

$$\mathcal{L}(\psi, \psi^*, \partial_\mu \psi, \partial_\mu \psi^*) = \frac{\hbar^2}{2m} \left( (\partial_0 \psi^*) (\partial_0 \psi) - (\partial_x \psi^*) (\partial_x \psi) - \lambda^2 \psi^* \psi \right)$$

$$0 = \frac{\delta \mathcal{L}}{\delta \psi^*} - \partial_\mu \left( \frac{\delta \mathcal{L}}{\delta (\partial_\mu \psi^*)} \right)$$

$$= \frac{\delta \mathcal{L}}{\delta \psi^*} - \partial_0 \left( \frac{\delta \mathcal{L}}{\delta (\partial_0 \psi^*)} \right) - \partial_x \left( \frac{\delta \mathcal{L}}{\delta (\partial_x \psi^*)} \right) - \partial_y \left( \frac{\delta \mathcal{L}}{\delta (\partial_y \psi^*)} \right) - \partial_z \left( \frac{\delta \mathcal{L}}{\delta (\partial_z \psi^*)} \right)$$

$$\frac{\delta \mathcal{L}}{\delta \psi^*} = -\frac{\hbar^2}{2m} \lambda^2 \psi$$

$$\frac{\delta \mathcal{L}}{\delta (\partial_0 \psi^*)} = \frac{\hbar^2}{2m} \partial_0 \psi$$

$$\frac{\delta \mathcal{L}}{\delta (\partial_x \psi^*)} = \frac{\hbar^2}{2m} \partial_x \psi$$

$$0 = \left( -\frac{\hbar^2}{2m} \lambda^2 - \frac{\hbar^2}{2m} \partial_0^2 + \frac{\hbar^2}{2m} \partial_x^2 \right) \psi$$

$$\Rightarrow 0 = \left( \partial_x^2 - \frac{1}{c^2} \partial_t^2 - \lambda^2 \right) \psi$$

Dies ist die Klein-Gordon-Gl (1d), sie beschreibt Spin-0 Teilchen.

**A2** Pauli-Gleichung

$$H = \left( H_0 - \frac{q}{2m} \underbrace{(\vec{F} \times \vec{p})}_{\vec{L}} \cdot \vec{B} \right) \mathbb{1} - \frac{q}{m} \frac{\hbar}{2} \vec{S} \cdot \vec{B}$$

$$\vec{B} = B \vec{e}_z \Rightarrow \vec{L} \cdot \vec{B} = B L_z$$

$$\vec{S} \cdot \vec{B} = B S_z$$

$$H |n, m_l, m_s\rangle = \left[ H_0 - \frac{q}{2m} B l_z \right] |n, m_l, m_s\rangle$$

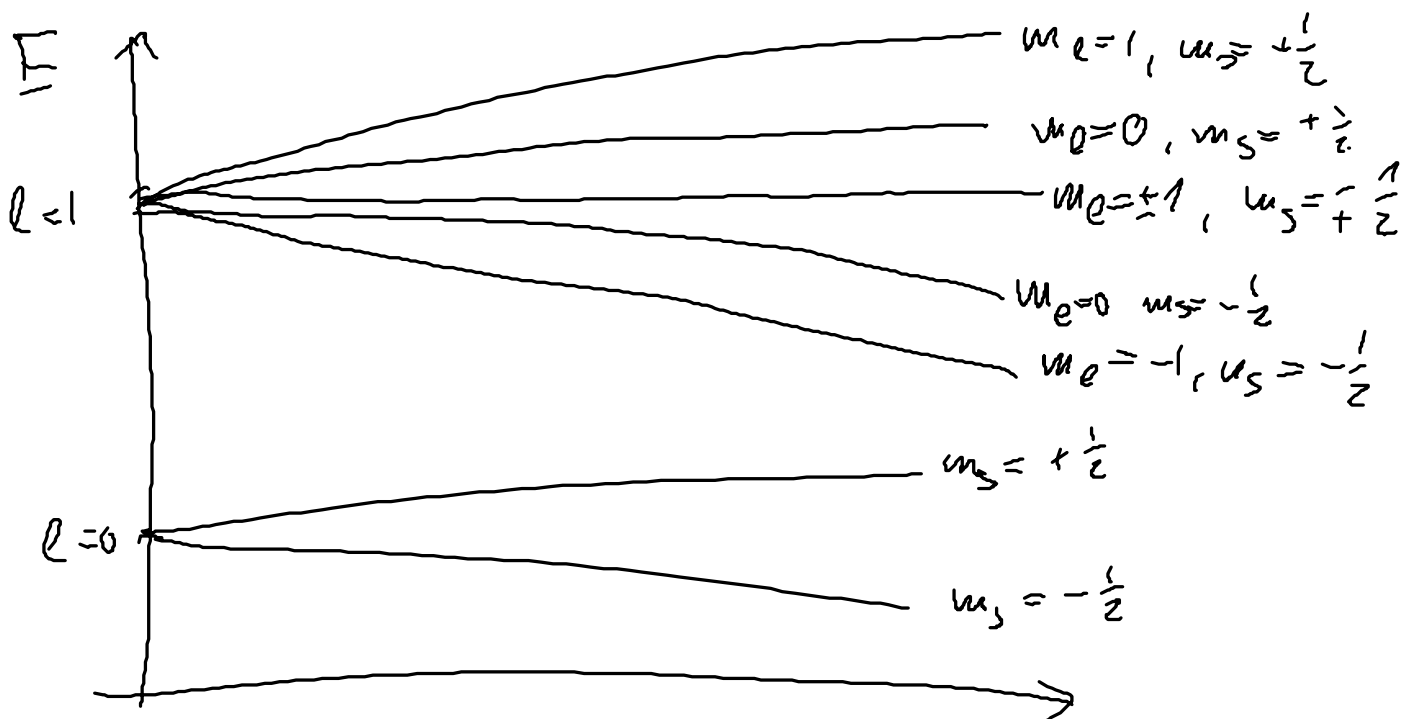
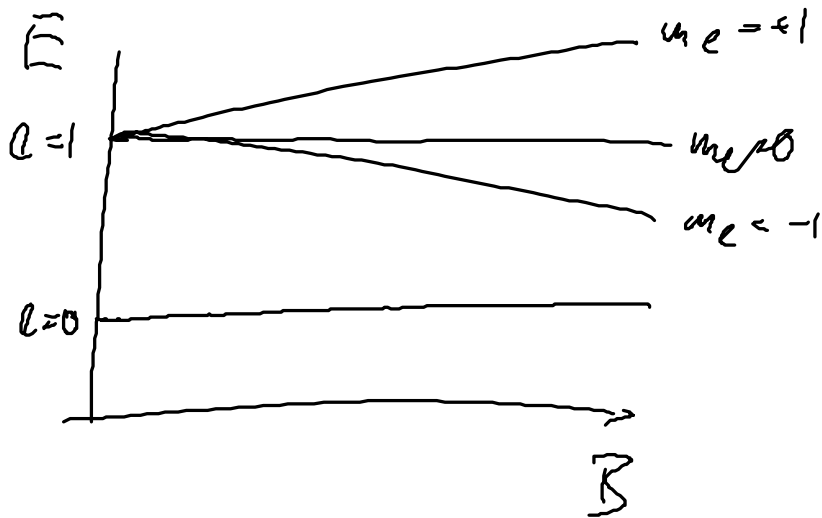
$$= \left[ -\frac{\hbar^2}{2m} - \frac{q}{2m} B \hbar m_l - \frac{q}{m} B \hbar m_s \right] |n, m_l, m_s\rangle$$

$$\Rightarrow E = -\frac{\hbar^2}{2m} - \frac{q B \hbar}{2m} (m_l + 2m_s)$$

$$\Rightarrow E = -\frac{\hbar^2}{2m} + \frac{e B \hbar}{2m} (m_l + 2m_s)$$

$q = -e$

$S = 0$ , ohne Spin



→ Energieaufspaltung nach Spin & Bahndrehimpuls

**A3** gestörter harmonischer Oszillator

$$b = \frac{1}{\sqrt{2}} \left( \sqrt{\frac{m\omega}{\hbar}} x + i \frac{1}{\sqrt{2m\omega}} p_x \right)$$

$$\Rightarrow x = \sqrt{\frac{\hbar}{2m\omega}} (b^\dagger + b)$$

$$(i) \Rightarrow V = \frac{\hbar\omega}{\sqrt{2}} (b^\dagger + b)$$

$$\langle m | V | n \rangle = \frac{\hbar\omega}{\sqrt{2}} \left( \sqrt{nm} \delta_{m, n+1} + \sqrt{n} \delta_{m, n-1} \right)$$

$$\begin{aligned} (ii) | \psi_n \rangle &\approx | \varphi_n \rangle^0 + | \varphi_n \rangle^{(1)} \\ &= | \varphi_n \rangle^0 + \sum_{m \neq n} \frac{\langle m | \lambda V | n \rangle}{E_n^0 - E_m^0} | m \rangle^0 \\ &= | \varphi_n \rangle^0 - \lambda \sqrt{\frac{n+1}{2}} | n+1 \rangle + \lambda \sqrt{\frac{n}{2}} | n-1 \rangle \end{aligned}$$

$$\begin{aligned} (iii) E_n &\approx E_n^0 + E_n^{(1)} + E_n^{(2)} \\ &= \hbar\omega \left( n + \frac{1}{2} \right) + \sum_{m \neq n} \frac{\langle m | \lambda V | n \rangle^2}{E_n^0 - E_m^0} \\ &= \hbar\omega \left( n + \frac{1}{2} - \frac{\lambda^2}{2} \right) \end{aligned}$$

**A4** Zwei-Niveaus-System

$$H = \sum_i \hbar \nu_i a_i^\dagger a_i + \sum_{i,j} \hbar \Omega_{ij} a_i^\dagger a_j$$

$$= \hbar \nu_1 a_1^\dagger a_1 + \hbar \nu_2 a_2^\dagger a_2 + \hbar \Omega_{12} a_1^\dagger a_2 + \hbar \Omega_{21} a_2^\dagger a_1$$



$$(i) E_g = 2 E_0 + E_1 = \frac{5}{2} \hbar \omega$$

(ii) 2-fakt, da  $n=1$  entartet Spin ↑ o. ↓

(iii)

$$\psi_g = \frac{1}{\sqrt{3!}} \begin{vmatrix} \varphi_{\alpha_1}(r_1) & \varphi_{\alpha_2}(r_1) & \varphi_{\alpha_3}(r_1) \\ \varphi_{\alpha_1}(r_2) & \varphi_{\alpha_2}(r_2) & \varphi_{\alpha_3}(r_2) \\ \varphi_{\alpha_1}(r_3) & \varphi_{\alpha_2}(r_3) & \varphi_{\alpha_3}(r_3) \end{vmatrix}$$

$$= \frac{1}{\sqrt{6}} \left( \varphi_{\alpha_1}(r_1) \varphi_{\alpha_2}(r_2) \varphi_{\alpha_3}(r_3) \right. \\ \left. + \varphi_{\alpha_2}(r_1) \varphi_{\alpha_3}(r_2) \varphi_{\alpha_1}(r_3) \right. \\ \left. + \varphi_{\alpha_3}(r_1) \varphi_{\alpha_1}(r_2) \varphi_{\alpha_2}(r_3) \right. \\ \left. - \varphi_{\alpha_3}(r_1) \varphi_{\alpha_2}(r_2) \varphi_{\alpha_1}(r_3) \right. \\ \left. - \varphi_{\alpha_2}(r_1) \varphi_{\alpha_3}(r_2) \varphi_{\alpha_1}(r_3) \right. \\ \left. - \varphi_{\alpha_1}(r_1) \varphi_{\alpha_3}(r_2) \varphi_{\alpha_2}(r_3) \right)$$

$$\text{dabei } (\alpha_1, \alpha_2, \alpha_3) = (0 \uparrow, 0 \downarrow, 1 \uparrow)$$

$$\text{oder } (\alpha_1, \alpha_2, \alpha_3) = (0 \uparrow, 0 \downarrow, 1 \downarrow)$$