6.1 Erzeugungs- und Vernichtungsoperatoren

Orthogonalität

$$\langle n_1 n_2 \dots | n_1' n_2' \rangle = \dots\dots \langle \psi_{\nu_1}(1)\psi_{\nu_2}(2)\dots | \psi_{\nu_1'}(1)\psi_{\nu_2'}(2)\dots \rangle$$

$$= \dots \langle \psi_{\nu_1}(1)|\psi_{\nu_1'}(1)\rangle \langle \psi_{\nu_2}(2)|\psi_{\nu_2'}(2)\rangle \dots \langle \psi_{\nu_N}(N)|\psi_{\nu_N'}(N)\rangle$$

Normierung : $N = 2$

$$|n_1 n_2 \dots\rangle = \frac{1}{\sqrt{2}}\left[\psi_{\nu_1}(1)\psi_{\nu_2}(2) - \psi_{\nu_2}(1)\psi_{\nu_1}(2)\right]$$

$$\langle n_1 n_2 \dots | n_1 n_2 \dots\rangle = \frac{1}{2}\Big[\underbrace{\langle \psi_{\nu_1}(1)|\psi_{\nu_1}(1)\rangle \langle \psi_{\nu_2}(2)|\psi_{\nu_2}(2)\rangle}_{=1} - \underbrace{\langle \psi_{\nu_1}(1)|\psi_{\nu_2}(1)\rangle \langle \psi_{\nu_2}(2)|\psi_{\nu_1}(2)\rangle}_{=0} - 0 + 1\Big] = 1$$

$$A(i)|n_1 n_2 \dots\rangle = \dots \sum_{P\in S} (\pm 1)^p\, T_p \{\psi_{\nu_1}(1)\dots A(i)\psi_{\nu_i}(i)\dots \psi_{\nu_N}(N)\}$$

$$A(i)\psi_{\nu_i}(i) = \sum_{\lambda=1}^{\infty} \psi_\lambda(i)\, A_{\lambda\nu_i} \quad \text{mit } A_{\lambda\nu_i} = \langle \psi_\lambda(i)|A(i)|\psi_{\nu_i}(i)\rangle$$

1) $\lambda = \nu_i \longrightarrow A_{\lambda\lambda} \leftrightarrow |n_1 n_2 \dots \rangle$

2) $\lambda \neq \nu_i \longrightarrow A_{\lambda\nu_i} \leftrightarrow |n_1 n_2 \dots n_\lambda + 1 \dots n_{\nu_i} - 1, \dots\rangle$

<u>Def.:</u>

$$a_\lambda |n_1 n_2 \dots n_\lambda \dots\rangle = \sqrt{n_\lambda}\, |n_1 n_2 \dots n_\lambda - 1, \dots\rangle$$

$$a_\lambda^+ |n_1 n_2 \dots n_\lambda \dots\rangle = \sqrt{n_\lambda + 1}\, |n_1 n_2 \dots n_\lambda + 1 \dots\rangle$$

$$\dots\dots A_{\lambda\nu_i}\, a_\lambda^+ a_{\nu_i} |n_1 n_2 \dots n_\lambda \dots n_{\nu_i} \dots\rangle$$

$$a_\lambda a_\lambda^+ |n_1 n_2 \dots\rangle = a_\lambda \sqrt{n_\lambda+1}\, |n_1 n_2 \dots n_\lambda+1 \dots\rangle$$
$$= (n_\lambda+1)\, |n_1 n_2 \dots\rangle$$

$$\Rightarrow [a_\lambda, a_\mu^+] = \delta_{\lambda\mu} \mathbb{1}$$

6.2 Feldoperatoren

$$\hat{\psi}(x) = \sum_\nu \psi_\nu(x) a_\nu$$
$$\hat{\psi}^+(x) = \sum_\mu \psi_\mu^*(x) a_\mu^+$$

Kommutator:

$$[\psi_\nu(x), \psi_\mu^+(x')] = \sum_{\nu,\mu} \psi_\nu(x)\psi_\mu^*(x') \underbrace{[a_\nu, a_\mu^+]}_{= \delta_{\nu\mu} \mathbb{1}}$$
$$= \sum_\nu \psi_\nu(x)\psi_\nu^*(x') \mathbb{1} = \delta(x-x') \mathbb{1}$$

$$\psi(x) = \sum_\nu \psi_\nu(x) c_\nu \quad \text{mit } c_\nu = \int \psi_\nu^*(x')\psi(x')dx'$$
$$= \int \underbrace{\sum_\nu \psi_\nu(x)\psi_\nu^*(x')}_{= \delta(x-x')} \psi(x')dx' = \int \delta(x-x')\psi(x')dx'$$
$$= \psi(x)$$