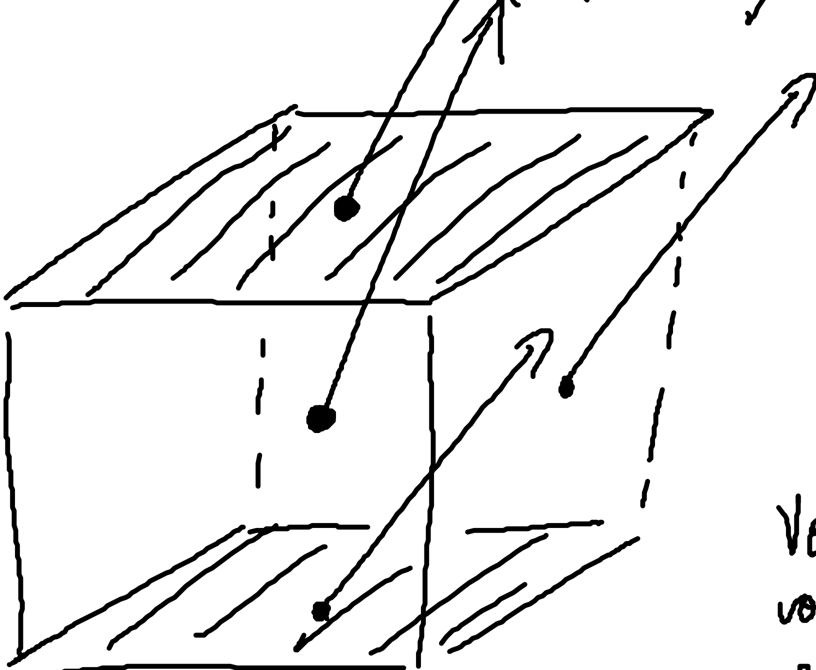
grad

rot

Die Quellen der Felder

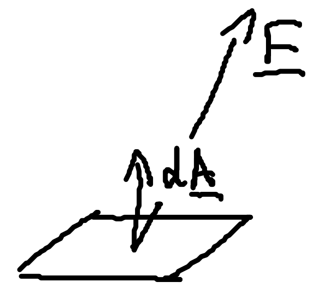


Betrachte kubisches Volumen (inf. klein)

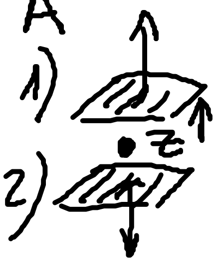


Volumen V wird
von Fläche A
eingeschlossen

Definiere $\lim\limits_{V \to 0} \frac{1}{V} \int_A \underline{F}\, d\underline{A}$



Beitrag von Flächen



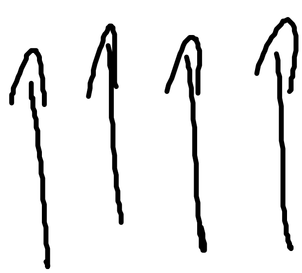
$$\underline{F}\, d\underline{A} = F_z\, dx\, dy$$

$$\underline{1) + 2)} = \left[\int_{1)} dx\, dy\, F_z\left(x, y, z + \frac{\Delta z}{2}\right) - \int_{2)} dx\, dy\, F_z\left(x, y, z - \frac{\Delta z}{2}\right) \right] \frac{1}{\Delta x \Delta y \Delta z}$$

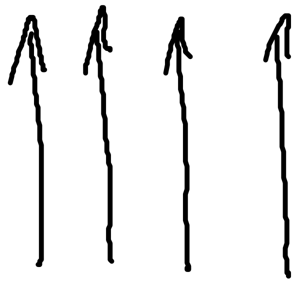
$$= \frac{\partial}{\partial z} \underbrace{\int dx\, dy\, F_z(x, y, z)}_{\to\, \Delta x \Delta y\, F_z(x, y, z)} \frac{\Delta z}{\Delta x \Delta y \Delta z} + O\left(\Delta z^2\right)$$

$$= \underline{\frac{\partial}{\partial z} F_z(x, y, z)}$$

Insgesamt: $\left\| \operatorname{div} \underline{F} = \frac{\partial}{\partial x} F_x + \frac{\partial}{\partial y} F_y + \frac{\partial}{\partial z} F_z \right\|$



hat div $F \neq 0$



$$\underline{F} = (2,0,0)$$

$$\Rightarrow \quad \operatorname{div} \underline{F} = 0.$$

Integralsatz von Gauß

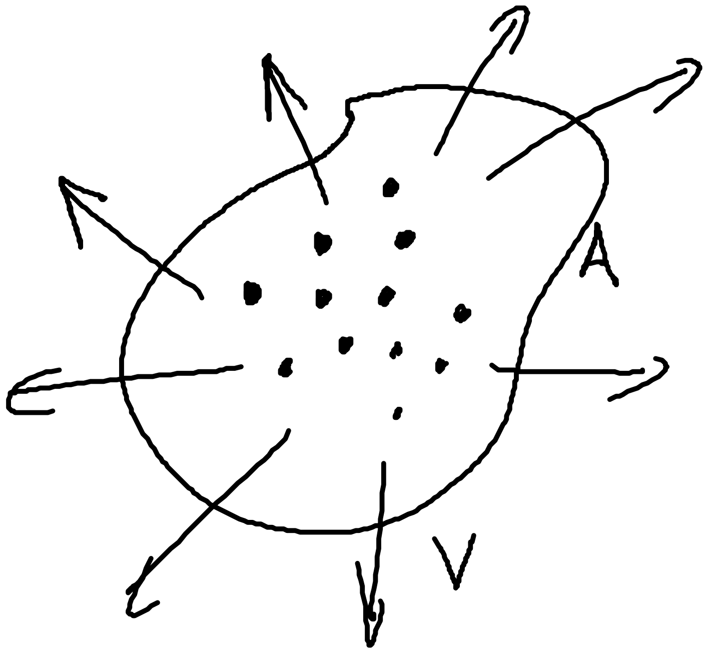
beliebiges Volumen V durch kleine „Würfel" approximieren

$$\int \underline{F}\, d\underline{A} = \sum_i V_i \underbrace{\frac{1}{V_i}\int_{A_i} \underline{F}\, dA_i}_{\to \operatorname{div}\underline{F}}$$

$$\to \int \operatorname{div} \underline{F}\, dV$$

$$\boxed{\int \underline{F}\, d\underline{A} = \int \operatorname{div}\underline{F}\, dV}$$ Gauß'scher Integralsatz

Beispiel Elektrostatik: Ladungsverteilung $\rho(\underline{x})$



$$\frac{1}{\varepsilon_0}\int \rho\, dV = \underbrace{\int_A \underline{E}\, d\underline{A}}_{\int \operatorname{div} \underline{E}\, dV}$$

Gauß'sches Gesetz der E-statik

$$\boxed{\operatorname{div} \underline{E} = \frac{1}{\varepsilon_0}\rho}$$

2. Maxwell'sche Gleichung

$$\varepsilon_0 = 8.85 \cdot 10^{-12}\,\frac{C^2}{Nm^2}$$

elektr. Feldkonstante

(SI-Einheiten).

$$\boxed{\operatorname{div} \underline{B} = 0}$$

4. Maxwellsche Gleichung

"Es gilt keine magnetischen Monopole"

$$\mathrm{rot}\,\underline{E} = -\frac{\partial}{\partial t}\underline{B}$$

$$\mathrm{div}\,\underline{E} = \frac{1}{\varepsilon_0}\rho$$

$$\mathrm{rot}\,\underline{B} = \mu_0 \underline{J} + \frac{1}{c^2}\frac{\partial}{\partial t}\underline{E}$$

$$\mathrm{div}\,\underline{B} = 0$$

Maxwellsche Gleichung

Klausur

1) - komplexe Zahl
   - Zeichnen (Funktion)
   - Re z < ...

2) gDGL (lineare)

Zwischenbem: : nicht schummeln !

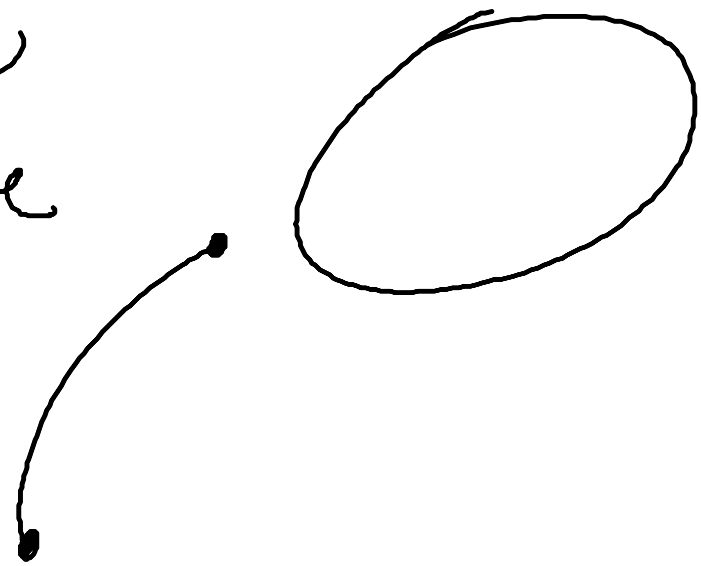
3) Fourierreihe

4) Eigenschwingungen $\boxed{\begin{matrix} EW \\ EV \end{matrix}}$

5) pDGL z.B. Diffusionsgl.

6) Fouriertrafo,

7) eine Kurve



$r \cos \omega t$
$r \sin \omega t$

8) gradient, z.B. krummlinige