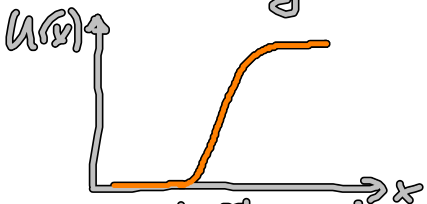
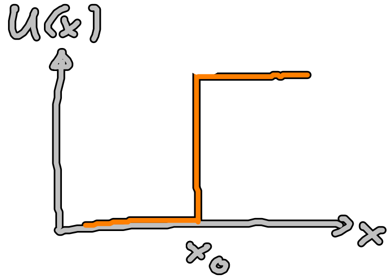
c) Anschlußbedingungen für unstetiges $U(x)$

• Idealisierung:



$\Delta x \ll \lambda = \frac{h}{p}$

$\longrightarrow$



• Vorteil: Bereiche mit konstantem U, SG: $\varphi'' + \frac{2m}{\hbar^2}(E-U)\varphi = 0$ (8.9)

Lösung: (i) $E > U$: $E - U = \frac{\hbar^2 k^2}{2m}$, $k > 0$

$$\longrightarrow \varphi(x) = A e^{ikx} + A' e^{-ikx} \quad \text{(Streuzustände)}$$

$$= A'' \sin kx + A''' \cos kx \quad \text{(gebundene Zustände)}$$

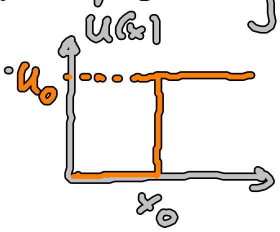
(ii) $E < U$: klassisch nicht erlaubt

$E - U = -\frac{\hbar^2 \kappa^2}{2m}$, $\kappa > 0$

$$\longrightarrow \varphi(x) = B e^{\kappa x} + B' e^{-\kappa x}$$

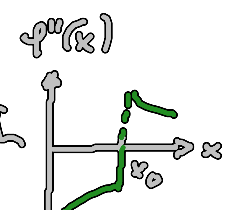
Bestimme $A, A', B, B' \dots$ aus Anschlußbedingungen!

• Anschlußbedingungen:

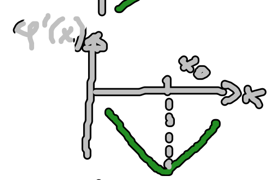
1. 

$$\lim_{\varepsilon \to 0} [U(x_0+\varepsilon) - U(x_0-\varepsilon)] = U_0 < \infty$$

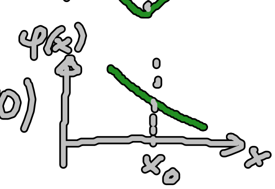
$$\varphi(x) \text{ endlich} \xrightarrow{(8.9)} \lim_{\varepsilon \to 0} [\varphi''(x_0+\varepsilon) - \varphi''(x_0-\varepsilon)] = \text{endlich}$$



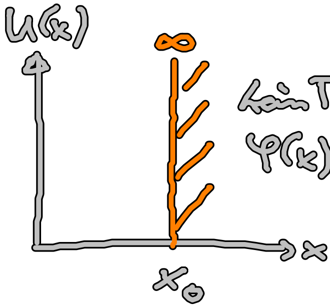
$$\lim_{\varepsilon \to 0} \int_{x_0-\varepsilon}^{x_0+\varepsilon} (8.9)\, dx \longrightarrow \lim_{\varepsilon \to 0} [\varphi'(x_0+\varepsilon) - \varphi'(x_0-\varepsilon)] = 0$$



$$\boxed{\begin{array}{l} \longrightarrow \varphi'(x)\big|_{x_0} \text{ stetig} \\ \longrightarrow \varphi(x)\big|_{x_0} \text{ stetig \& glatt} \end{array}} \quad (8.10)$$



2.



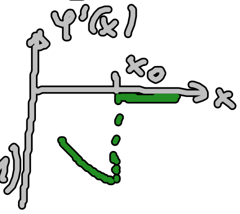
kein Teilchen
$\varphi(x) = 0$

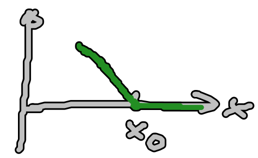
$$\lim_{\varepsilon\to 0}\left[U(x_0+\varepsilon) - \underbrace{U(x-\varepsilon)}_{0}\right] \text{“}=\infty\text{”}$$

$$\lim_{\varepsilon\to 0}\int_{x_0-\varepsilon}^{x+\varepsilon}(8.9)\,dx \longrightarrow \lim_{\varepsilon\to 0}\left[\varphi'(x_0+\varepsilon) - \underbrace{\varphi'(x_0-\varepsilon)}_{=0}\right] = \text{endlich}$$

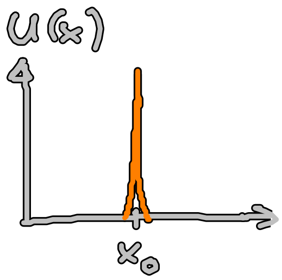
$$\boxed{\begin{array}{l} \longrightarrow \varphi'(x)\big|_{x_0} \text{ unstetig} \\ \longrightarrow \varphi(x)\big|_{x_0} = 0 \text{ \& stetig} \end{array}}$$

(8.11)

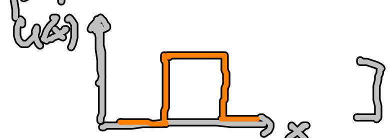




3.



$U(x) = U_0\,\delta(x-x_0)$ [Grenzfall von



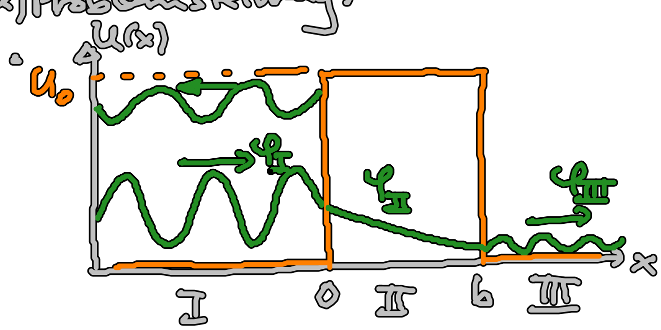
]

$$\lim_{\varepsilon\to 0}\int_{x_0-\varepsilon}^{x_0+\varepsilon}(8.9)\,dx \longrightarrow \boxed{\begin{array}{l} \lim_{\varepsilon\to 0}\left[\varphi'(x+\varepsilon) - \varphi'(x_0-\varepsilon)\right] = \frac{2m}{\hbar^2}U_0\,\varphi(x)\big|_{x_0} \\ \longrightarrow \varphi'(x)\big|_{x_0} \text{ unstetig} \\ \longrightarrow \varphi(x)\big|_{x_0} \text{ stetig} \end{array}}$$

(8.12)

8.3 Potentialschwelle – Tunneleffekt

a) Problemstellung:



Löse: $\hat{H}\varphi = E\varphi$

⟶ Ergebnis: Kontinuum von Zuständen (unendliches System!)

$E < U_0$: $\quad \varphi_I,\ \underbrace{\varphi_{II} \neq 0,\ \varphi_{III} \neq 0}$

Tunneleffekt = QT-Resultat

Klassik: Teilchen kann Pot. barriere nicht überwinden, wegen $E < U_0$

$E > U_0$: Transmission & Reflexion ($\neq$ Klassik)

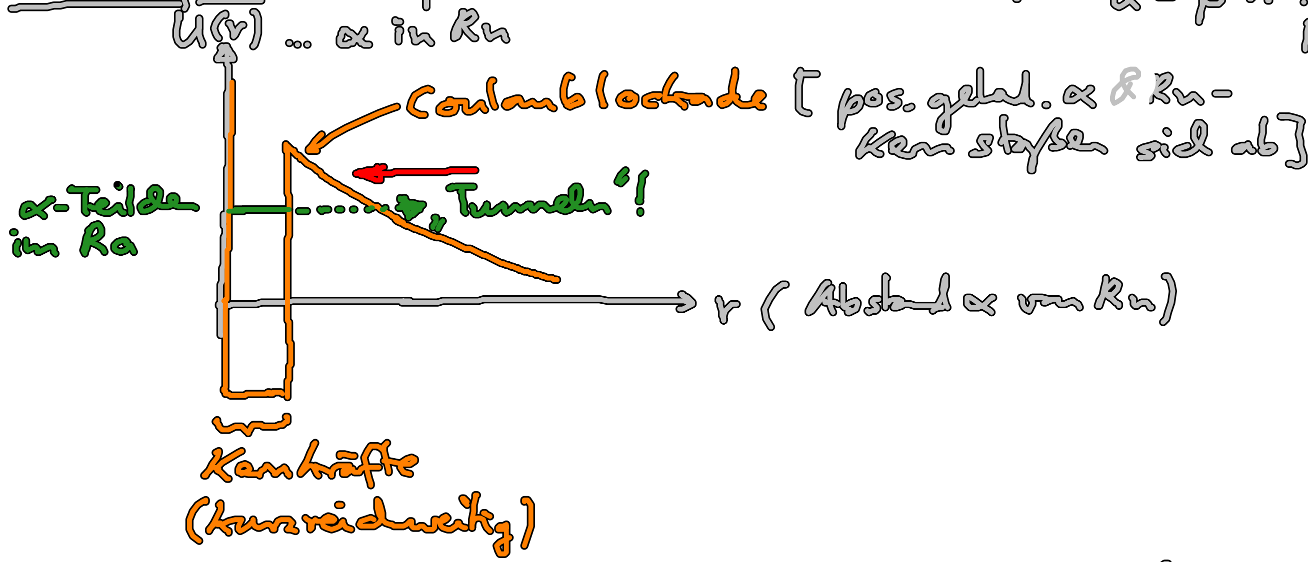
↳ Resonanzen

- Realität: „Bastle" Wellenpakete aus s Eigenzuständen

b) Tunneleffekt

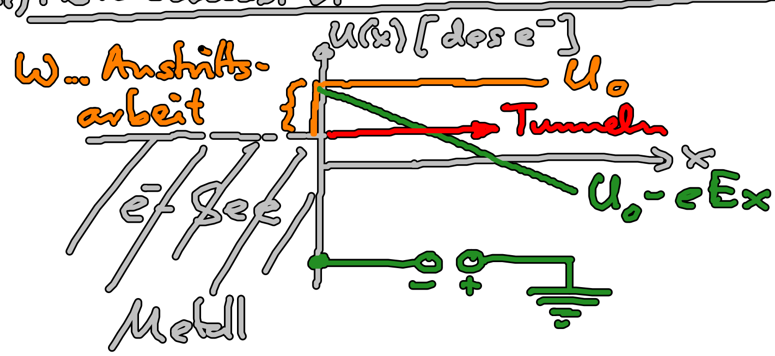
Kernladungszahl, Radium (Erdalkalimetall), Radon (Edelgas)

(i) α-Zerfall: Bsp: $^{88}Ra \longrightarrow {}^{86}Rn + \alpha$, $\alpha = p^2 n^2$ … Helium-Kern

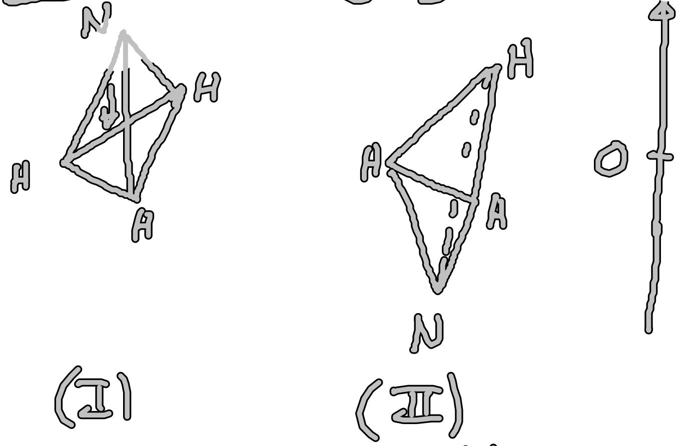


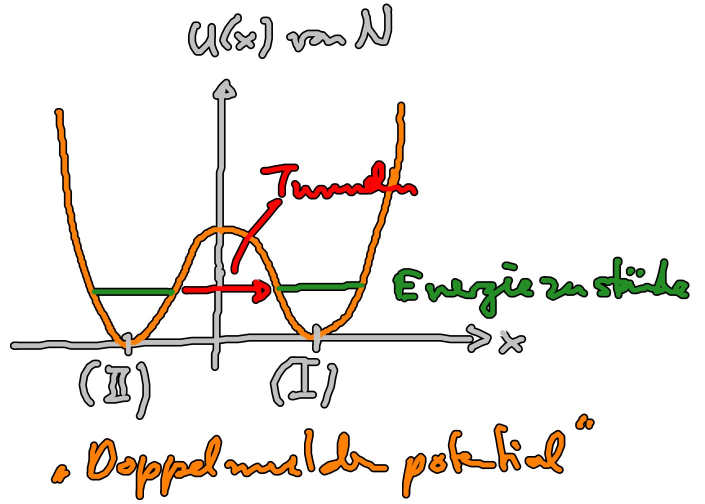
Umkehrung: Vereinigung gleichgeladener Kerne (Bsp: Kernfusion)

(ii) Feldemission von e^- in Metallen:

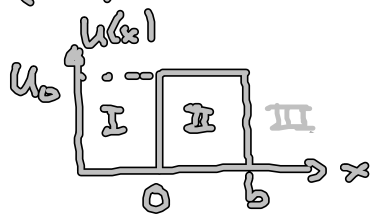


(iii) Molekülschwingung: NH_3





c) Rechnung für



• EW-Gln: (8.2)/(8.9): $\varphi''(x) - \frac{2m}{\hbar^2}(U-E)\varphi(x) = 0$

$$(8.13)\begin{cases} \text{Bereiche I und III } (U=0): & \varphi'' + k^2\varphi = 0, \quad \boxed{k^2 = \frac{2mE}{\hbar^2}} \\ \text{Bereich II } (U_0 \neq 0): & \varphi'' - \varkappa^2\varphi = 0, \quad \boxed{\varkappa^2 = \frac{2m}{\hbar^2}(U_0 - E)} = \begin{cases} >0, & E<U_0 \\ <0, & E>U_0 \end{cases} \end{cases}$$

$$\longrightarrow \boxed{\eta = \frac{\varkappa}{k}} \quad (8.14)$$

• Lösung:

$$(8.15)\begin{cases} (I)\ (-\infty < x < 0): & \varphi_I(x) = A_1 e^{ikx} + B_1 e^{-ikx} \\ (II)\ (0 < x < b): & \varphi_{II}(x) = A_2 e^{-\varkappa x} + B_2 e^{\varkappa x} \\ (III)\ (b < x\ \infty): & \varphi_{III}(x) = A_3 e^{ikx} + \cancel{B_3 e^{-ikx}} \end{cases}$$

$[\varkappa > 0,\ E < U_0$
$\varkappa = ik',\ E > U_0]$

Exp: einlaufende Welle von $-\infty$: A_1
reflekt. " nach $-\infty$: B_1
transm. " nach ∞: A_3

• Bestimmung der A_i, B_i:

Anschlußbed. (8.10):

$$\left.\underbrace{\begin{matrix} \varphi_I(0) = \varphi_{II}(0), & \varphi_I'(0) = \varphi_{II}'(0) \\ \varphi_{II}(b) = \varphi_{III}(b), & \varphi_{II}'(b) = \varphi_{III}'(b) \end{matrix}}_{4 \text{ lineare Gln. für } A_1, B_1, A_2, B_2, A_3}\right\} (8.16)$$

$$\left.\begin{aligned}
A_1 + B_1 &= A_2 + B_2 && \text{(a)}\\
ik A_1 - ik B_1 &= -\varkappa A_2 + \varkappa B_2 && \text{(b)}\\
A_3 e^{ikb} &= A_2 e^{-\varkappa b} + B_2 e^{\varkappa b} && \text{(c)}\\
ik A_3 e^{ikb} &= -\varkappa A_2 e^{-\varkappa b} + \varkappa B_2 e^{\varkappa b} && \text{(d)}
\end{aligned}\right\} \quad (8.17)$$

<u>Exp:</u> $|A_1|^2 \ldots$ frei wählbare Intensität der einlaufenden Welle

für Rechnung: A_3

$(8.17)(c,d) \longrightarrow A_2(A_3),\ B_2(A_3)$

$\longrightarrow (8.17)(a,b) \longrightarrow A_1(A_3),\ B_1(A_3)$

$\xrightarrow{\text{o.B.}}$

$$\boxed{\begin{aligned}
A_1 &= e^{ikb}\left[\cosh(\varkappa b) + \frac{i}{2}\left(\eta - \frac{1}{\eta}\right)\sinh(\varkappa b)\right] A_3\\
B_1 &= -\frac{i}{2} e^{ikb}\left(\eta + \frac{1}{\eta}\right)\sinh(\varkappa b) A_3\\
A_2 &= \frac{1}{2} e^{ikb} e^{\varkappa b}\left(1 - \frac{i}{\eta}\right) A_3\\
B_2 &= \frac{1}{2} e^{ikb} e^{-\varkappa b}\left(1 + \frac{i}{\eta}\right) A_3
\end{aligned}} \quad (8.18)$$

d) <u>Diskussion</u>

- Transmissionskoeffizient:

$$T = \left|\frac{A_3}{A_1}\right|^2 \underset{(8.18)}{=} \frac{1}{1 + \frac{1}{4}\left|\eta + \frac{1}{\eta}\right|^2 |\sinh(\varkappa b)|^2} \leq 1\ ! \quad (8.19)$$

Reflexionskoeffizient

$$R = \left|\frac{B_1}{A_1}\right|^2 \underset{(8.18)}{=} 1 - T \quad \ldots \text{ Erhaltung der Gesamtwahrscheinlichkeit}$$