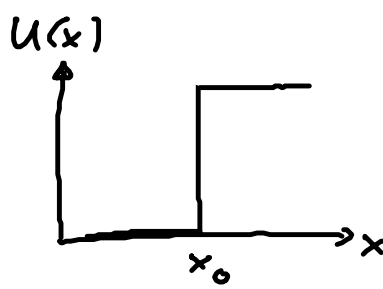
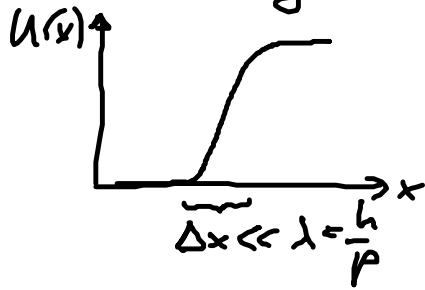


c) Anschlußbedingungen für unstetiges $U(x)$

• Idealisierung:



• Vorteil: Bereiche mit konstantem U , SG: $\psi'' + \frac{2m}{\hbar^2}(E-U)\psi = 0$ (8.9)

Lösung: (i) $E > U$: $E-U = \frac{\hbar^2 k^2}{2m}$, $k > 0$

$$\begin{aligned} \rightarrow \psi(x) &= A e^{ikx} + A' e^{-ikx} \quad (\text{Streuzustände}) \\ &= A'' \sin kx + A''' \cos kx \quad (\text{gebundene Zustände}) \end{aligned}$$

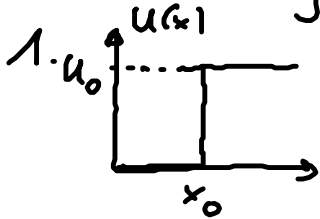
(ii) $E < U$: klassisch nicht erlaubt

$$E-U = -\frac{\hbar^2 \kappa^2}{2m}, \quad \kappa > 0$$

$$\rightarrow \psi(x) = B e^{\kappa x} + B' e^{-\kappa x}$$

Bestimme A, A', B, B' ... aus Anschlussbedingungen!

• Anschlussbedingungen:



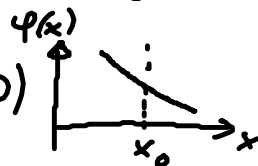
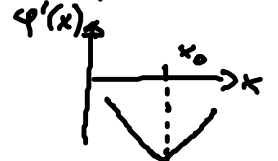
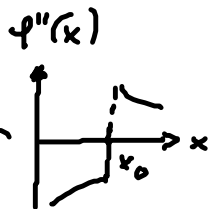
$$\lim_{\varepsilon \rightarrow 0} [U(x_0 + \varepsilon) - U(x_0 - \varepsilon)] = U_0 < \infty$$

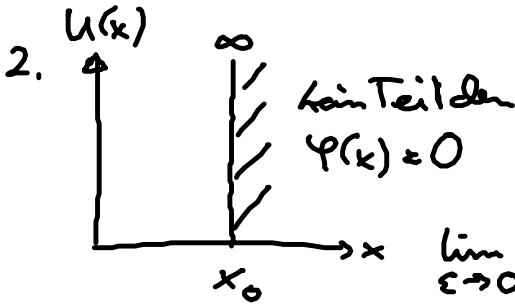
$$\psi(x) \text{ endlich} \xrightarrow{(8.9)} \lim_{\varepsilon \rightarrow 0} [\psi'(x_0 + \varepsilon) - \psi'(x_0 - \varepsilon)] = \text{endlich}$$

$$\lim_{\varepsilon \rightarrow 0} \int_{x_0 - \varepsilon}^{x_0 + \varepsilon} (8.9) dx \rightarrow \lim_{\varepsilon \rightarrow 0} [\psi(x_0 + \varepsilon) - \psi(x_0 - \varepsilon)] = 0$$

$$\begin{aligned} \rightarrow \psi'(x)|_{x_0} &\text{ stetig} \\ \rightarrow \psi(x)|_{x_0} &\text{ stetig \& glatt} \end{aligned}$$

(8.10)

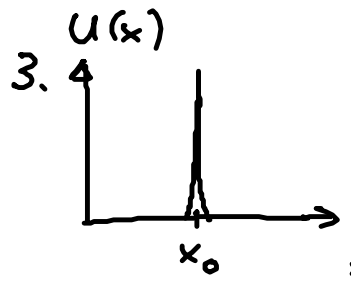
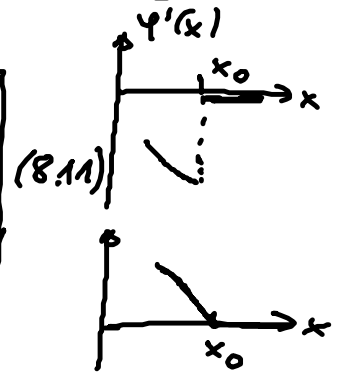




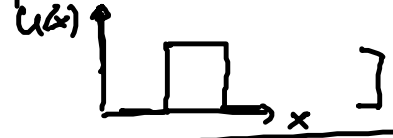
$$\lim_{\epsilon \rightarrow 0} [U(x_0 + \epsilon) - \underbrace{U(x_0 - \epsilon)}_0] = \infty$$

$$\lim_{\epsilon \rightarrow 0} \int_{x_0 - \epsilon}^{x_0 + \epsilon} (\text{P.S.}) dx \rightarrow \lim_{\epsilon \rightarrow 0} [\underbrace{\psi'(x_0 + \epsilon)}_0 - \psi'(x_0 - \epsilon)] = \text{endlich}$$

$\rightarrow \psi'(x)|_{x_0}$ unstetig
 $\rightarrow \psi(x)|_{x_0} = 0$ & stetig



$$U(x) = U_0 \delta(x - x_0) \quad [\text{Grenzfall von } U(x)]$$

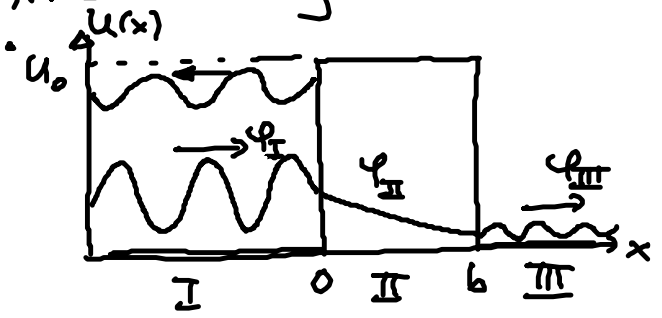


$$\lim_{\epsilon \rightarrow 0} \int_{x_0 - \epsilon}^{x_0 + \epsilon} (\text{P.S.}) dx \rightarrow \lim_{\epsilon \rightarrow 0} [\psi'(x_0 + \epsilon) - \psi'(x_0 - \epsilon)] = \frac{2m}{\hbar^2} U_0 \psi(x)|_{x_0}$$

$\rightarrow \psi'(x)|_{x_0}$ unstetig
 $\rightarrow \psi(x)|_{x_0}$ stetig

8.3 Potentialschwelle - Tunnel effekt

a) Problemstellung:



Löse: $A\psi = E\psi$

\rightarrow Ergebnis: Kontinuum von Zuständen (unendliches System!)

$$E < U_0: \underbrace{\psi_I, \psi_{II} \neq 0, \psi_{III} \neq 0}$$

Tunnel effekt = QT-Resultat

Klassik: Teilchen kann Pot. barriere nicht überwinden, wegen $E < U_0$

$E > U_0$: Transmission & Reflexion (\neq Klassik)

Resonanzen

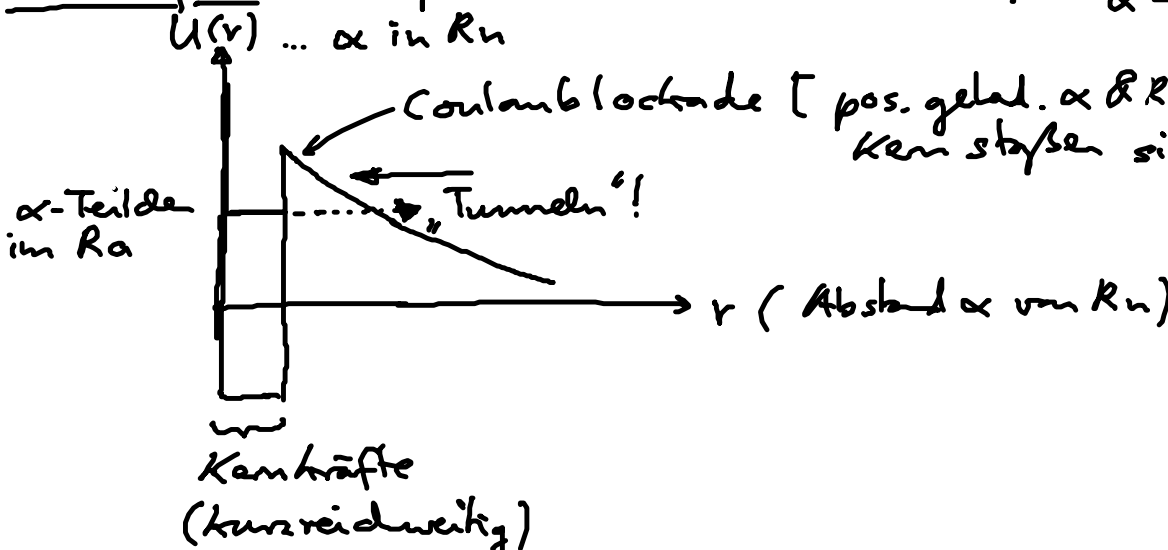
• Realität: „Pakete“ Wellenpakete aus s Eigenzuständen

b) Tunnel effekt

(i) α -Zerfall:

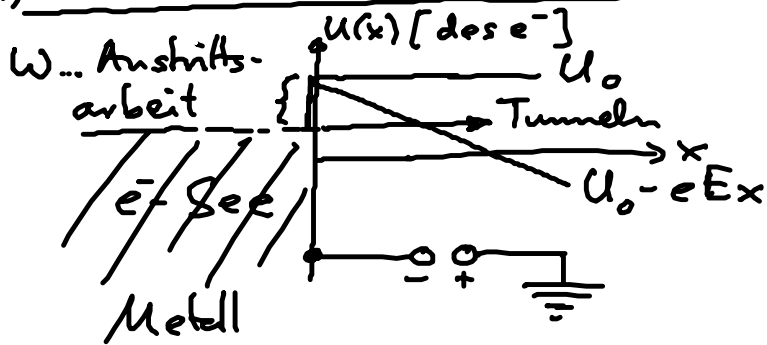


Kernladungszahl
 Radium (Erdalkalimetall)
 Radon (Edelgas)

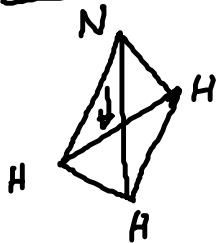


Umkehrung: Vereinigung gleich geladener Kerne (Bsp: Kernfusion)

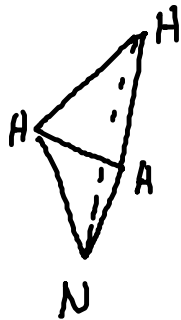
(ii) Feldemission von e^- in Metallen:



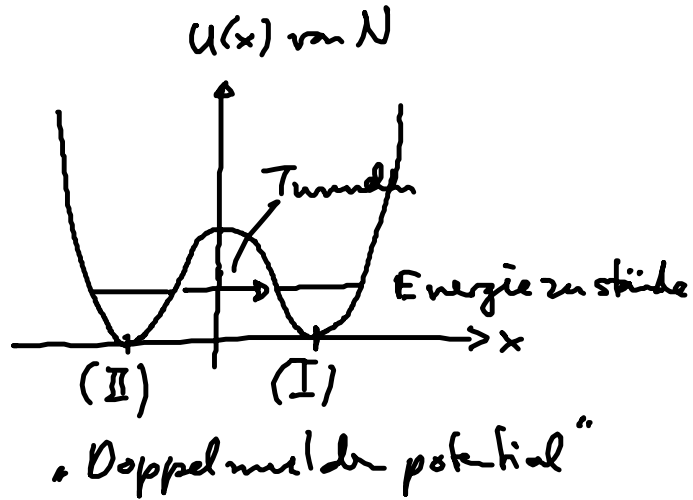
(iii) Molekülschwingung: NH_3



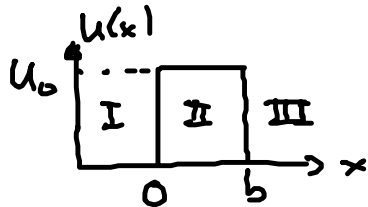
(I)



(II)



c) Rechnung für



• FW-Gln: (8.2)/(8.3): $\psi''(x) - \frac{2m}{\hbar^2} (U-E) \psi(x) = 0$

(8.13) $\left\{ \begin{array}{l} \text{Bereich I und III (U=0): } \psi'' + k^2 \psi = 0, \quad k^2 = \frac{2mE}{\hbar^2} \\ \text{Bereich II (U=U_0): } \psi'' - \kappa^2 \psi = 0, \quad \kappa^2 = \frac{2m}{\hbar^2} (U_0 - E) = \begin{cases} > 0, E < U_0 \\ < 0, E > U_0 \end{cases} \end{array} \right.$

$\rightarrow \left\{ \begin{array}{l} \kappa = \frac{\kappa}{k} \end{array} \right. \quad (8.14)$

• Lösung:

(8.15) $\left\{ \begin{array}{l} \text{(I) } (-\infty < x < 0): \psi_{\text{I}}(x) = A_1 e^{ikx} + B_1 e^{-ikx} \\ \text{(II) } (0 < x < b): \psi_{\text{II}}(x) = A_2 e^{-\kappa x} + B_2 e^{\kappa x} \\ \text{(III) } (b < x < \infty): \psi_{\text{III}}(x) = A_3 e^{ikx} + B_3 e^{-ikx} \end{array} \right. \quad \left[\begin{array}{l} \kappa > 0, E < U_0 \\ \kappa = iK, E > U_0 \end{array} \right]$

Exp: einlaufende Welle von $-\infty$: A_1
 reflekt. " nach $-\infty$: B_1
 transm. " nach ∞ : A_3

• Bestimmung der A_i, B_i :

Anschluß bed. (8.10): $\left. \begin{array}{l} \psi_{\text{I}}(0) = \psi_{\text{II}}(0), \quad \psi'_{\text{I}}(0) = \psi'_{\text{II}}(0) \\ \psi_{\text{II}}(b) = \psi_{\text{III}}(b), \quad \psi'_{\text{II}}(b) = \psi'_{\text{III}}(b) \end{array} \right\} (8.16)$

↳ lineare Gln. für A_1, B_1, A_2, B_2, A_3

$$\left. \begin{aligned}
 A_1 + B_1 &= A_2 + B_2 & (a) \\
 ikA_1 - ikB_1 &= -\kappa A_2 + \kappa B_2 & (b) \\
 A_3 e^{ikb} &= A_2 e^{-\kappa b} + B_2 e^{\kappa b} & (c) \\
 ikA_3 e^{ikb} &= -\kappa A_2 e^{-\kappa b} + \kappa B_2 e^{\kappa b} & (d)
 \end{aligned} \right\} (8.17)$$

Exp: $|A_1|^2 \dots$ frei wählbare Intensität der einlaufenden Welle
 für Redy: A_3

$$(8.17) (c, d) \rightarrow A_2(A_3), B_2(A_3)$$

$$\rightarrow (8.17) (a, b) \rightarrow A_1(A_3), B_1(A_3)$$

o.B. \rightarrow

$$\left. \begin{aligned}
 A_1 &= e^{ikb} \left[\cosh(\kappa b) + \frac{i}{2} \left(\eta - \frac{1}{\eta} \right) \sinh(\kappa b) \right] A_3 \\
 B_1 &= -\frac{i}{2} e^{ikb} \left(\eta + \frac{1}{\eta} \right) \sinh(\kappa b) A_3 \\
 A_2 &= \frac{1}{2} e^{ikb} e^{\kappa b} \left(1 - \frac{i}{\eta} \right) A_3 \\
 B_2 &= \frac{1}{2} e^{ikb} e^{-\kappa b} \left(1 + \frac{i}{\eta} \right) A_3
 \end{aligned} \right\} (8.18)$$

d) Diskussion

• Transmissionskoeffizient:

$$T = \left| \frac{A_3}{A_1} \right|^2 \stackrel{(8.18)}{=} \frac{1}{1 + \frac{1}{4} \left| \eta + \frac{1}{\eta} \right|^2 |\sinh(\kappa b)|^2} \leq 1!$$

Reflexionskoeffizient

$$R = \left| \frac{B_1}{A_1} \right|^2 \stackrel{(8.18)}{=} 1 - T \quad \dots \quad \text{Erhaltung der Gesamtwahrscheinlichkeit}$$

(8.19)