

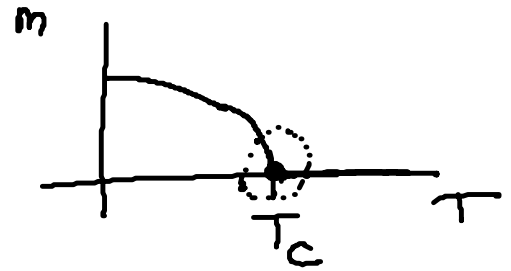
30.1.07

$$m = \tanh(\beta J m + \beta h)$$

Zustandsgleichung  $\beta h = m(1-\tau) + m^3(\tau - \tau^2 + \tau^3/3 + \dots)$

$$\tau \equiv T_c/T$$

$$t \equiv \frac{T-T_c}{T_c} = \frac{1}{\tau} - 1$$



Ordnungsparameter  $\eta = m$  Magnetisierung (pro Spin)

$$m = \langle \sigma_i \rangle$$

homogen

$$\tau = \frac{1}{1+t} = 1-t+t^2-t^3+\dots \quad t \text{ gilt die Abweichung von } T_c \text{ an}$$

$$|t| \ll 1$$

$$1-\tau = t + O(t^2)$$

$$\tau - \tau^2 + \frac{1}{3}\tau^3 = 1-t - (1-2t) + \frac{1}{3}(1-t)(1-2t) + O(t^2)$$

$$= \frac{1}{3} + O(t^2)$$

Zustandsgl:

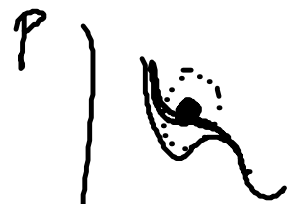
$$\beta h = \eta t + \frac{1}{3}\eta^3 + \dots, \quad |t| \ll 1$$

$$|\eta| \ll 1$$

van-der-Waals-fas

Kritische Punkt

$T_c, P_c, V_c$



Definiert  $p = \frac{P}{P_c} - 1$   $\overline{\quad\quad\quad}$   
 $v$

$$v = \frac{V}{V_c} - 1 \quad t = \frac{T}{T_c} - 1$$

$$p = 4t - 6tv - \frac{3}{2}v^3 + O(tv^2)$$

AUFGABE

Ordnungsparameter  $\eta \equiv \frac{p - p_c}{p_c} = \frac{V_c}{V} - 1$

$$= \frac{1}{v+1} - 1 = \frac{-v}{v+1}$$

$$\Rightarrow \boxed{p = 4t + 6t\eta + \frac{3}{2}\eta^3 + O(t\eta^2), |\eta| \ll 1, |t| \ll 1} \approx -v + O(v^2)$$

Zustandsgleichung aus Minimierung des Landau-Funktional (des Ordnungsparameters)

Ansatz:  $\mathcal{L}[\eta] = a_0 + a_1\eta + a_2\eta^2 + a_3\eta^3 + a_4\eta^4 + \dots$

(setzt Zustandsgl. aus  $0 = \mathcal{L}'[\eta]$ )

$$\Rightarrow 0 = a_1 + 2a_2\eta + 3a_3\eta^2 + 4a_4\eta^3 + \dots$$

$$\| \mathcal{L}_{\text{ISING}}[\eta] = a_0 - \beta h \eta + \frac{t}{2}\eta^2 + \frac{1}{12}\eta^4 + \dots \|$$

$$\Rightarrow 0 = -\beta h + t\eta + \frac{1}{3}\eta^3 + \dots$$

$$\| \mathcal{L}_{\text{WAALS}}[\eta] = a_0 + (4t - p)\eta + 3t\eta^2 + \frac{3}{8}\eta^4 + \dots \|$$

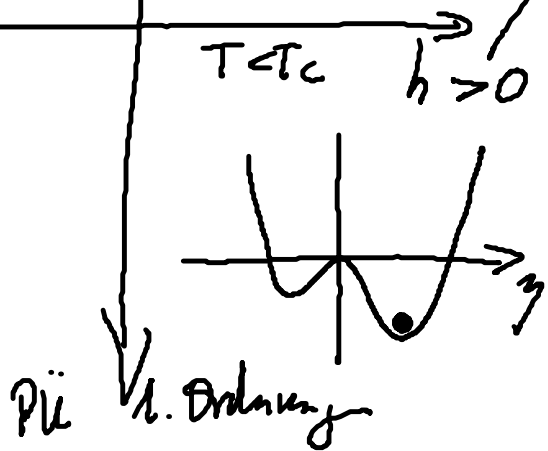
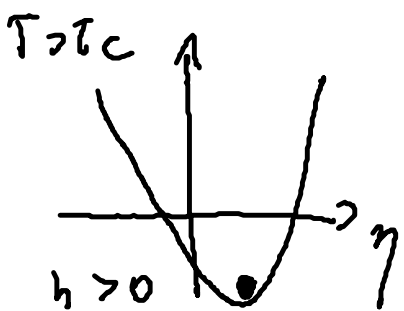
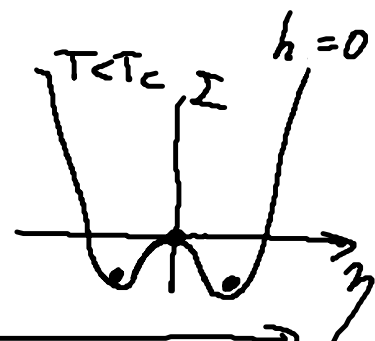
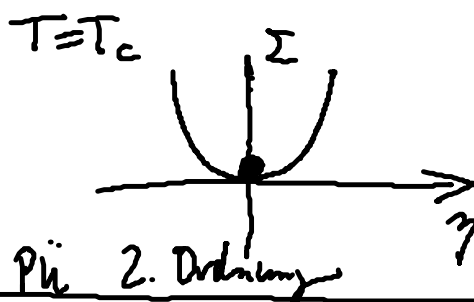
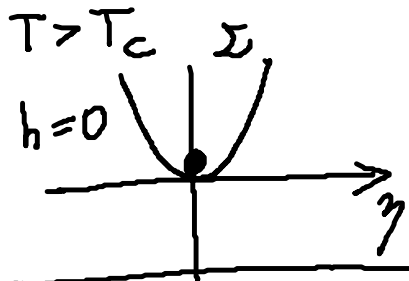
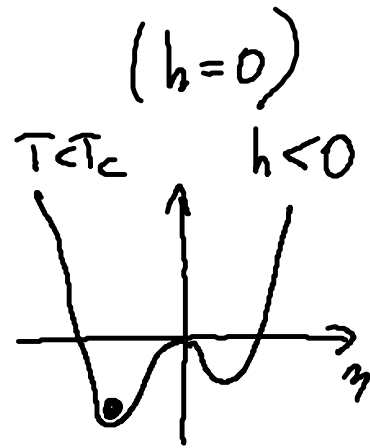
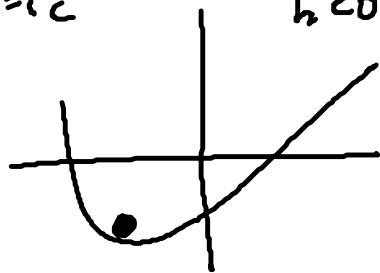
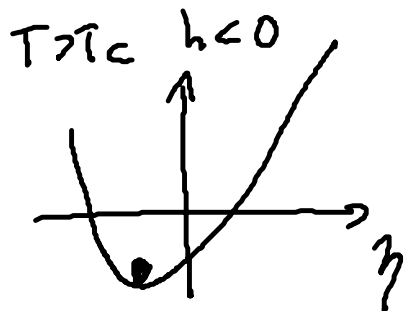
- Koll.  $a_4 > 0$

- Koeff.  $a_3 = 0$

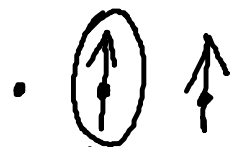
- Koeff.  $a_2 \propto t \equiv \frac{T - T_c}{T_c}$  wechselt Vorzeichen

Wir skizzieren  $\mathcal{J}[\eta]$

$$\mathcal{J}_{\text{Ising}} = a_0 + \frac{t}{2} \eta^2 + \frac{1}{12} \eta^4$$



Fluktuationen



$$p(\sigma_1 \dots \sigma_N) = \frac{e^{-\beta \mathcal{H}[\sigma_i]} \sigma_i}{\mathcal{Z}}$$

$$p(\sigma_1) = \sum_{\sigma_2 \sigma_3 \dots \sigma_N} p(\sigma_1 \dots \sigma_N) \quad \text{Einteilchen-Verteilung} \rightarrow$$

z.B.  $\langle \sigma_1 \rangle$

n-tes Moment  $\langle \sigma_1^n \rangle = \sum_{\sigma_1} \sigma_1^n p(\sigma_1)$

Mehr-Teilchen-Verteilungen,  
z.B. für zwei Spins

$$p(\sigma_i, \sigma_k) = \sum_{\sigma_l (l \neq i, k)} p(\sigma_1, \dots, \sigma_N)$$

- sehr schwer auszurechnen

Zwei-Teilchen-Korrelationen

Korrelationsmatrix

$$\langle \sigma_i \sigma_j \rangle$$

$$= \frac{\sum_{\sigma_1 \dots \sigma_N} \sigma_i \sigma_j e^{-\beta \mathcal{H}}}{Z}$$

$$Z = \sum_{\sigma_1 \dots \sigma_N} e^{-\beta \left[ -\sum_{ij} J_{ij} \sigma_i \sigma_j - \sum_i h_i \sigma_i \right]}$$

$$= \frac{1}{Z} \frac{\partial^2 Z}{\partial \beta \sigma_i \partial \beta \sigma_j}$$

Integraldarstellung

$$Z = \int \mathcal{D}[H] e^{-S[H]} =$$

$$= \frac{1}{\sqrt{\det \beta J}} \int \frac{dH_1 \dots dH_N}{(2\pi)^{N/2}} e^{-S(H_1 \dots H_N)}$$

$$S(H_1 \dots H_N) = \frac{1}{2\beta} \sum_{i,j} H_i (\beta^{-1})_{ij} H_j - \sum_i \ln 2 \cosh(H_i + \beta h_i)$$

Zielt  $\frac{\partial^2}{\partial \beta h_i \partial \beta h_j} \prod_{l=1}^N 2 \cosh(H_l + \beta h_l)$ ,

z.B.  $\frac{\partial}{\partial \beta h_j} \cosh(H_i + \beta h_i) = \delta_{ij} \sinh(H_i + \beta h_i)$

$$= \delta_{ij} \cosh(H_i + \beta h_i) \tanh(H_i + \beta h_i)$$

$$= \tanh(H_i + \beta h_i) \tanh(H_j + \beta h_j) \prod_{l=1}^N 2 \cosh(\dots)$$

$i \neq j$

$$\langle \sigma_i \sigma_j \rangle = \frac{\int \mathcal{D}[H] \tanh(H_i + \beta h_i) \tanh(H_j + \beta h_j) e^{-S}}{\int \mathcal{D}[H] e^{-S}}$$

- MF-Näherung:  $H_i = \bar{H}_i$ ,  
 $m_i = \tanh(\bar{H}_i + \beta h_i) = \tanh(H_i + \beta h_i)$

$$\langle \sigma_i \sigma_j \rangle_{MF} = \frac{\int \mathcal{D}[H] m_i m_j e^{-S}}{\int \mathcal{D}[H] e^{-S}} = m_i m_j$$

$$= \langle \sigma_i \rangle_{MF} \langle \sigma_j \rangle_{MF}$$

$$\Rightarrow \text{Fluktuation } \langle \delta \sigma_i \delta \sigma_j \rangle \equiv$$

$$\equiv \langle (\sigma_i - \langle \sigma_i \rangle) (\sigma_j - \langle \sigma_j \rangle) \rangle = \langle \sigma_i \sigma_j \rangle - \langle \sigma_i \rangle \langle \sigma_j \rangle$$

$$= 0. \quad \text{Das ist unrealistisch.}$$

Um das zu verbessern, muß die quadratische Entwicklung von  $S$  beachtet werden:

Entwickeln um die MF-Lösung

$$H_i = \bar{H}_i + \varphi_i \quad \text{nach kleinen } \varphi_i$$

$$S[\bar{H}_i + \varphi_i] = S[\bar{H}_i] + \sum_i \frac{\partial S}{\partial H_i}[\bar{H}_i] \varphi_i + \frac{1}{2} \sum_{i,j} \frac{\partial^2}{\partial H_i \partial H_j} S[\bar{H}] \varphi_i \varphi_j$$

Matrix der 2. Ableitung

$$S[\bar{H}_i] = \frac{1}{2\beta} \sum_{ij} \bar{H}_i (\bar{F}^{-1})_{ij} \bar{H}_j - \sum_i \ln 2 \cosh(\bar{H}_i + \beta h_i)$$

$$A_{ij} \equiv \frac{\partial^2}{\partial \bar{H}_i \partial \bar{H}_j} S = \frac{1}{2\beta} [(\bar{F}^{-1})_{ij} + (\bar{F}^{-1})_{ji}] - \frac{\delta_{ij}}{\cosh^2(\bar{H}_i + \beta h_i)}$$
$$= \frac{1}{\beta} \bar{F}^{-1}_{ij} - (1 - m_i^2) \delta_{ij}$$

Jetzt  $\tanh(\bar{H}_i + \varphi_i + \beta h_i) =$

$$= \underbrace{\tanh(\bar{H}_i + \beta h_i)}_{m_i} + \underbrace{\tanh'(\bar{H}_i + \beta h_i)}_{(1 - m_i^2)} \varphi_i + \dots$$

$$\langle \delta \sigma_i \delta \sigma_j \rangle = (1 - m_i^2)(1 - m_j^2) \langle \varphi_i \varphi_j \rangle$$

$$\langle \varphi_i \varphi_j \rangle \equiv \frac{\int \mathcal{D}[\varphi] \varphi_i \varphi_j e^{-S''[\varphi]}}{\int \mathcal{D}[\varphi] e^{-S''[\varphi]}}$$

$$S''[\varphi] \equiv \frac{1}{2} \sum_{ij} \varphi_i A_{ij} \varphi_j$$