Dispersionsrelation für Welle in einem Medium

$$\boxed{k^2 = \epsilon(\omega)\,\frac{\omega^2}{c^2}\left(1 + i\,\frac{1}{\omega\tau}\right)}$$

$$\tau = \frac{\epsilon\epsilon_0}{\sigma}$$

$$\underline{E}(\underline{r},t) = \underline{E}_0\, e^{i(\underline{k}\underline{r} - \omega t)}$$

Isolator (disp. Dielektrikum):

$$\boxed{k^2 \approx \epsilon(\omega)\,\frac{\omega^2}{c^2}}$$

$$\tilde{n}(\omega) = n(\omega) + i\varkappa(\omega)$$

$$\tilde{n}(\omega)^2 = \epsilon(\omega) \equiv \epsilon' + i\epsilon''$$

$$\boxed{\left.\begin{matrix}\varkappa = \\ n = \end{matrix}\right\}\left\{\frac{1}{\sqrt{2}}\left(\sqrt{\epsilon'^2 + \epsilon''^2} \mp \epsilon'\right)\right\}^{1/2}}$$

Abs. koeff.
reeller Brech. index

(i) Absorption

a) $\epsilon'' = 0 \implies \varkappa = 0$, $n = \sqrt{\epsilon'}$ $\implies$ ungedämpfte Welle

b) $\epsilon'' > 0 \implies \varkappa > 0$ (gedämpfte Welle)

Transparenzgebiet der Substanz: $\boxed{\epsilon'' < \epsilon'}$

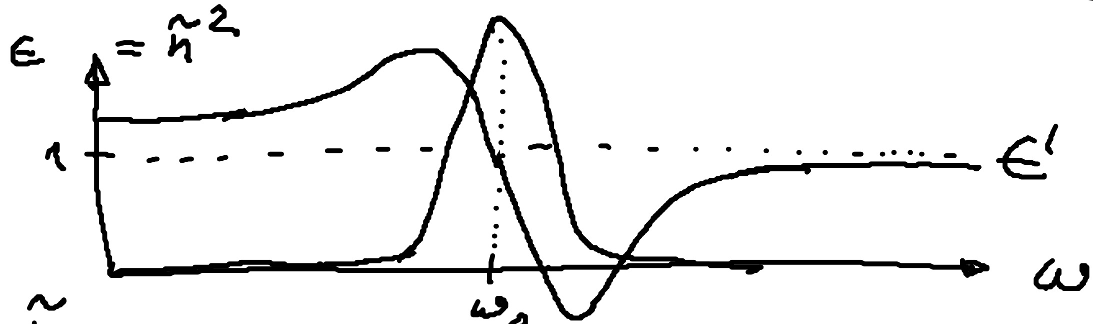
(ii) Dispersion

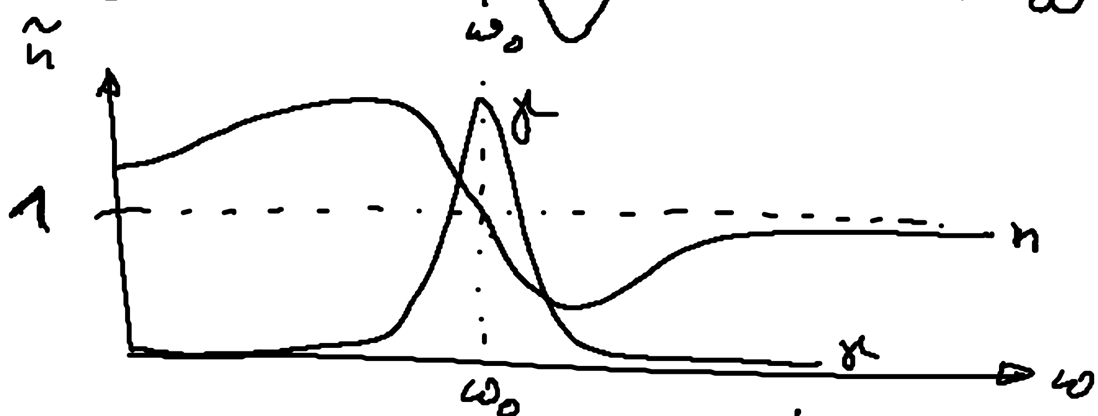
$\mathrm{Re}\,k \equiv k' = \frac{\omega}{c}\,n(\omega)$ $\implies$ nichtlin. Dispersion

Gruppengeschw. $v_g := \frac{d\omega}{dk} = \frac{1}{\frac{dk'}{d\omega}} = \frac{c}{\frac{d}{d\omega}[\omega n(\omega)]}$

Phasengeschw. $v_{ph} := \frac{\omega}{k}$

$$\boxed{v_g = \frac{c}{n + \omega \frac{dn}{d\omega}} \neq \frac{c}{n(\omega)} = v_{ph}}$$

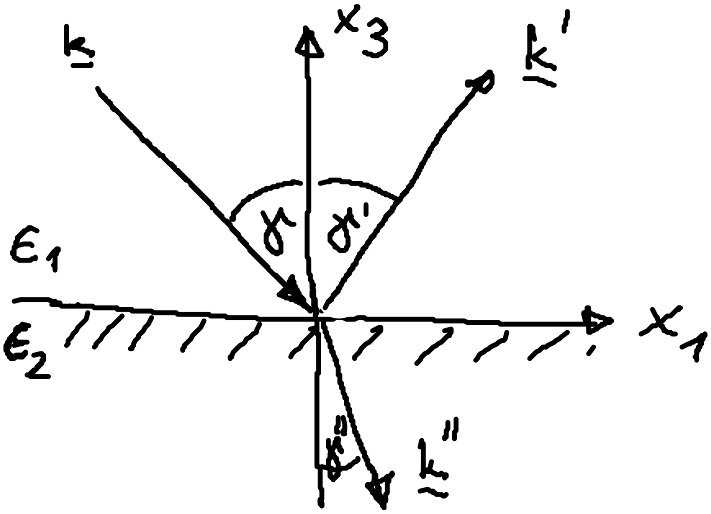




normale Dispersion $\frac{dn}{d\omega} > 0$ (Transparenz-gebiet)

anomale Dispersion $\frac{dn}{d\omega} < 0$ (Abs.)

5.7 Brechung und Reflexion



$\epsilon_{1,2} \in \mathbb{R}$

$$\frac{\omega}{c_1} = |\underline{k}| \overset{s.u.}{=} |\underline{k}'| = \frac{\omega'}{c_1}$$

$$|\underline{k}''| = \frac{\omega''}{c_2}$$

$$c_i = \frac{c}{n_i} = \frac{c}{\sqrt{\epsilon_i}}$$

Einfallende Welle $\underline{E} = \underline{E}_0 e^{i(\underline{k}\underline{r} - \omega t)}$

reflekt. " $\underline{E}' = \underline{E}_0' e^{i(\underline{k}'\underline{r} - \omega' t)}$

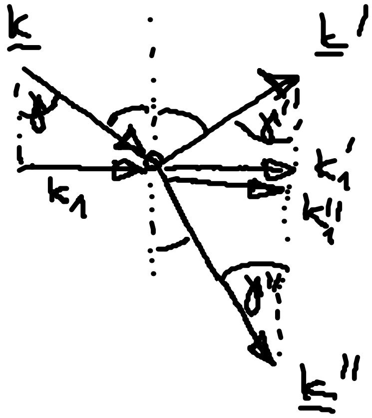
transmitt. " $\underline{E}'' = \underline{E}_0'' e^{i(\underline{k}''\underline{r} - \omega'' t)}$

grenz bed. für $\underline{E}$: $\left. E_1 + E_1' \right|_{x_3=0} = \left. E_1'' \right|_{x_3=0}$

$$\underline{r} = 0: \; E_{01} e^{-i\omega t} + E_{01}' e^{-i\omega' t} = E_{01}'' e^{-i\omega'' t} \quad \forall t$$

$$\boxed{\begin{aligned}\omega &= \omega' = \omega'' \\ E_{01} + E'_{01} &= E''_{01}\end{aligned}}$$

$$t=0: \quad E_{01} e^{ik_1 x_1} + E'_{01} e^{ik'_1 x_1} = E''_{01} e^{ik''_1 x_1} \quad \forall x_1$$



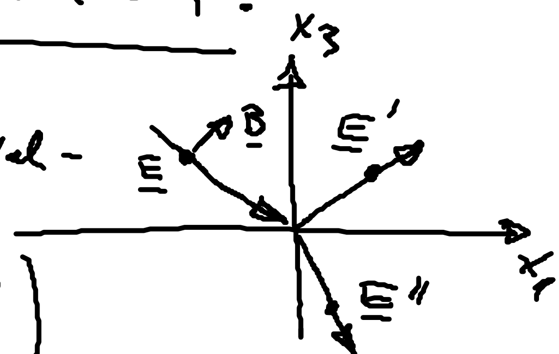
$$\boxed{k_1 = k'_1 = k''_1}$$

$$\underbrace{k \sin\gamma = k' \sin\gamma'}_{\frac{\omega''}{c_1} \quad \frac{\omega''}{c_1}} = \underset{\frac{\omega''}{c_2}}{k'' \sin\gamma''}$$

<u>Reflexionsgesetz</u> $\qquad \boxed{\sin\gamma = \sin\gamma'}$

<u>Brechungsgesetz</u>
(Snellius) $\qquad \boxed{\frac{\sin\gamma''}{\sin\gamma} = \frac{c_2}{c_1} = \frac{n_1}{n_2}}$

Bestimmung der Amplituden:



(1) $\boxed{E_{02} + \underline{E'_{02}} = \underline{\underline{E''_{02}}}}$ Tangential-komp.

$$\underline{B}_0 = \frac{c}{\omega}(\underline{k} \times \underline{E}_0) = \frac{c}{\omega} E_{02} \begin{pmatrix} -k_3 \\ 0 \\ k_1 \end{pmatrix}$$

$$B_{01} + B'_{01} = B''_{01} \Rightarrow k_3 E_{02} + k'_3 E'_{02} = k''_3 E''_{02}$$

Reflex. $k_3 = -k'_3 \Rightarrow \boxed{k_3(E_{02} - \underline{E'_{02}}) = k''_3 \underline{\underline{E''_{02}}}}$ (2)

$$\Rightarrow \quad \frac{E'_{02}}{E_{02}} = \frac{k_3 - k''_3}{k_3 + k''_3} \qquad \frac{E''_{02}}{E_{02}} = \frac{2k_3}{k_3 + k''_3}$$

$$k''_3 = |\underline{k}''| \cos\gamma'' = |\underline{k}| \underbrace{\frac{n_2}{n_1}}_{\frac{\sin\gamma}{\sin\gamma''}} \cos\gamma''$$

$$k_3 = |\underline{k}| \cos\gamma$$

$$\boxed{\begin{aligned} \frac{E'_{02}}{E_{02}} &= \frac{\sin(\gamma'' - \gamma)}{\sin(\gamma'' + \gamma)} \\[2ex] \frac{E''_{02}}{E_{02}} &= \frac{2 \sin\gamma'' \cos\gamma}{\sin(\gamma'' + \gamma)} \end{aligned}}$$

Fresnel'sche Formeln

Intensitätsverhältnisse

$$\langle \underline{S} \rangle = \frac{1}{T} \int_0^T dt \, \underline{E} \times \underline{H} \sim |\underline{E}_0|^2$$

Reflexionsvermögen

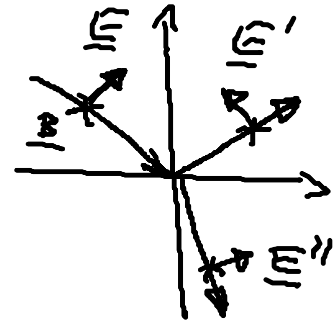
$$R_\perp = \left| \frac{E'_{02}}{E_{02}} \right|^2 = \frac{\sin^2(\gamma'' - \gamma)}{\sin^2(\gamma'' + \gamma)}$$

Transmissionsvermögen

$$T_\perp = \left| \frac{E''_{02}}{E_{02}} \right|^2 = \frac{4 \sin^2\gamma'' \cos^2\gamma}{\sin^2(\gamma'' + \gamma)} = 1 - R_\perp$$

(b) Polarisation von $\underline{E} \parallel$ Einfallsebene

$$\boxed{R_{\parallel} = 1 - T_{\parallel} = \frac{\tan^2(\gamma'' - \gamma)}{\tan^2(\gamma'' + \gamma)}}$$

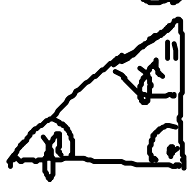


(i) Speziell $R_{\parallel} = 0$:

$\boxed{\gamma'' + \gamma = \frac{\pi}{2}}$: $\tan(\gamma'' + \gamma) = \infty \Rightarrow R_{\parallel} = 0$

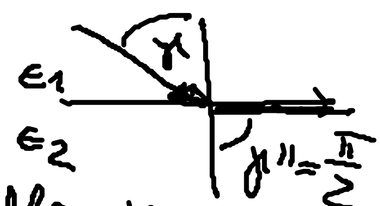
Brewster-Winkel γ_B so, daß $\tan \gamma_B = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$

$$\left(\tan \gamma = \frac{\sin \gamma}{\cos \gamma} = \frac{\sin \gamma}{\sin \underbrace{\left(\frac{\pi}{2} - \gamma\right)}_{\gamma''}} = \frac{n_2}{n_1} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \right)$$



(ii) Totalreflexion

$\epsilon_2 < \epsilon_1$



Grenzwinkel der Totalreflexion

$\gamma'' = \frac{\pi}{2}$, $R_{\perp} = R_{\parallel} = 1$

$T_{\perp} = T_{\parallel} = 0$

$$\frac{\sin \gamma}{\sin \gamma''} \overset{!}{=} \frac{\sin \gamma}{\underbrace{\sin \frac{\pi}{2}}_{1}} = \frac{n_2}{n_1} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \Rightarrow \boxed{\sin \gamma_G = \sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

Für $\gamma > \gamma_G$: $k_3'' = i/d$

$$\underline{E}'' = \underline{E}_0'' \, e^{-|x_3|/d} \, e^{i(k_1'' x_1 - \omega t)}$$