

3.7.2008

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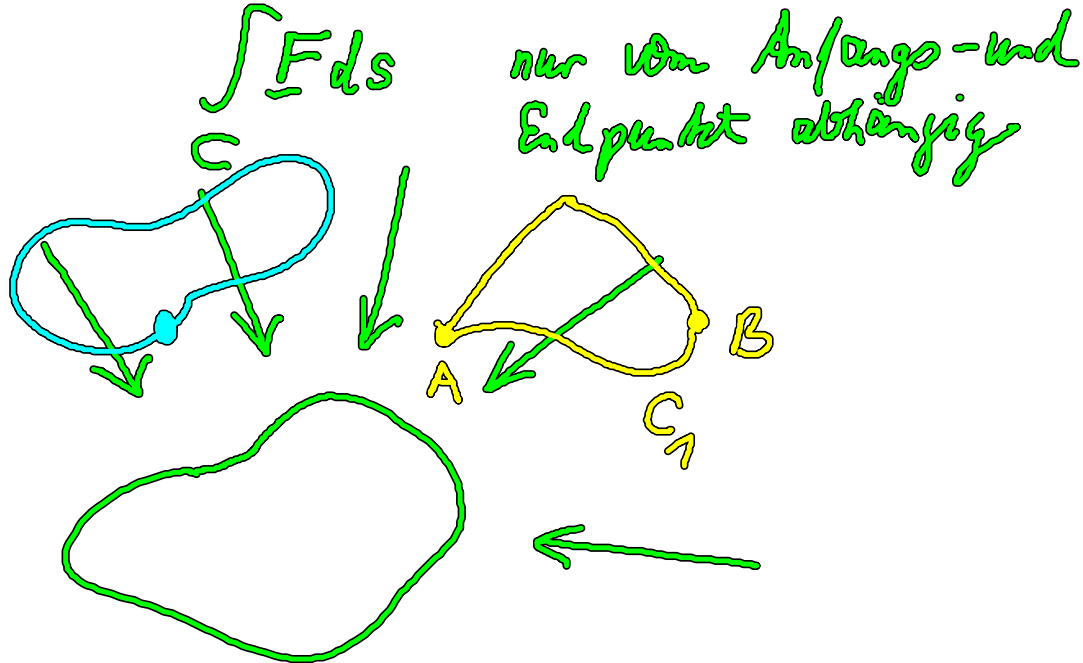
Gradient, Rotation, Divergenz

$$\underline{F} = -\underline{\nabla} \phi$$

$\underline{\nabla}$ : grad

$\phi$ : Potential

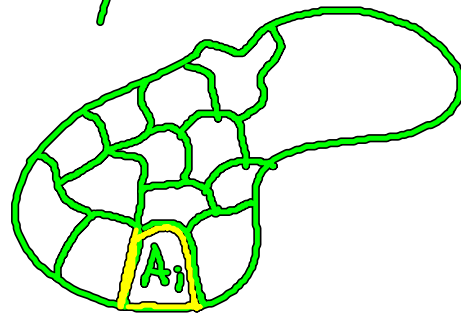
Konservative Kräfte :



$$\oint_C \underline{F} ds \equiv w[C] \quad \text{Wirbelstärke}$$

Für konservative Kräfte ist  $w[C] = 0$ .

Rotation  $\text{rot } \underline{F}$  als lokales Maß für die Wirbelstärke



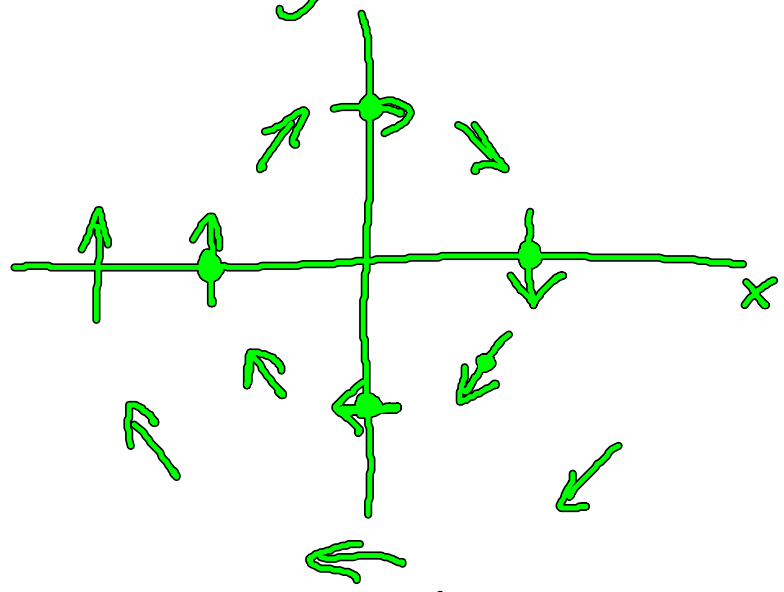
$$(\text{rot } \underline{F}) \cdot \underline{n} \equiv \lim_{A_i \rightarrow 0} \frac{\oint_{C_i} \underline{F} ds}{A_i}$$

Ausrechnen für kart. KO:

$$\Rightarrow \text{rot } \underline{F} \equiv \underline{\nabla} \times \underline{F} = \begin{vmatrix} \underline{e}_x & \underline{e}_y & \underline{e}_z \\ \partial_x & \partial_y & \partial_z \\ F_x & F_y & F_z \end{vmatrix}$$

wobei  $\underline{F} = (F_x, F_y, F_z)$

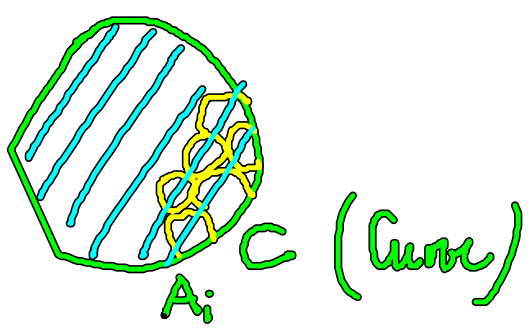
Beispiel:  $\underline{f}(x,y,z) = (y, -x, 0)$



$$\text{rot } \underline{f} = \begin{vmatrix} \dot{\partial}_x & \dot{\partial}_y & \dot{\partial}_z \\ y & -x & 0 \end{vmatrix} = (0, 0, -2)$$

Integralsatz von Stokes

$$W[C] \equiv \oint_C \underline{F}(\underline{x}) d\underline{x} = \sum_i A_i \underbrace{\frac{W_i}{A_i}}_{\text{rot } \underline{F}}$$



$$\rightarrow \int_A \text{rot } \underline{F} dA$$

$$\oint_C \underline{F} d\underline{x} = \int_A \text{rot } \underline{F} dA$$

Induktionsgesetz:

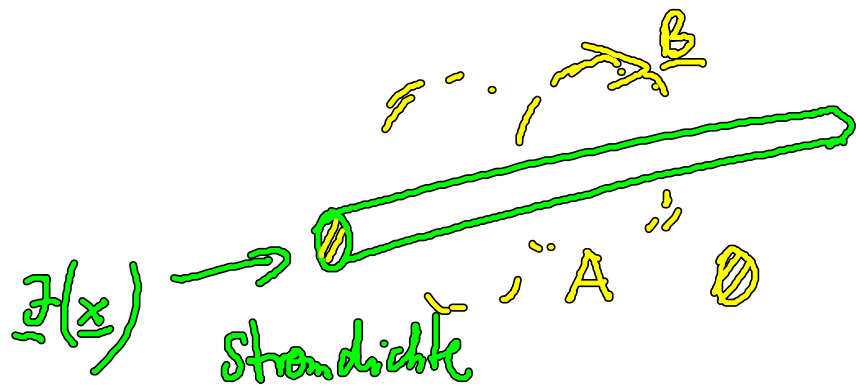
$$-\frac{\partial}{\partial t} \int_A \underline{B} \, d\underline{A} = \oint_C \underline{E} \, d\underline{s}$$

$$= \int_A \operatorname{rot} \underline{E} \, d\underline{A}$$

$$-\frac{\partial}{\partial t} \underline{B} = \operatorname{rot} \underline{E}$$

1. Maxwell'sche  
Gleichung  
(Induktionsgesetz)

Magnetostatik:



$$\oint_C \underline{B} \, d\underline{s} = \mu_0 \int_A \underline{J} \, d\underline{A}$$

$$\int_A \text{rot } \underline{B} \, dA$$

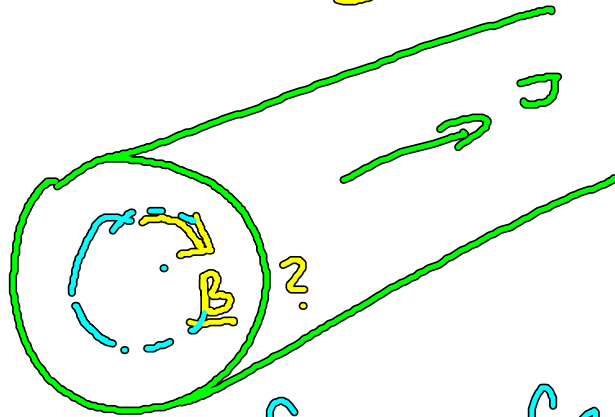
$$\text{rot } \underline{B} = \mu_0 \underline{J} + \frac{1}{c^2} \frac{\partial}{\partial t} \underline{E}$$

$$\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{Vs}}{\text{Am}}$$

3. Maxwell'sche Gleichung.  
magnetische Feldkonstante.  
 $\rightarrow \underline{B}?$

AUFGABE:

$$\text{rot } \underline{B} = \mu_0 \underline{J}$$



$$\oint \underline{B} \, ds = \mu_0 \int \underline{J} \, dA$$

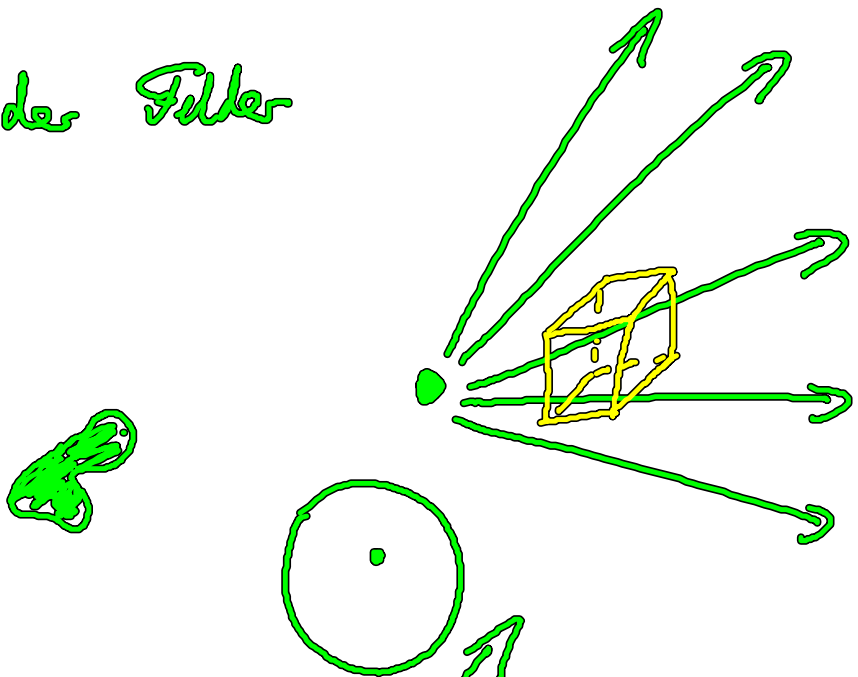
Berkeley Physik-Kurs Band II  
(E. n. Purcell)

bestes Physikbuch.

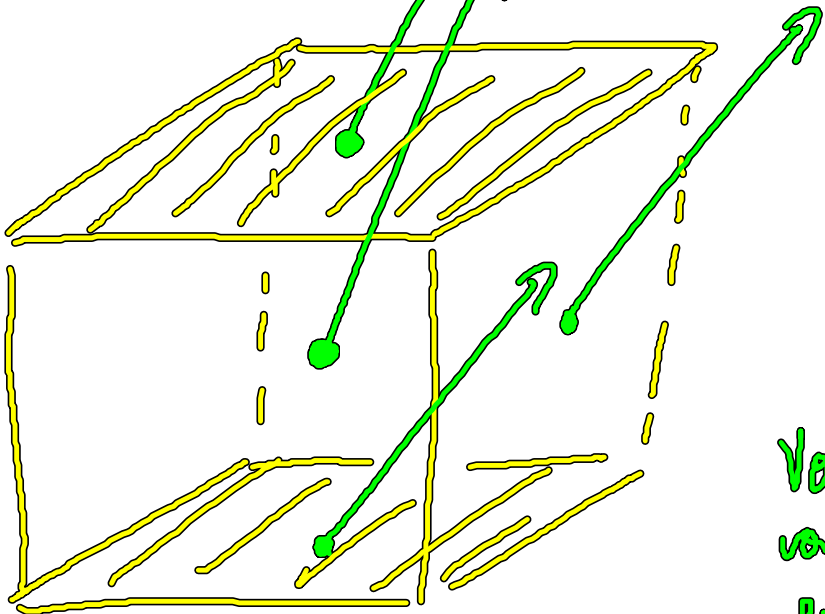
grad

$\sqrt{A}$

Die Quellen der Felder



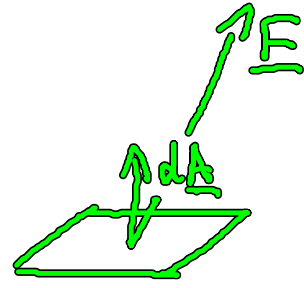
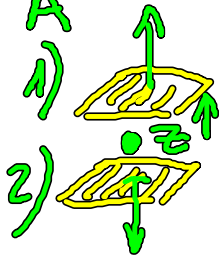
Betrachte kubisches Volumen (inf. klein)



Volumen  $V$  wird von Fläche  $A$  eingeschlossen

Definiere  $\lim_{V \rightarrow 0} \frac{1}{V} \int_A \underline{F} dA$

Beitrag von Flächen



$$1)+2) = \left[ \begin{array}{l} 1) \int dx dy F_z(x,y, z + \frac{\Delta z}{2}) \\ - \int dx dy F_z(x,y, z - \frac{\Delta z}{2}) \end{array} \right] \frac{1}{\Delta x \Delta y \Delta z}$$

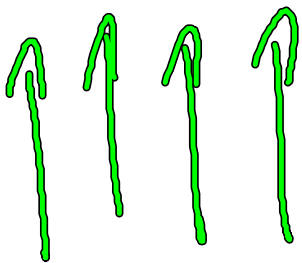
$\underline{F} dA = F_z dx dy$

$$= \frac{\partial}{\partial z} \int dx dy F_z(x,y,z) \frac{\Delta z}{\Delta x \Delta y \Delta z} + O(\Delta z^2)$$

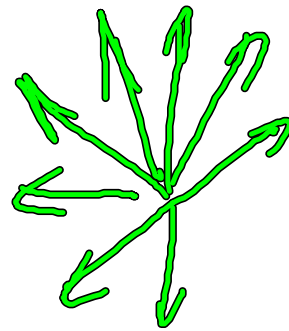
$$\rightarrow \Delta x \Delta y F_z(x,y,z)$$

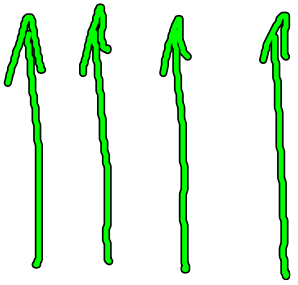
$$= \frac{\partial}{\partial z} F_z(x,y,z)$$

Insights:  $\| \operatorname{div} \underline{F} = \frac{\partial}{\partial x} F_x + \frac{\partial}{\partial y} F_y + \frac{\partial}{\partial z} F_z \|$



hat  $\operatorname{div} F \neq 0$

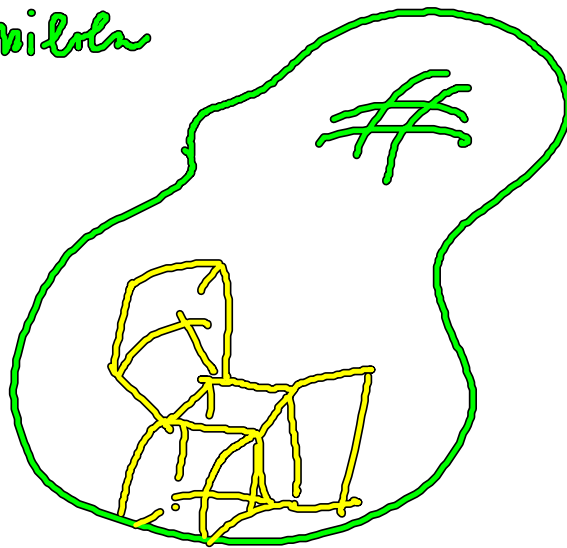




$$\underline{E} = (2, 0, 0)$$
$$\Rightarrow \operatorname{div} \underline{E} = 0.$$

## Integralsatz von Gauß

beliebiges Volumen  $V$  durch kleine „Würfel“  
approximieren



$$\int \underline{E} d\underline{A} = \sum_i V_i \underbrace{\frac{1}{V_i} \int_{A_i} \underline{E} dA_i}_{\rightarrow \operatorname{div} \underline{E}}$$

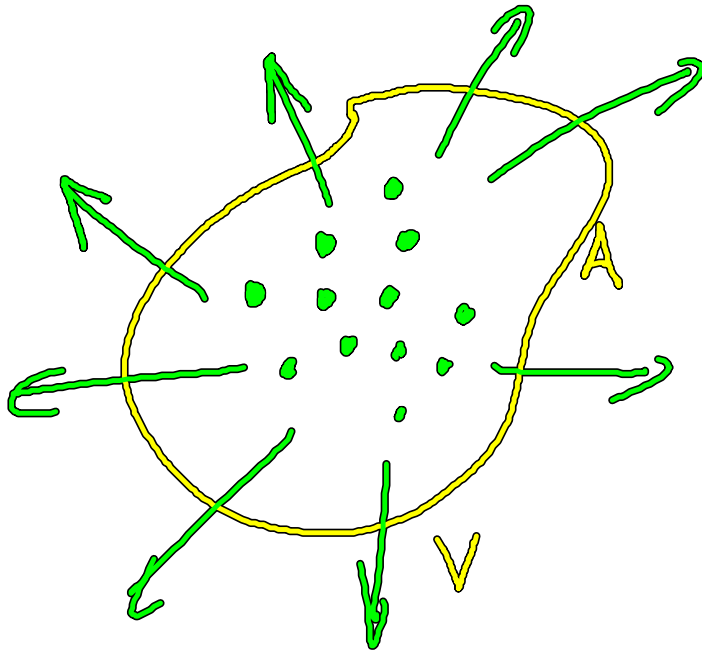
$$\rightarrow \int \operatorname{div} \underline{E} dV$$

$$\boxed{\int \underline{E} d\underline{A} = \int \operatorname{div} \underline{E} dV}$$

Gauß'scher Integralsatz



Beispiel Elektrostatik: Ladungsverteilung  $\rho(\underline{x})$



$$\frac{1}{\epsilon_0} \int \rho dV = \int_A \underline{E} dA$$

$\int \text{div } \underline{E} dV$

Gauß'sches Gesetz  
der E-Statik

$$\boxed{\text{div } \underline{E} = \frac{1}{\epsilon_0} \rho}$$

$$\epsilon_0 = 8.85 \cdot 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}$$

(SI-Einheiten).

$$\boxed{\text{div } \underline{B} = 0}$$

2. Maxwell'sche  
Gleichung  
elektr. Feldkonstante

4. Maxwell'sche  
Gleichung

" Es gibt keine magnetischen  
Monopole "

$$\operatorname{rot} \underline{E} = -\frac{\partial}{\partial t} \underline{B}$$

$$\operatorname{div} \underline{E} = \frac{1}{\epsilon_0} \rho$$

$$\operatorname{rot} \underline{B} = \mu_0 \underline{J} + \frac{1}{c^2} \frac{\partial}{\partial t} \underline{E}$$

$$\operatorname{div} \underline{B} = 0$$

Maxwellsche Gleichung

Klausur

- 1) - komplexe Zahl  
- Zeichen (Funktion)  
-  $\operatorname{Re} z < \dots$

- 2)  $\partial$  DGL (linear)

Zwischenben: :

nicht schummeln !

3)

Fourierreihe

4)

Eigenwertprobleme



5)

PDE

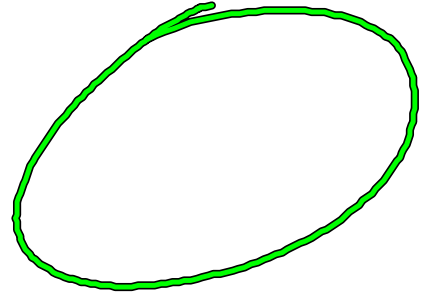
z.B. Diffusionsgl.

6)

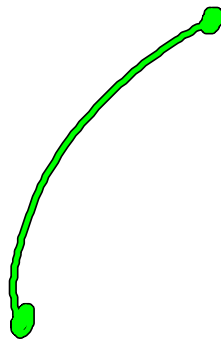
Fouriertrafo ,

7)

eine Kurve



Gamma mit  
Gamma nicht



8)

Gradient ,

z.B. Kantenlänge

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