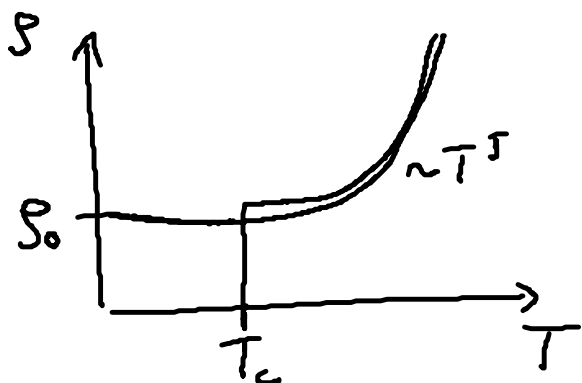


Chapter 10: Superconductivity

- So far: - Solids as a many-body "unit" of ions and electrons
 - Born-Oppenheimer approximation
 - independent e^- approximation
 - this always exists (OFT)
 - but there may not be meaningful physical objects in a solid ("quasiparticles") to look at

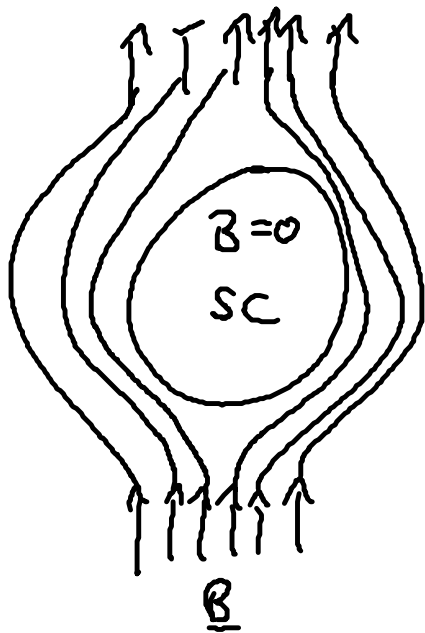
- Superconductivity: 1911 H. Kamerlingh Onnes
below T_c , resistivity of Hg, Sn, Pb drops to zero abruptly:



	T_c
Hg	4.15 K
Sn	3.72 K
Pb	7.19 K

- Perfect conductivity should not exist at finite T
- Record for persistent ring current (Ashcroft): $2\frac{1}{2}$ yrs.

1933 Meissner-Ochsenfeld : Perfect diamagnets



$$\underline{B} = \mu_0 \mu \underline{H}$$

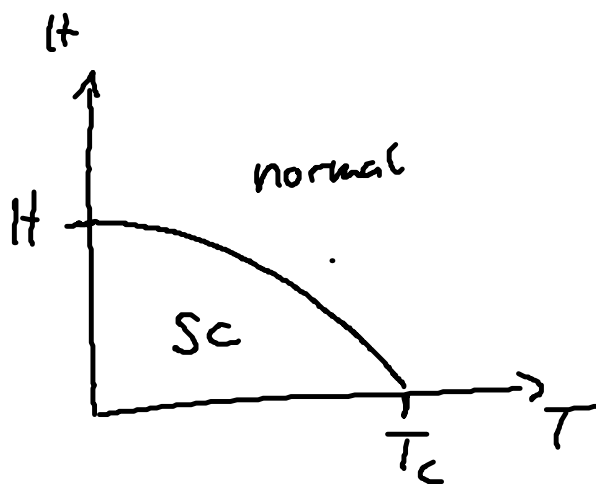
$$= \mu_0 (1 + \chi) \underline{H} = 0$$

$$\chi = -1$$

works only up to a certain critical field H_c

→ SC reverts to normal conductivity, beyond $H > H_c$

"phase diagram"



This is odd :

Consider a perfect conductor :

Drift model without resistance :

$$m \cdot \frac{d\underline{v}_s}{dt} = -e \cdot \underline{E}$$

$$\frac{d}{dt} \underline{j} = -\frac{d}{dt} (n_s \cdot e \cdot \underline{v}_s) = \frac{n_s \cdot e^2}{m} \cdot \underline{E}$$

$$n_s = \frac{N_s}{V} \quad \text{number of s.c. particles per volume}$$

But: induction

$$\underline{\nabla} \times \underline{E} = -\frac{1}{c} \frac{\partial \underline{B}}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t} \left(\underline{\nabla} \times \underline{j} + \frac{n_s e^2}{m c} \underline{B} \right) = 0$$

This means: Any change in B will be screened by corresponding currents - but this would also allow finite B to remain unchanged!

However, what if:

$$\underline{\nabla} \times \underline{j} + \frac{n_s e^2}{m c} \underline{B} = 0 \quad \left[\text{not just } \frac{\partial}{\partial t} \right]$$

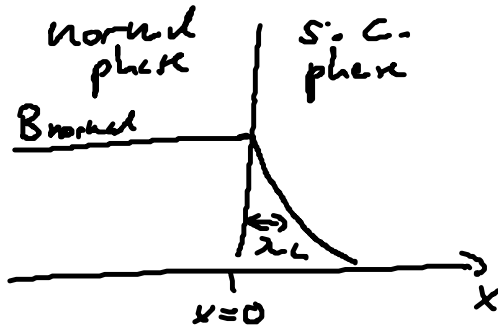
Then: through Maxwell's

$$\underline{\nabla} \times \underline{B} = \frac{4\pi}{c} \underline{j}$$

$$\Rightarrow \underline{\nabla} \times (\underline{\nabla} \times \underline{B}) + \frac{4\pi n_s e^2}{m c^2} \underline{B} = 0$$

2nd order d.e. in B.

For example: If



$$\underline{\nabla} \times (\underline{\nabla} \times \underline{B}) \rightarrow \frac{d^2}{dx^2} B(x)$$

$$\Rightarrow B''(x) + \frac{4\pi n_s e^2}{mc^2} B(x) = 0$$

$$\Rightarrow B(x) \sim e^{-\frac{x}{\lambda_L}}$$

$$\lambda_L = \sqrt{\frac{mc^2}{4\pi n_s e^2}}$$

London equation:

$$\underline{\nabla} \times \underline{j} + \frac{n_s e^2}{mc} \underline{B} = 0$$

λ_L "London penetration depth"

$$\sim 4.19 \text{ \AA} \left(\frac{\rho_s}{a_0}\right)^{3/2} \cdot \left(\frac{\hbar}{n_s}\right)$$

typically $\lambda_L \sim 10^2 - 10^3 \text{ \AA}$

Questions:

- Need phase transition of the electronic structure that "ignores" all scattering events
- Can we justify the microscopic London Eq. "beyond mere perfect conductivity"?
- What other interesting physics happens here?

For this lecture:

- principle of superconductivity
 - phenomenologically
 - microscopically: "weak coupling" (BCS)
"Bardeen - Cooper - Schrieffer"

Textbooks "beyond":

Ashcroft - Mermin - qualitative, well explained (old)

Tinkham "Introduction to superconductivity"
mathematical but s.t. not rigorous

Alexandrov (2006): "Theory of Superconductivity"
Clear, rigorous, up to most recent developments

Overview of superconductors:

- "Conventional superconductors"
(captured by BCS)
→ simple metals, compounds with $T_c < 30\text{K}$

- High- T_c S.C.

- Cu-O based ($\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$)
($\text{YBa}_2\text{Cu}_3\text{O}_7$)

~ 1987 ; $T_c \sim 30$

$\rightarrow \sim 160\text{K}$ (record today)

characterized by Cu-O planes (brittle)

antiferromagnetic insulators above T_c , gap $\sim 2\text{eV}$.

- "heavy fermion" S.C. CeCu_2Si_2
 $\text{PuCoGa}_5 \sim T_c \sim 18\text{K}$

- Lately: "R0FeAs" (2008)

LaOFeAs , F-doped $T_c \sim 30\text{K}$

present: $> 50\text{K}$ and going up

high critical fields $\sim 60\text{T}$

(Nb_3Sn : $\sim 25\text{T}$)

How to understand this zoo?

- Phenomenological (but general)

- Microscopic theory (next lecture
not "finished" field for high T_c)

Phenomenological Theories

London Theory

quantum mechanics

$$\underline{j}(\underline{r}) = \frac{ie}{2m} (\psi^* \underline{\nabla} \psi - \psi \underline{\nabla} \psi^*) - \frac{e^2}{m} \underline{A} \psi^* \psi$$

$$\underline{A} = \underline{A}(\underline{r}) \text{ vector potential} \quad \underline{\nabla} \times \underline{A} = \underline{B}$$

For electrons, this Ansatz would yield London's equation in the electron gas

Now, assume bosons as carriers: charge e^*
mass m^{**}

$$-\frac{1}{2m^{**}} (\underline{\nabla} + ie^* \underline{A})^2 \phi(\underline{r}) = E \phi(\underline{r})$$

Gauge: can choose $\underline{\nabla} \cdot \underline{A} = 0$ (Maxwell's gauge)

$$\Rightarrow \left(-\frac{1}{2m^{**}} \underline{\nabla}^2 - \frac{ie^*}{m^{**}} \underline{A} \cdot \underline{\nabla} + \frac{e^{*2} \underline{A}^2}{2m^{**}} \right) \phi(\underline{r}) = E \phi(\underline{r})$$

Small \underline{A} : lowest state

$$\phi(\underline{r}) = \phi_0(\underline{r}) + \phi_1(\underline{r})$$

↓
g.s. without
field

↓ weakly pert.
through \underline{A}

But: below T_c

all bosons tend to condense into ϕ

$$\text{w/o field } \phi_0(\underline{r}) = \frac{1}{\sqrt{V}} \quad ; \quad k=0, \epsilon_0=0$$

First order in \underline{A} , $\phi_1 =$

$$-\frac{1}{2m^{**}} \nabla^2 \phi_0 - \frac{1}{2m^{**}} \nabla^2 \phi_1 - \underbrace{\frac{ie}{m^{**}} \underline{A} \cdot \nabla \phi_0}_0$$

$$= E_0 \phi_1(\underline{r}) + E_1 \phi_0(\underline{r})$$

↓
0

↓
scalar; meaningful

$$1st\ order = \sim \nabla \cdot \underline{A} \approx 0$$

⇒ in first order:

$$\phi_1(\underline{r}) = 0 \text{ vanishes}$$

however:

$$j(\underline{r}) = \frac{ie^{*2}}{2m^{**}} \underbrace{(\psi^* \nabla \psi - \psi \nabla \psi^*)}_0 \text{ for } \phi_0 - \frac{e^{*2}}{m^{**}} \underline{A} \psi^* \psi$$

$$\Rightarrow j(\underline{r}) = -\frac{e^{*2} N_s}{m^{**} V} \underline{A}$$

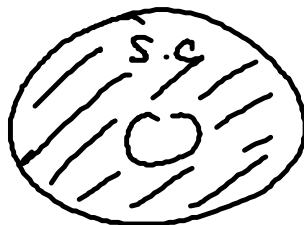
$$\text{and } \nabla \times j \sim \underline{B} \Rightarrow$$

$$\underline{B} + \lambda_L^2 \nabla \times \nabla \times \underline{B} = 0$$

London equation

Flux quantization

Imagine a superconductor with hole:



If condensate of bosons

→ collective wave function as $\phi(\underline{r}) = \sqrt{n_s} \cdot e^{i\Phi}$
phase factor
 $\bar{\Phi} = \bar{\Phi}(\underline{r})$

Magnetic flux through hole

$$\bar{\Phi}_B = \int_{\text{surface}} d\underline{s} \cdot \underline{B} = \oint_{\text{surface rim}} d\underline{l} A(\underline{r})$$

$\underline{B} = \underline{\nabla} \times \underline{A}$

But inside s.c.

can always find a contour where $\underline{B}, \underline{j} = 0$ (London)

$$0 = \underline{j}(\underline{r}) = \frac{ie^*}{2m^*} (\phi^* \underline{\nabla} \phi - \phi \underline{\nabla} \phi^*) - \frac{e^* \hbar^2}{m^*} \underline{A} \phi^* \phi$$

$$\Rightarrow A(\underline{r}) = - \frac{\underline{\nabla} \bar{\Phi}}{e^*} \leftarrow \text{phase}$$

But phase change along a contour:

$$\bar{\Phi}_B = - \oint_{\text{contour}} d\underline{l} \frac{\underline{\nabla} \bar{\Phi}}{e^*} = \frac{\oint \bar{\Phi}}{e^*}$$

Closed contour:

$$\oint \bar{\Phi} = 2\pi \cdot p \quad \text{where } p \text{ integer!}$$

$$\text{So } \bar{\Phi}_B = \frac{2\pi \hbar c}{e^*} p \quad (\text{in SI units})$$

$$\text{Expt: } e^* = 2e \quad \text{qu~~ant~~ized!}$$

Electron pairing?

Ogg 1946 + Schafroth et al. later:

Electron pairs as bosons.

Idea: Couple real-space electrons somehow?

to yield bosons in $S=0$ or $S=1$ states

NB: Theory of superfluid ^3He does this

BUT: - same theory with e^- yields $T_c \sim 10^4 \text{ K}$

• what could overcome real-space Coulomb interaction??

No answers! → Forgotten

Pippard non-local electrodynamics

Observation: Exp. penetration depths of \underline{B} , $j \neq \lambda_L$

$$\text{London: } j(\underline{r}) = -\frac{1}{4\pi} \frac{1}{\lambda_L^2} A(\underline{r})$$

$$\frac{1}{\lambda_L^2} = \frac{4\pi e^* h_s}{m^{**}}$$

Pippard: How about non-local interaction between carriers, field?

$$\text{set } j(\underline{r}) = -\frac{3}{16\pi^2} \frac{1}{\lambda_L^2} \int d^3r' (\underline{r}-\underline{r}') \cdot \frac{A(\underline{r}') \cdot (\underline{r}-\underline{r}')}{|\underline{r}-\underline{r}'|^4} e^{-|\underline{r}-\underline{r}'|/\xi}$$

ξ : Second length scale in the problem

"coherence length" $[\xi(r)]$

$\lambda_L \gg \xi$: $A(r)$ varies slowly in penetration range
for covered by $e^{-|r-r|/\xi}$
can simply take out $A(r)$ from $\int d^3r^{-1}$
 \rightarrow London limit

$\lambda_L \ll \xi$: Actual penetration depth λ_H is

where $A(r)$ varies

\rightarrow simply weigh London limit by
volume ratio $\frac{\lambda_H}{\xi}$

$$\Rightarrow j(r) \approx -\frac{\lambda_H}{\xi} \cdot \frac{e^{*2} n_s}{m^{*k}} A(r)$$

$$\Rightarrow \frac{1}{\lambda_H^2} \approx \frac{4\pi \lambda_H e^{*2} n_s}{\xi m^{*k}}$$

$$\Rightarrow \lambda_H = \lambda_L \left(\frac{\xi}{\lambda_L} \right)^{1/3} > \lambda_L$$