## 2.3. Magnetostat. Feldgleichungen

$$\underline{F} = q\underline{v} \times \underline{B}(\underline{r}) \qquad \text{Lorentzkraft}$$

$$\boxed{\underline{B}(\underline{r}) = \frac{\mu_0}{4\pi} \int d^3r' \, \underline{j}(\underline{r}') \times \frac{\underline{r}-\underline{r}'}{|\underline{r}-\underline{r}'|^3}} \qquad \text{Ampère-Gesetz}$$

Mit dem Vektorpotential

$$\underline{A}(\underline{r}) = \frac{\mu_0}{4\pi} \int_{\mathbb{R}^3} d^3_{r'} \frac{\underline{j}(\underline{r}')}{|\underline{r}-\underline{r}'|}$$

läßt sich

$$\boxed{\underline{B}(\underline{r}) = \text{rot}\ \underline{A}}$$

schreiben.

<u>Beweis</u>:

$$\underline{\nabla} \times \underline{A} = \frac{\mu_0}{4\pi} \int d^3_{r'} \left( \underline{\nabla}_r \frac{1}{|\underline{r}-\underline{r}'|} \right) \times \underline{j}(\underline{r}')$$

$$- \frac{(\underline{r}-\underline{r}')}{|\underline{r}-\underline{r}'|^3}$$

$$= \frac{\mu_0}{4\pi} \int d^3_{r'} \underline{j}(\underline{r}') \times \frac{(\underline{r}-\underline{r}')}{|\underline{r}-\underline{r}'|^3} \overset{!}{=} \underline{B}$$

Folgendes ist äquivalent:

(i) $\underline{B}(\underline{r}) = \text{rot}\ \underline{A}$ (Vektorpot.)

$\Updownarrow$

(ii) $\boxed{\text{div}\ \underline{B} = 0}$ $\qquad \underline{\nabla} \cdot (\underline{\nabla} \times \underline{A}) =$

Es gibt keine Quelle der magn. Induktion.
(„magn. Ladungen" = „Monopol")

$\Updownarrow$

(iii) $\oint_{\partial V} \underline{B} \cdot d\underline{f} = 0$

## Zusammenhang zwischen $\underline{B}$ und $\underline{j}$:

nichtstationär

$$\text{rot}\ \underline{B} = \underline{\nabla} \times (\underline{\nabla} \times \underline{A}) = \underline{\nabla}(\underline{\nabla} \cdot \underline{A}) - \Delta \underline{A}$$

$$\underline{a} \times (\underline{b} \times \underline{c}) = \underline{b}(\underline{a}\underline{c}) - \underline{c}(\underline{a}\underline{b})$$

$$\underline{\nabla}\cdot\underline{A} = \frac{\mu_0}{4\pi}\int d^3r'\,\underline{\nabla}_r\cdot\left(\frac{\underline{j}(\underline{r}')}{|\underline{r}-\underline{r}'|}\right)$$

$$= \frac{\mu_0}{4\pi}\int d^3r'\,\underline{j}(\underline{r}')\cdot\underbrace{\underline{\nabla}_r\frac{1}{|\underline{r}-\underline{r}'|}}_{-\underline{\nabla}_{r'}\frac{1}{|\underline{r}-\underline{r}'|}}$$

$$= \frac{\mu_0}{4\pi}\int_{\mathbb{R}^3} d^3r'\left[\underbrace{-\underline{\nabla}_{r'}\cdot\left(\frac{\underline{j}(\underline{r}')}{|\underline{r}-\underline{r}'|}\right)}_{\text{gauß}} + \frac{1}{|\underline{r}-\underline{r}'|}\underbrace{\underline{\nabla}_{r'}\cdot\underline{j}(\underline{r}')}_{-\frac{\partial}{\partial t}\rho\ \ \frac{\text{Kont.}}{\text{gl.}}}\right]$$

$$= \underbrace{-\frac{\mu_0}{4\pi}\int_{S(r\to\infty)} d\underline{f}'\,\frac{\underline{j}(\underline{r}')}{|\underline{r}-\underline{r}'|}}_{0} - \frac{\partial}{\partial t}\underbrace{\frac{\mu_0}{4\pi}\int d^3r'\,\frac{\rho(\underline{r}',t)}{|\underline{r}-\underline{r}'|}}_{\mu_0\epsilon_0\,\Phi(\underline{r},t)}$$

$$\underline{\nabla}(\underline{\nabla}\cdot\underline{A}) = \underline{\mu_0\epsilon_0\frac{\partial}{\partial t}\underline{E}}$$

$$\Delta\underline{A} = \frac{\mu_0}{4\pi}\int d^3r'\,\underline{j}(\underline{r}')\underbrace{\Delta_r\frac{1}{|\underline{r}-\underline{r}'|}}_{-4\pi\delta(\underline{r}-\underline{r}')} = -\mu_0\,\underline{j}(\underline{r},t)$$

Also $\boxed{\mathrm{rot}\,\underline{B} = \mu_0\underline{j} + \mu_0\epsilon_0\frac{\partial}{\partial t}\underline{E}}$

Verschiebungsstromdichte

= nichtstationär

## Für stationäre Strom- und Ladungsverteil.:

$$\boxed{\mathrm{rot}\,\underline{B} = \mu_0 \underline{j}}$$

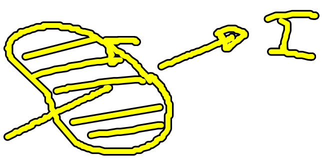
differentielle Form des Ampère-Gesetzes

Integration über Fläche F:


$$\int_F d\underline{f}\,\mathrm{rot}\,\underline{B} \overset{\text{Stokes}}{=} \oint_{\partial F} d\underline{s}\cdot\underline{B} = \mu_0 \underbrace{\int_F d\underline{f}\cdot \underline{j}(\underline{r})}_{I \ \text{Strom}}$$

$$\boxed{\oint_{\partial F} d\underline{s}\cdot\underline{B}(\underline{r}) = \mu_0 I}$$

Integralform (Durchflutungsgesetz)




## Zusammenfassung

### Magnetostatik

$$\boxed{\begin{array}{c} \mathrm{div}\,\underline{B} = 0 \\ \Updownarrow \\ \underline{B} = \mathrm{rot}\,\underline{A} \end{array}}$$

quellenfrei

$$\boxed{\begin{array}{c} \mathrm{rot}\,\underline{B} = \mu_0 \underline{j} \\ \Updownarrow \\ \oint d\underline{s}\cdot\underline{B} = \mu_0 I \end{array}}$$

Ampère

$$\Updownarrow$$

### Elektrostatik

$$\boxed{\begin{array}{c} \mathrm{rot}\,\underline{E} = 0 \\ \Updownarrow \\ \underline{E} = -\underline{\nabla}\phi \end{array}}$$

wirbelfrei

$$\boxed{\begin{array}{c} \varepsilon_0\,\mathrm{div}\,\underline{E} = \rho \\ \Updownarrow \\ \varepsilon_0 \oint_{\partial V} d\underline{f}\cdot\underline{E} = Q \end{array}}$$

diff.

integral

Gauß

$$\Updownarrow$$

$$\boxed{\Delta \underline{A} = -\mu_0 \underline{j}}$$

gilt nur, falls $\boxed{\mathrm{div}\, \underline{A} = 0}$
Coulomb-Eichung

$$\boxed{\Delta \phi = -\frac{1}{\epsilon_0}\rho(\underline{r})}$$

Poisson-Gl.

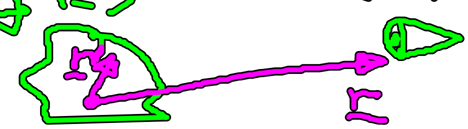
## 2.4 Magnetische Multipole (stationär)

Ausgangspunkt $\underline{A}(\underline{r}) = \frac{\mu_0}{4\pi}\int_{\mathbb{R}^3} d^3r' \, \frac{\underline{j}(\underline{r}')}{|\underline{r}-\underline{r}'|}$

$\mathrm{div}\, \underline{A} = 0$ Coulomb-Eichung

$\underline{A}(\underline{r}) \to 0$ für $r \to \infty$

Taylorentw. von $\frac{1}{|\underline{r}-\underline{r}'|}$ für räuml. lokalisierte stationäre Stromvert. $\underline{j}(\underline{r}')$ mit $r' \ll r$




$$\frac{1}{|\underline{r}-\underline{r}'|} = \frac{1}{r} + \frac{1}{r^3}(\underline{r}\cdot\underline{r}') + \dots$$

$$\underline{A}(\underline{r}) = \underbrace{\frac{\mu_0}{4\pi r}\int_{\mathbb{R}^3} d^3r' \, \underline{j}(\underline{r}')}_{\text{Monopol } = 0} + \frac{\mu_0}{4\pi r^3}\int_{\mathbb{R}^3} d^3r' \, (\underline{r}\cdot\underline{r}')\, \underline{j}(\underline{r}') + \dots$$

$$\nabla_{r'} \cdot [x'_k \underline{j}(\underline{r}')] = x'_k \underbrace{(\underline{\nabla}\cdot\underline{j})}_{0 \text{ stationär (Kont.gl.)}} + \underline{j}\cdot(\underline{\nabla}\, x'_k) = j_k$$

$$\int d^3r' \, j_k(\underline{r}') = \int_{\mathbb{R}^3} d^3r' \, \nabla[x'_k \underline{j}] \overset{\text{Gauß}}{=} \int_{S_\infty} d\underline{f}\cdot x'_k \underline{j} = 0$$

# Dipol-Term

$$\underline{A}(\underline{r}) = \frac{\mu_0}{4\pi r^3}\,\underline{m}\times\underline{r}$$

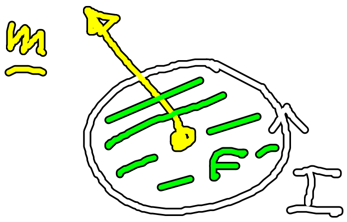
$$\underline{m} := \frac{1}{2}\int d^3r'\,\underline{r}'\times\underline{j}(\underline{r}')$$

magn. Dipolmoment

$$\left( \phi(\underline{r}) = \frac{1}{4\pi\varepsilon_0 r^3}\,\underline{p}\cdot\underline{r} \qquad \underline{p} := \int d^3r'\,\underline{r}'\rho(\underline{r}') \right)$$

el. Dipolmoment

Beispiel (i) ebene Leiterschleife




Ringstrom $\rightarrow$ magn. Dipolmoment $\underline{m}$

$$\underline{m} = I F \underline{n}$$