5.7 Paramagnetismus

Modell

N ortsfeste (unterscheidbare) Teilchen
mit Drehimpuls $\underline{L}$ im Magnetfeld der Ind. $\underline{B}$

Drehimpulsquantis.
$$m_\ell = -\ell, -\ell+1, \dots, \ell$$

Energie $E = -\mu B m_\ell$
$$\mu := g \frac{e\hbar}{2m} = g \mu_{Bohr}$$

z.B. Spin $\ell = \frac{1}{2}$, $g = 2$, $m_\ell = \pm\frac{1}{2}$

Bahn $\ell = 1$, $g = 1$, $m_\ell = -1, 0, +1$

1-Teilchen-Zustandssumme:

$$Z = \sum_{m_\ell = -\ell}^{\ell} \exp\{\beta\mu B m_\ell\}$$

$\nu = m_\ell + \ell$

$$Z = \exp\{-\beta\mu B \ell\} \sum_{\nu=0}^{2\ell} \left(e^{\beta\mu B}\right)^\nu$$

$$= e^{-\beta\mu B\ell} \frac{e^{\beta\mu B(2\ell+1)} - 1}{e^{\beta\mu B} - 1} = \frac{\sinh[\beta\mu B(\ell+\frac{1}{2})]}{\sinh[\frac{1}{2}\beta\mu B]}$$

z.B. $\ell = \frac{1}{2}$:
$$\boxed{Z = \frac{\sinh(\beta\mu B)}{\sinh(\frac{1}{2}\beta\mu B)} = 2\cosh(\tfrac{1}{2}\beta\mu B)}$$

Magnetisierung M = $\frac{\text{mittleres magn. Moment}}{\text{Volumen}}$

$$M = \frac{N}{V} \sum_{m_\ell=-\ell}^{\ell} \mu m_\ell Z^{-1} e^{\beta \mu B m_\ell}$$

$$= \frac{N}{V} = \frac{1}{Z} \sum_{m_\ell=-\ell}^{\ell} \mu m_\ell e^{\beta \mu B m_\ell}$$

$$= \frac{N}{V} \frac{1}{\beta} \frac{\partial}{\partial B} \ln Z$$

$$= \frac{N}{V} \mu \left\{ (\ell+\tfrac{1}{2}) \coth\left[\beta \mu B (\ell + \tfrac{1}{2})\right] - \tfrac{1}{2} \coth\left[\tfrac{1}{2} \beta \mu B\right] \right\}$$

(Brillouin-Fkt.)

z.B. $\ell = \frac{1}{2}$: $\boxed{M = \frac{N}{V} \mu \frac{1}{2} \tanh\left(\frac{1}{2} \beta \mu B\right)}$

(Langevin-Fkt.)

$\hat{=}$ therm. Zustandsgl. $M(T, V, B)$

Hohe Temp.: $kT \gg \mu B$ (z.B. $B = 1T$: $T \gg 1K$)

Entw. $\coth x \approx \frac{1}{x} + \frac{x}{3} + \ldots$ für $x \ll 1$

$$\Longrightarrow \quad M = \frac{N}{V} \frac{\ell(\ell+1)}{3} \beta \mu^2 B \quad \text{lin. in } B$$

speziell $\ell = \frac{1}{2}$: $\boxed{M(T, V, B) = \frac{N}{V} \frac{\mu^2 B}{4kT}}$ Curie-Gesetz

magn. Suszept. χ_m : $M = \chi_m H$ (H Magnetfeld)

$$B = \mu_0 (H + M) = \mu_0 (1 + \chi_m) H$$

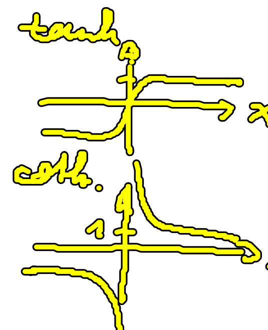
$$\Rightarrow \quad M = \frac{1}{\mu_0} \frac{\chi_m}{1+\chi_m} B \approx \frac{1}{\mu_0} \chi_m B$$

$$\Rightarrow \quad \chi_m = \mu_0 \frac{N}{V} \frac{l(l+1)}{3} \frac{\mu^2}{kT} = \frac{C}{T}$$

(Curie-Konst. C)

Tiefe Temp., hohe Magnetfelder : $kT \ll \mu B$

$\coth x \approx 1 \qquad x \to \infty$



$$\Rightarrow \quad M = \frac{N}{V} \mu \left\{ \left(l+\tfrac{1}{2}\right) - \tfrac{1}{2} \right\} = \frac{N}{V} \mu l$$

vollständige Ausrichtung aller Momente $\underline{\mu} \uparrow\uparrow \underline{B}$

Energie u. Entropie

Entropie S für $l = \frac{1}{2}$ (N-Teilchen-Zustandssumme Z^N)

$$S \overset{*}{=} k\,(\ln Z^N + \beta U) \quad \text{für kanon. Vert. } Z^{-1} e^{-\beta H}$$

$$U = -\frac{\partial}{\partial \beta} \ln Z^N$$

$$= -N \frac{\partial}{\partial \beta} \ln \left[\cosh\left(\frac{\beta \mu B}{2} \right) \right]$$

$$= -\frac{N \mu B}{2} \frac{\sinh(\frac{\beta}{2} \mu B)}{\cosh(\frac{\beta}{2} \mu B)}$$

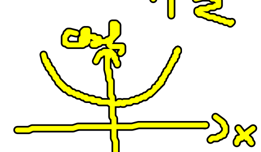
$$\boxed{U(T) = -N \frac{\mu B}{2} \tanh\left(\frac{\beta}{2} \mu B\right)}$$

kalor. Zustandsgl.
$U(T, B)$

$$* \quad S(T) = kN \left(\ln Z - \beta \frac{\partial}{\partial \beta} \ln Z \right)$$

$$= kN \left[\ln 2 + \ln \cosh\left(\frac{\beta \mu B}{2}\right) - \frac{\beta \mu B}{2} \tanh\left(\frac{\beta \mu B}{2}\right) \right]$$

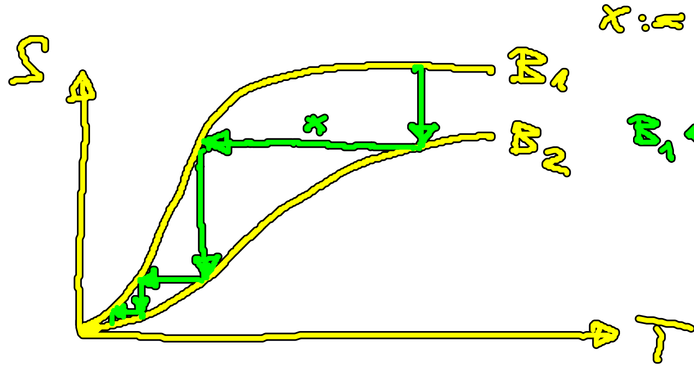
$T \to \infty \quad : \quad S \to kN \ln 2$



$$T \to 0 \quad : \quad S \to kN \left[\ln 2 + \ln \frac{e^x}{2} - x\,(1 - 2e^{-2x}) \right]$$

$$= 2kN\, x e^{-2x} \to 0$$

$$x := \frac{\mu B}{2kT}$$



$B_1 < B_2$

Adiabat. Entmagnetisierung *

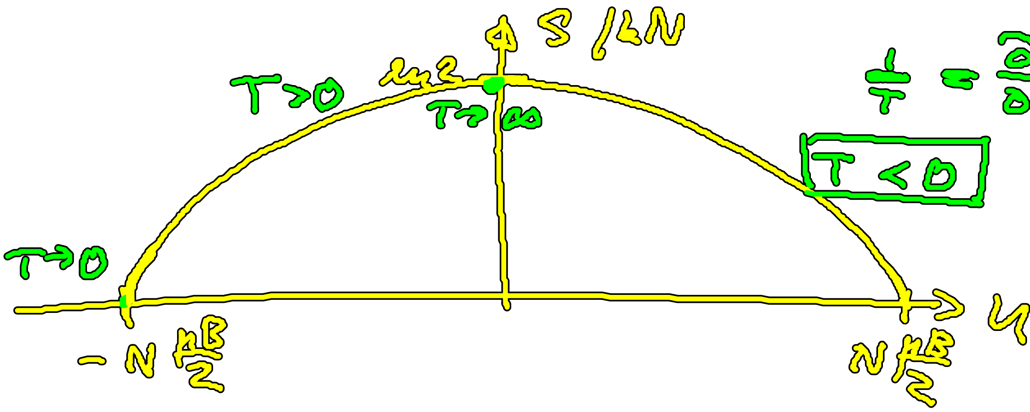
Elim. von T zugunsten von U:

$$u := \frac{2}{N\mu B} U = -\tanh x \qquad x = \frac{\mu B}{2kT}$$

$$x = -\operatorname{artanh} u = \frac{1}{2} \ln \frac{1-u}{1+u}$$

$$\cosh x = \frac{1}{\sqrt{1-\tanh^2 x}} = \frac{1}{\sqrt{1-u^2}}$$

$$\Rightarrow \boxed{S(U) = kN \left\{ \ln 2 - \frac{1-u}{2} \ln(1-u) - \frac{1+u}{2} \ln(1+u) \right\}}$$



$$\frac{1}{T} = \frac{\partial S}{\partial U}$$

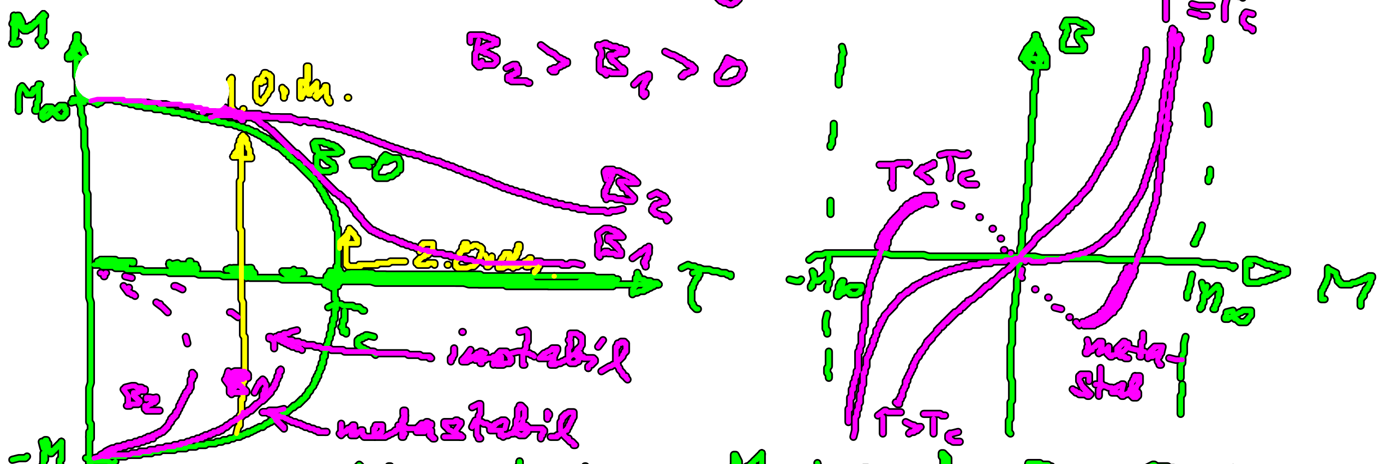
5.8 Ferromagnetismus

WW zwischen Elementarmagneten (P. Weiss):

eff. Feld $\underline{B}' = \underline{B} + w\underline{M}$ ($\underline{B}$ extern, w Weiss-Faktor)

$\Rightarrow \boxed{\underline{M} = \underline{M}_\infty \tanh \frac{m}{kT}(B + wM)}$

implizite Zustandsgl.



spontane Magnetisierung $M \neq 0$ für $B=0$, $T<T_c$:

2 ferromagn. Phasen ($M>0$, $M<0$)

$\Rightarrow$ spontane Symmetriebrechung

$T = T_c$ (Curie-Temp.): Phasenüberg. 2. Ordn. ($B=0$)

$B \neq 0$ zerstört Phasenüberg. 2. Ordn.

mikroskop. Theorie des Ferromagn.:

Ising-Modell (Ising 1925)

$$H = -\mu B \sum_{i=1}^{N} s_i - \epsilon \sum_{ij} s_i s_j \qquad s_i = g\tilde{s}_i = \pm 1$$

nächste Nachbar-WW

$\{s_i\}$ $\quad \uparrow \; \downarrow \; \downarrow \; \uparrow \; \uparrow$

$\quad\quad\; \uparrow \; \uparrow \; \downarrow \; \downarrow \; \uparrow$

$$Z(T,B) = \sum_{\{s_i\}} e^{-\beta H}$$

- exakte Lösung in 1, 2D möglich
- Computersimulation: Metropolis-Algorithmus

  $\longrightarrow$ auswürfeln der Spin-Konfigurationen

(i) Berechne Gesamtenergie für geg. Konfig.

(ii) 1 Spin flip

- ja, falls Energie abgesenkt wird
- ? mit Wahrsch. $e^{-\beta \Delta E}$, falls Energie um ΔE erhöht wird

kinet. Monte-Carlo-Simulation zur Lösung einer Mastergl.

$\Longrightarrow$ Applet Ising-Modell

s. homepage der Vorlesung