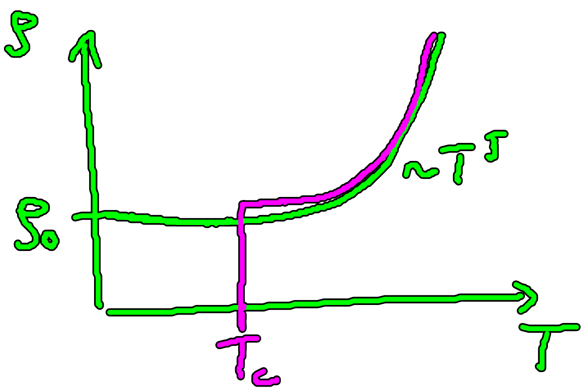
Chapter 10: Superconductivity

- So far: - Solids as a many-body "mix" of ions and electrons
  - Born-Oppenheimer approximation
  - independ.-e^- approximation
    → this always exists (DFT)
    → but these may not be meaningful physical objects in a solid ("quasiparticles") to look at

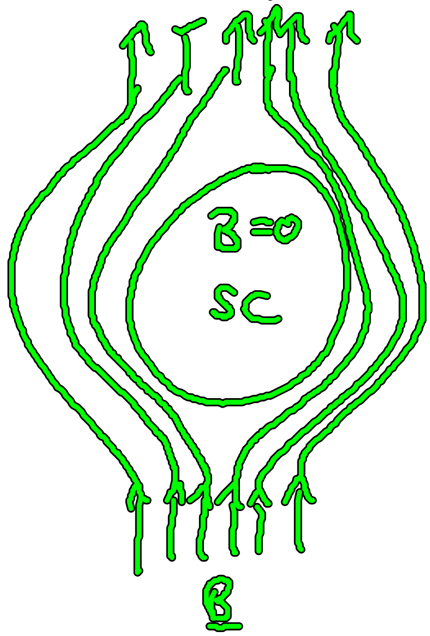
- Superconductivity: 1911 H. Kamerlingh Onnes
  below T_c, resistivity of Hg, Sn, Pb drops to zero abruptly:



	T_c
Hg	4.15 K
Sn	3.72 K
Pb	7.19 K

- Perfect conductivity should not exist at finite T
- Record for persistent ring current (Ashcroft): $2\frac{1}{2}$ yrs.

1933 Meissner–Ochsenfeld: Perfect diamagnets



$\underline{B} = \mu_0 \mu \, \underline{H}$

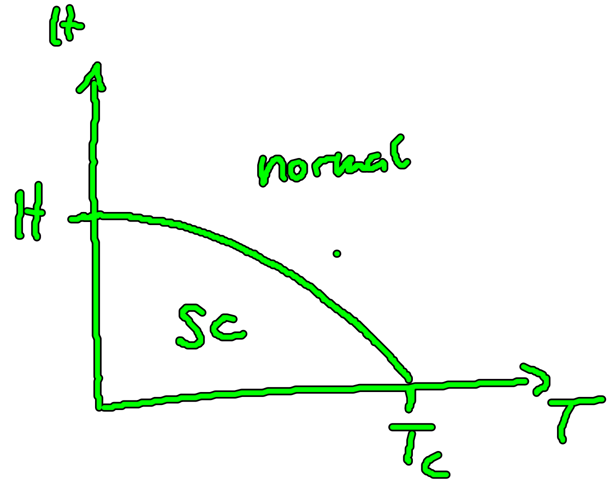
$= \mu_0 (1+\chi) \underline{H} = 0$

$\chi = -1$

works only up to a certain critical field H_c

→ SC reverts to normal conductivity beyond $H > H_c$

"phase diagram"



This is <u>odd</u>:

Consider a perfect conductor:

Drift model without resistance:

$$m \cdot \frac{d\underline{v}_s}{dt} = -e \cdot \underline{E}$$

$$\frac{d}{dt} \underline{j} = -\frac{d}{dt}\left(n_s \cdot e \cdot \underline{v}_s\right) = \frac{n_s \cdot e^2}{m} \cdot \underline{E}$$

$$n_s = \frac{N_s}{V}$$ number of s.c. particles per volume

But: induction

$$\underline{\nabla} \times \underline{E} = -\frac{1}{c}\frac{\partial B}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t}\left(\underline{\nabla} \times \dot{j} + \frac{n_s e^2}{mc}\underline{B}\right) = 0$$

This means: Any <u>change</u> in $\underline{B}$ will be screened by corresponding currents - but this would <u>also</u> allow finite $\underline{B}$ to remain unchanged!

However, what if:

$$\underline{\nabla} \times j + \frac{n_s e^2}{mc}\underline{B} = 0 \qquad \left[\text{\underline{not} just } \frac{\partial}{\partial t}\right]$$

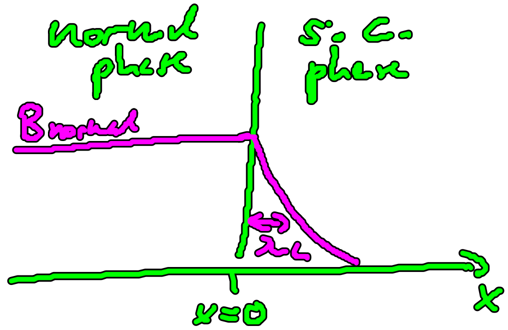
Then: through Maxwell's

$$\underline{\nabla} \times \underline{B} = \frac{4\pi}{c}\dot{j}$$

$$\Rightarrow \underline{\nabla} \times (\underline{\nabla} \times \underline{B}) + \frac{4\pi n_s e^2}{mc^2}\underline{B} = 0$$

2nd order d.e. in $\underline{B}$.

For example: If



$$\underline{\nabla} \times (\underline{\nabla} \times \underline{B}) \rightarrow \frac{d^2}{dx^2} B(x)$$

$$\Rightarrow B''(x) + \frac{4\pi n_s e^2}{mc^2} B(x) = 0$$

$$\Rightarrow B(x) \sim e^{-\frac{x}{\lambda_L}}$$

$$\lambda_L = \sqrt{\frac{mc^2}{4\pi n_s e^2}}$$

London equation: $\boxed{\underline{\nabla} \times \underline{j} + \frac{n_s e^2}{mc} \underline{B} = 0}$

λ_L "London penetration depth"

$\sim 41.9 \text{Å} \left(\frac{r_s}{a_0}\right)^{3/2} \cdot \left(\frac{n}{n_s}\right)$

; typically $\lambda_L \sim 10^2 - 10^3 \text{Å}$

Questions:

- Need <u>phase transition</u> of the electronic structure that "ignores" all scattering events
- Can we justify the microscopic London Eq. "beyond mere perfect conductivity"
- What other interesting physics happens here?

For this lecture:

- principle of superconductivity
  - phenomenologically
  - microscopically: "Weak coupling" (BCS) "Bardeen - Cooper - Schrieffer"

Textbooks "beyond":

Ashcroft - Mermin – qualitative, well explained (old)

Tinkham "Introduction to superconductivity".
mathematical but s.t. not rigorous

Alexandrov (2006): "Theory of Superconductivity"
Clear, rigorous, up to most recent developments

Overview of superconductors:

- "Conventional superconductors" (captured by BCS)
  → simple metals, compounds with $T_c < 30K$

– High-T_c S.C.

• Cu–O based ($La_{2-x}Ba_xCuO_4$)
($YBa_2Cu_3O_7$)

~1987 ; $T_c \sim 30$
→ ~160 K (record today)

characterized by Cu–O planes (brittle)
antiferromagnetic insulators above T_c, gap ~2eV.

• "heavy fermion" s.c. $CeCu_2Si_2$
$PuCoGa_5$ ~ $T_c \sim 18K$

• Lately : "R O Fe As" (2008)

LaOFeAs , F-doped $T_c \sim 30K$
present : >50 K and going up
high critical fields ~60 T
(Nb_3Sn : ~25 T)

<u>How to understand this zoo ?</u>

– Phenomenological (but general)

– Microscopic theory (next lecture
not "finished" field for high T_c)

Phenomenological Theories

London Theory

quantum mechanics

$$\underline{j}(\underline{r}) = \frac{ie}{2m}\left(\psi^* \underline{\nabla}\psi - \psi\underline{\nabla}\psi^*\right) - \frac{e^2}{m}\underline{A}\psi^*\psi$$

$\underline{A} = \underline{A}(\underline{r})$ vector potential $\quad \underline{\nabla}\times\underline{A} = \underline{B}$

For electrons, this Ansatz would yield Landau diamagnetism in the electron gas

Now, assume <u>bosons</u> as carriers: charge e^*
mass m^{**}

$$-\frac{1}{2m^{**}}\left(\underline{\nabla} + ie^*\underline{A}\right)^2 \phi(\underline{r}) = E\,\phi(\underline{r})$$

Gauge: can choose $\underline{\nabla}\cdot\underline{A} = 0$ (Maxwell's gauge)

$$\Rightarrow \left(-\frac{1}{2m^{**}}\underline{\nabla}^2 - \frac{ie^*}{m^{**}}\underline{A}\cdot\underline{\nabla} + \frac{e^*\underline{A}^2}{2m^*}\right)\phi(\underline{r}) = E\,\phi(\underline{r})$$

Small $\underline{A}$: lowest state

$$\phi(\underline{r}) = \underbrace{\phi_0(\underline{r})}_{\text{g.s. without field}} + \underbrace{\phi_1(\underline{r})}_{\text{weakly pert. through } \underline{A}}$$

<u>But</u>: below T_c
all bosons tend to condense into ϕ

w/o field $\phi_0(\underline{r}) = \frac{1}{\sqrt{V}}$; $k=0$, $\epsilon_0 = 0$

First order in $\underline{A}$, ϕ_1 =

$$-\underbrace{\frac{1}{2m^{**}}\underline{\nabla}^2\phi_0}_{0} - \frac{1}{2m^{**}}\underline{\nabla}^2\phi_1 - \underbrace{\frac{ie}{m^{**}}\underline{A}\cdot\underline{\nabla}\phi_0}_{0}$$

$$= \underbrace{E_0}_{0}\,\phi_1(\underline{r}) + E_1\,\phi_0(\underline{r})$$

scalar; meaningful

1st order : $\sim \underline{\nabla}\cdot\underline{A} \approx 0$

$\Rightarrow$ in first order:

$\phi_1(\underline{r}) = 0$ vanishes

however:

$$\underline{j}(\underline{r}) = \frac{ie^*}{2m^{**}}\underbrace{\left(\psi^*\underline{\nabla}\psi - \psi\underline{\nabla}\psi^*\right)}_{0 \text{ for } \phi_0} - \frac{e^{*2}}{m^{**}}\underline{A}\,\psi^*\psi$$

$$\Rightarrow \quad \underline{j}(\underline{r}) = -\frac{e^{*2}N_s}{m^{**}V}\underline{A}$$

and $\underline{\nabla}\times\underline{j} \sim \underline{B}$ $\Rightarrow$

$$\underline{B} + \lambda_L^2\,\underline{\nabla}\times\underline{\nabla}\times\underline{B} = 0$$

London equation

Flux quantization

Imagine a superconductor with hole:

If condensate of bosons

$\rightarrow$ collective wave function as $\phi(\underline{r}) = \sqrt{n_s} \cdot \underbrace{e^{i\Phi}}_{\text{phase factor}}$

$\Phi = \Phi(\underline{r})$

Magnetic flux through hole

$$\Phi_B = \int_{\text{surface}} d\underline{s} \cdot \underline{B} \overset{\uparrow}{=} \oint_{\substack{\text{surface}\\ \text{rim}}} d\underline{l}\, A(\underline{r})$$

$\underline{B} = \underline{\nabla} \times \underline{A}$

But inside s.c.

can always find a contour where $\underline{B}, j = 0$ (London)

$$0 = j(\underline{r}) = \frac{ie^*}{2m^{**}} \left(\phi^* \underline{\nabla} \phi - \phi \underline{\nabla} \phi^*\right) - \frac{e^{*2}}{m^{**}} \underline{A}\, \phi^* \phi$$

$$\Rightarrow \quad A(\underline{r}) = -\frac{\underline{\nabla}\Phi}{e^*} \leftarrow \underline{\text{phase}}$$

But phase change along a contour:

$$\Phi_B = -\oint_{\text{contour}} d\underline{l}\, \frac{\underline{\nabla}\Phi}{e^*} = \frac{\delta\Phi}{e^*}$$

Closed contour:

$$\delta\Phi = 2\pi \cdot p \quad \text{where } p \text{ integer!}$$

So $\Phi_B = \dfrac{2\pi\hbar c}{e^*}\, p$ (in SI units)

$\underline{\text{quantized}}$!

Expt: $e^* = 2e$

Electron pairing ?

Ogg 1946 + Schafroth et al. later:

Electron pairs as bosons.

Idea: Couple real-space electrons somehow?

to yield bosons in $S=0$ or $S=1$ states

NB: Theory of superfluid ^3He does this

BUT: • same theory with e^- yields $T_c \sim 10^4 K$

• what could overcome real-space Coulomb

interaction ??

No answers! → Forgotten

Pippard non-local electrodynamics

Observation: Exp. penetration depths of $\underline{B}, j \neq \lambda_L$

London: $$j(\underline{r}) = -\frac{1}{4\pi}\frac{1}{\lambda_L^2} A(\underline{r})$$

$$\frac{1}{\lambda_L^2} = \frac{4\pi e^* n_s}{m^{**}}$$

Pippard: How about non-local interaction between carriers, field?

set $$j(\underline{r}) = -\frac{3}{16\pi^2}\frac{1}{\lambda_L^2 \xi}\int d^3r' \, (\underline{r}-\underline{r}') \cdot$$

$$\cdot \frac{A(\underline{r}')\cdot(\underline{r}-\underline{r}')}{|\underline{r}-\underline{r}'|^4} e^{-|\underline{r}-\underline{r}'|/\xi}$$

ξ : Second length scale in the problem
"coherence length" $[\xi(0)]$

$\lambda_L \gg \xi$: $\underline{A}(\underline{r})$ varies slowly in penetration range
for covered by $e^{-|\underline{r}'-\underline{r}|/\xi}$
can simply take out $\underline{A}(\underline{r})$ from $\int d^3r'$
$\rightarrow$ London limit

$\lambda_L \ll \xi$: Actual penetration depth λ_H ;
where $A(\underline{r})$ varies
$\rightarrow$ simply weigh London limit by
volume ratio $\frac{\lambda_H}{\xi}$

$$\Rightarrow \quad j(\underline{r}) \simeq -\frac{\lambda_H}{\xi} \cdot \frac{e^{*2} n_s}{m^{**}} \underline{A}(\underline{r})$$

$$\Rightarrow \quad \frac{1}{\lambda_H^2} \simeq \frac{4\pi \lambda_H e^{*} n_s}{\xi m^{**}}$$

$$\Rightarrow \quad \lambda_H = \lambda_L \left(\frac{\xi}{\lambda_L}\right)^{1/3} > \lambda_L$$