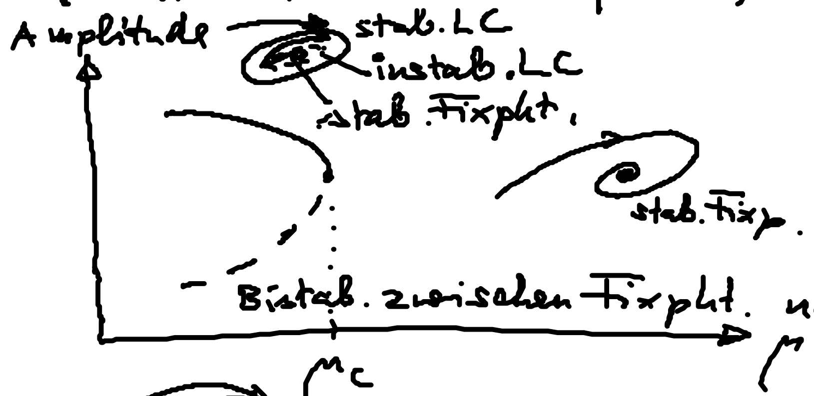
(C) Lokale Bifurkation von Grenzzyklen

Startpunkt jetzt: Grenzzyklus, nicht Fixpkt.
keine einfache lin. Stabilitätsanalyse

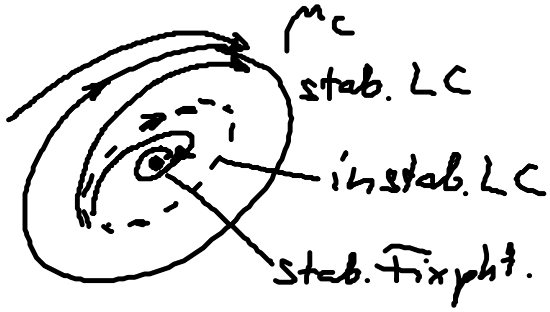
(C1) Sattel-Knoten-Bifurkation eines Grenzzyklus

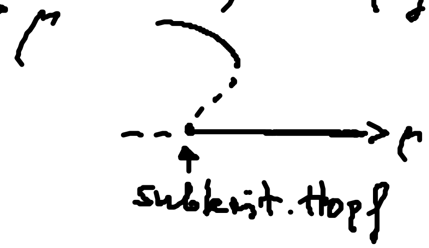
(Kondensation von Pfaden, fold bifurcation of LC)



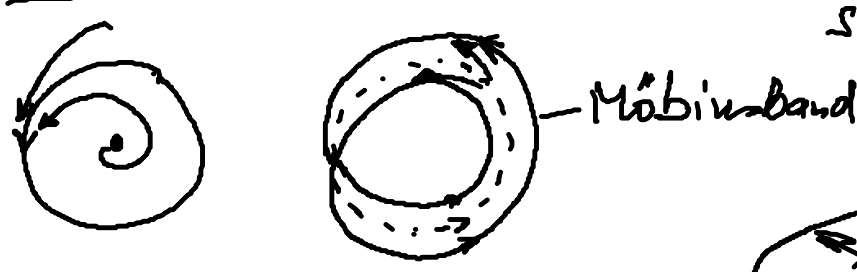
$\boxed{\begin{array}{l} \text{Amplitude} \neq 0 \\ \text{Frequenz} \neq 0 \end{array}}$

Bistab. zwischen Fixpkt. und LC, häufig:





(C2) Perioden-Verdopplung (flip-Bifurkation, subharmon. Bif.)



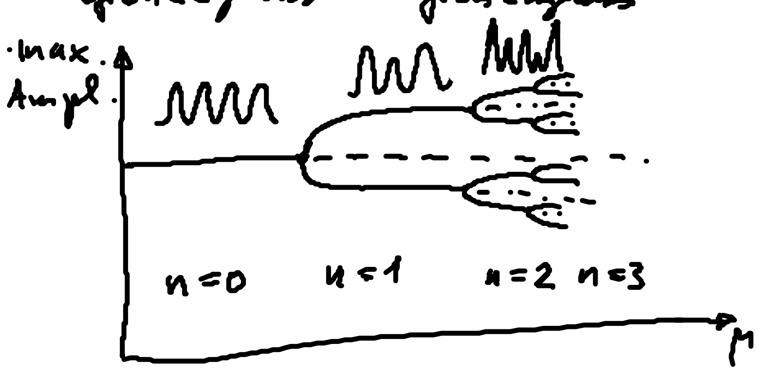
Periode-1 Grenzzyklus

Periode-2 Grenzzyklus

$\boxed{\text{mind. 3D Phasenraum}}$

phase flip π nach 1 Umlauf

Torsion benachbarter Traj.!



Floquet-Exp. $\Lambda = \lambda + i\omega$

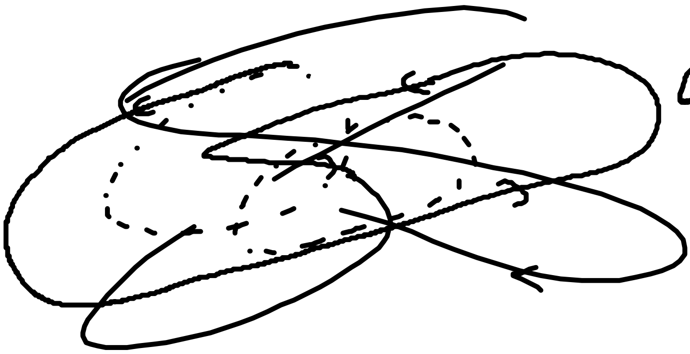
Bif.: $\lambda = 0$, $\omega T = \pi$

$\Rightarrow$ Floquet-Mult. $\boxed{\mu = e^{\Lambda T} = e^{i\pi}}$

$T \quad 2T \quad 4T \cdots 2^n T$ $\qquad\qquad$ $\boxed{\quad = -1}$

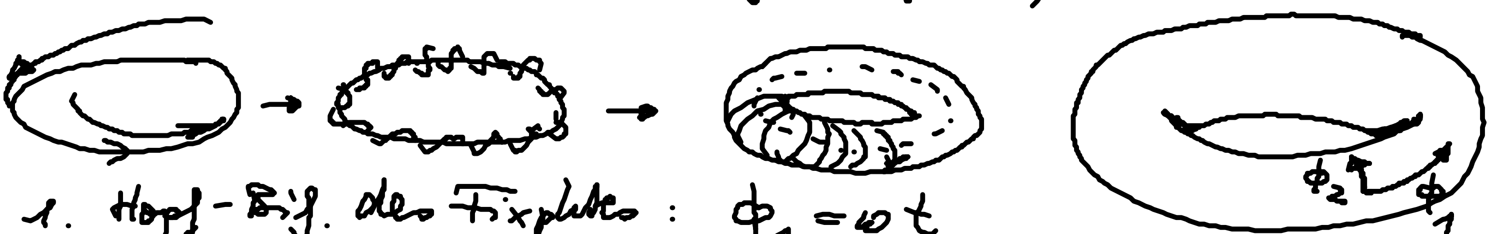
häufig Periodenverdopplungskaskade → <u>Chaos</u>
(Feigenbaum-Szenario)

⇒ unendlich viele instabile period. Orbits der Perioden $2^n T$
($n = 0, 1, 2, \ldots$)



(C3) Sacker-Neimark-Bifurkation

(Sekundäre Hopf-Bif. eines Grenzzyklus)



1. Hopf-Bif. des Fixpkts : $\phi_1 = \omega_1 t$
2. Hopf-Bif. des LC : $\phi_2 = \omega_2 t$

LC ⟶ 2-Torus $\qquad$ $\boxed{\text{mind. 3D Phasenraum}}$

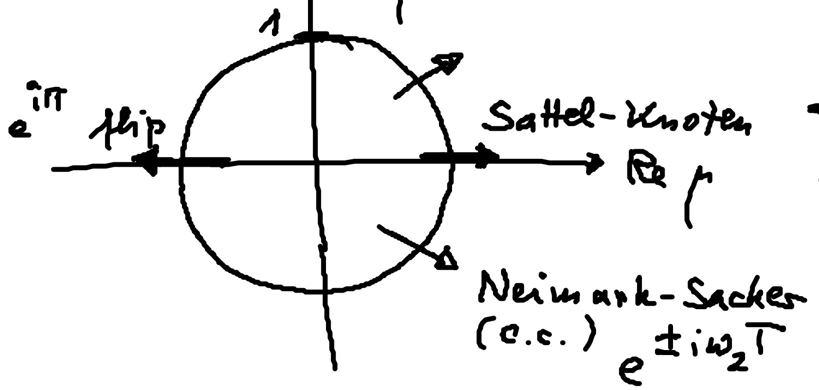
inkommensurable Frequenzen $\frac{\omega_1}{\omega_2} \notin \mathbb{Q}$
(<u>quasiperiodisch</u>) $\qquad$ irrational

⇒ Trajektorie schließt sich nicht, liegt dicht auf dem Torus

falls $\frac{\omega_1}{\omega_2} \in \mathbb{Q}$ ⇒ geschlossener Orbit = LC
(frequency locking)
Modenkopplung

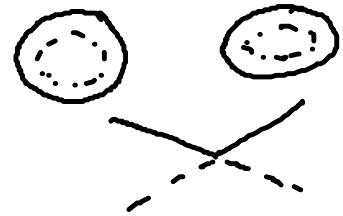
<u>Instabilität von Grenzzyklen</u> : Floquet-Multiplikator

$\uparrow \mathrm{Im}\,\mu$ $\qquad\qquad$ $\mu = e^{\Lambda T}$



Stab.-grenze $|\mu| = 1$

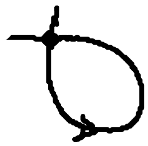
oder transkrit.



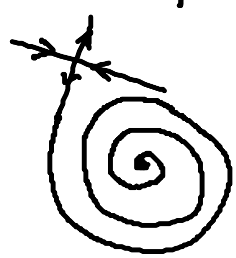
(D) globale Bifurkation von Grenzzyklen

globale qualitative Änderung des Phasenporträts,

z.B.



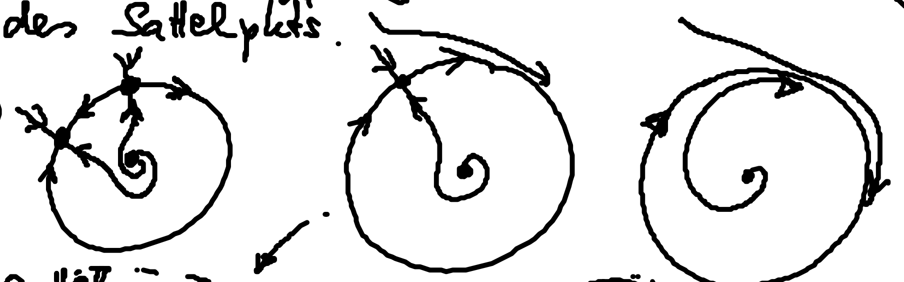
homoklin. Orbit $\longrightarrow$

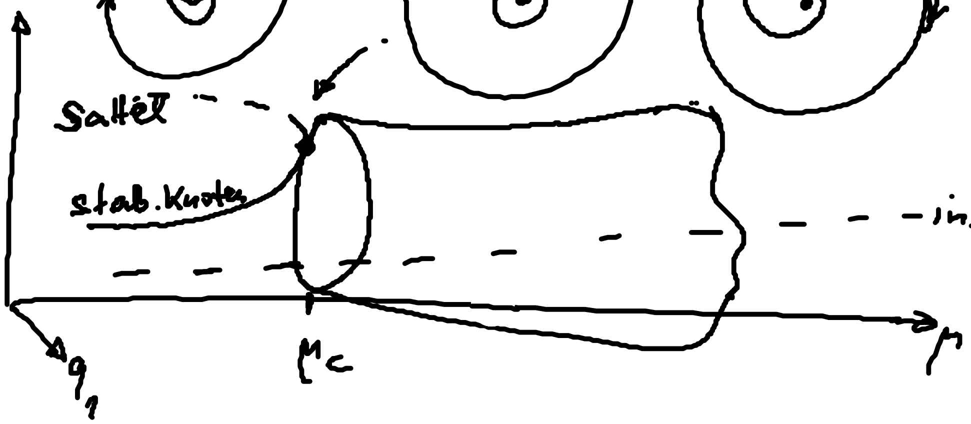


kein loop

invar. Mannigfaltigk.
des Sattelpkts.

(D1)



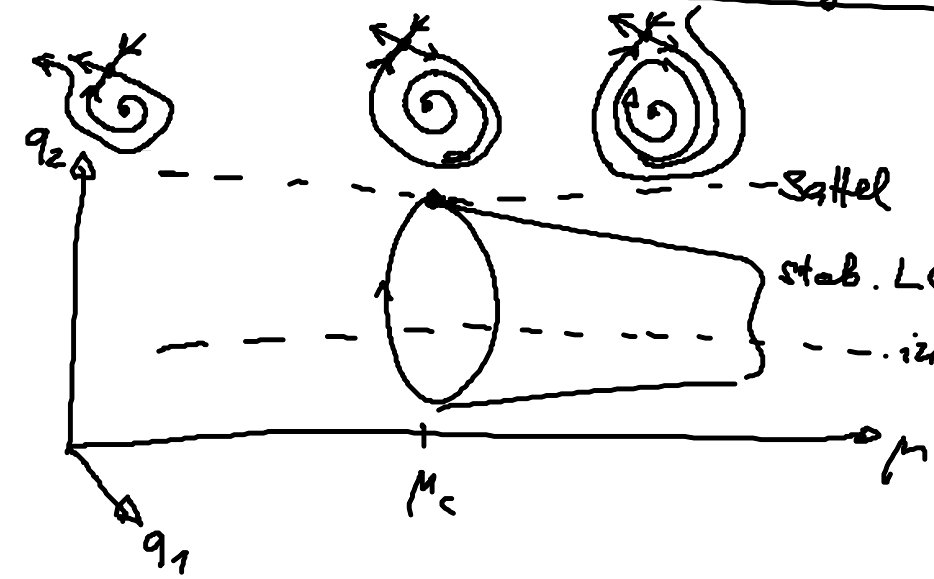


(D1) Sattel-Knoten-Bif. auf einem Grenzzyklus (Omega explosion)

saddle-node infinite period (SNIPER)
saddle-node on invariant cycle (SNIC)

$\boxed{\begin{array}{l} \text{Amplitude} \neq 0 \\ \text{Frequ.} \to 0 \end{array}}$

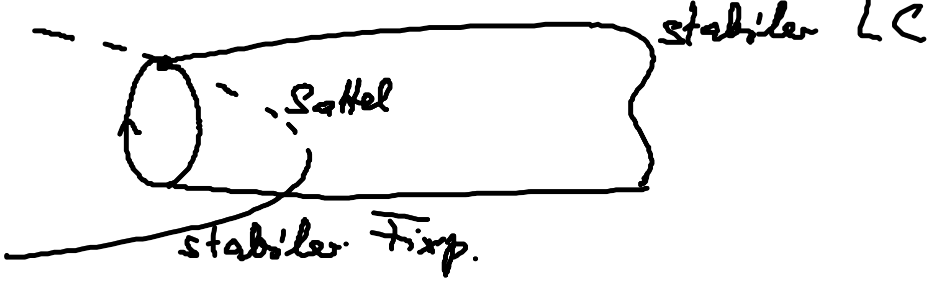
(D2) Homokline Bif. (blue-sky catastrophe)

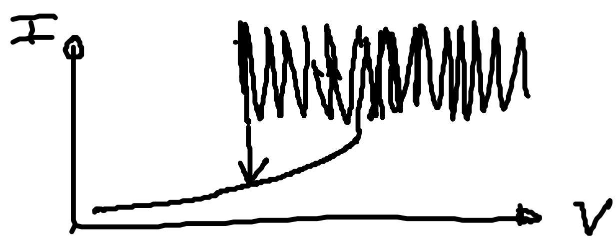


Sattelpkt. kollidiert mit LC: homokliner Orbit (saddle-to-saddle loop)

Ampl. $\neq 0$
Frequ. $\to 0$

häufig Bistab. zwischen Osz. u. Fixpkt., z.B.





Einfaches generisches Modell für SNIPER

(Ditzinger, Ning, Hu: PRE 50, 3508 (94)
Hu, Ditzinger, Ning, Haken: PRL 71, 87 (93)
Hizanidis, Aust, Schöll; Int. J. Bif. Chaos 18, 1759 (08)

$$\boxed{\begin{aligned} \dot{x} &= x(1-x^2-y^2) + y(x-b) \\ \dot{y} &= y(1-x^2-y^2) - x(x-b) \end{aligned}}$$

in Polarkoord. $x = r\cos\varphi$, $y = r\sin\varphi$:

$$\boxed{\begin{aligned} \dot{r} &= r(1-r^2) \\ \dot{\varphi} &= b - r\cos\varphi \end{aligned}} \quad b>0 \qquad \begin{pmatrix} \delta\dot{r} \\ \delta\dot{\varphi} \end{pmatrix} = \begin{pmatrix} 1-3r^2 & 0 \\ -\cos\varphi & r\sin\varphi \end{pmatrix}$$

Fixpkte: $r=0$ (immer instab. Fokus, ü)

$r = 1$, $b = \cos\varphi \Rightarrow \varphi = \arccos b \; (b<1)$

für $b>1$: LC mit $r=1$, $\dot\varphi = b - \cos\varphi > 0 \;\forall\varphi$

$$\frac{2\pi}{\sqrt{b^2-1}} = \int_0^{2\pi} \frac{d\varphi}{b-\cos\varphi} = \int_0^T dt = T$$

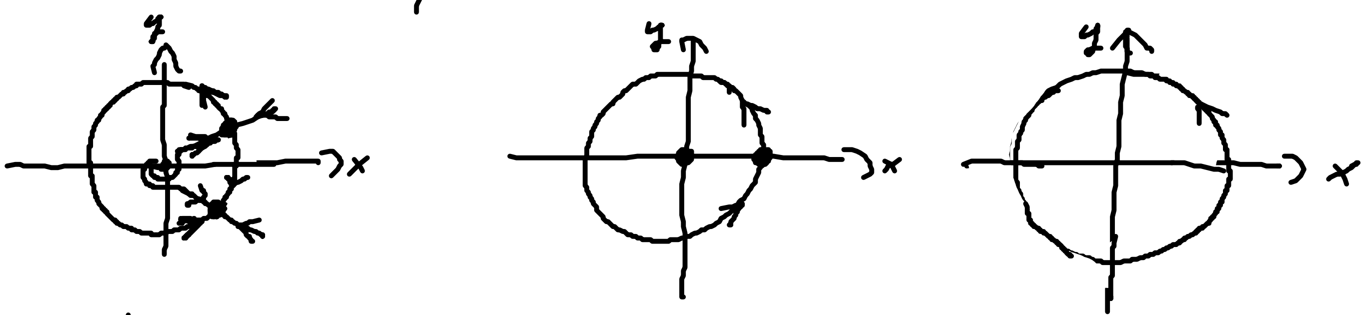
Bif.-pkt.: $b = 1 \Rightarrow T \to \infty$ (Frequ. $\to 0$)

Fixpkte auf dem Kreis $r=1$:

$(x^*, y^*) = (b, +\sqrt{1-b^2})$ Sattelpkt. $\left(\Lambda = \begin{cases} -2 \\ \sqrt{1-b^2} \end{cases}\right)$

$= (b, -\sqrt{1-b^2})$ stab. Kn. $\left(\Lambda = \begin{cases} -2 \\ -\sqrt{1-b^2} \end{cases}\right)$

Sattel-Knoten-Bif.: $b = 1$



$b<1$ $\qquad$ $b=1$ $\qquad$ $b>1$

(Anregbarkeit Typ I)

(E) Bifurkation von räumlichen Mustern

$$\boxed{\frac{\partial}{\partial t} \underline{q}(x,t) = F(\underline{q}, \mu) + D\Delta q}$$

Diff.konst. D

Lineare Stab. $\delta q \sim e^{\lambda t} e^{i\underline{k}\underline{x}}$
der Fixpkte. (räuml. homogen)

$\Rightarrow$ Dispersionsrel. $\lambda(\underline{k})$

$\Rightarrow$ $\mathrm{Re}\,\lambda(\underline{k})$ < 0 stabil

> 0 instabil

$= 0$ Bif. von räumlich-period. Lösungen mit Wellenvektor $\underline{k}$