

$$VR: [a_\nu, a_\mu^\dagger] = \delta_{\nu\mu} \mathbb{1}$$

$$[\hat{\psi}(\underline{x}), \hat{\psi}^\dagger(\underline{x}')] = \sum_{\nu, \mu} [\psi_\nu(\underline{x}) a_\nu, \psi_\mu^*(\underline{x}') a_\mu^\dagger]$$

$$= \sum_{\nu, \mu} \psi_\nu(\underline{x}) \psi_\mu^*(\underline{x}') [a_\nu, a_\mu^\dagger] = \sum_{\nu} \psi_\nu(\underline{x}) \psi_\nu^*(\underline{x}') = \delta(\underline{x} - \underline{x}')$$

Orthogonalität: $\langle \psi_\nu | \psi_\mu \rangle = \int \psi_\nu^*(\underline{x}) \psi_\mu(\underline{x}) d\tau = \delta_{\nu\mu}$

Vollständigkeit: $\sum_{\nu} \psi_\nu(\underline{x}) \psi_\nu^*(\underline{x}') = \delta(\underline{x} - \underline{x}')$

$$\psi(\underline{x}) = \sum_{\nu} \psi_\nu(\underline{x}) c_\nu \quad \text{mit} \quad c_\nu = \int \psi_\nu^*(\underline{x}') \psi(\underline{x}') d\tau'$$

$$= \int \underbrace{\sum_{\nu} \psi_\nu(\underline{x}) \psi_\nu^*(\underline{x}')}_{\delta(\underline{x} - \underline{x}')} \psi(\underline{x}') d\tau' = \psi(\underline{x})$$

$$\begin{aligned} \hat{N} &= \sum_{\lambda=1}^{\infty} a_\lambda^\dagger a_\lambda = \sum_{\lambda=1}^{\infty} \int d\tau \int d\tau' \underbrace{\psi_\lambda(\underline{x}) \hat{\psi}^\dagger(\underline{x}) \psi_\lambda^*(\underline{x}') \hat{\psi}(\underline{x}')}_{\delta(\underline{x} - \underline{x}')} \\ &= \int d\tau \hat{\psi}^\dagger(\underline{x}) \hat{\psi}(\underline{x}) \end{aligned}$$

$$a_\lambda = \int \psi_\lambda^*(\underline{x}) \hat{\psi}(\underline{x}) d\tau$$