7.1 Quantisierung freier elektromagnetischer Felder

$$\hat{A} = \frac{1}{\sqrt{2}}\sqrt{\frac{\hbar}{\varepsilon\omega}}\frac{1}{\sqrt{V}}\vec{u}\left[e^{i\vec{q}\cdot\vec{r}}c(t) + e^{-i\vec{q}\cdot\vec{r}}c^+\right]$$

$$\hat{\vec{\Pi}} = \frac{1}{\sqrt{2}}\sqrt{\frac{\hbar}{\varepsilon\omega}}\frac{1}{\sqrt{V}}\vec{u}\left[-i\omega e^{i\vec{q}\cdot\vec{r}'}c + i\omega e^{-iq\vec{r}'}c^+\right]$$

$$[\hat{A}_k,\hat{\Pi}_e] = \frac{1}{2}\frac{\hbar}{\varepsilon\omega}\underbrace{\frac{1}{V}u_k u_e}_{\delta_{ke}} i\omega\Big[\underbrace{-e^{iq(\vec{r}+\vec{r}')}c^2}_{\to 0} + \underbrace{e^{iq(\vec{r}-\vec{r}')}cc^+ - e^{-iq(r-\vec{r}')}}_{\delta(\vec{r}-\vec{r}')}c^+c \cdots \underbrace{cc}_{0}\Big]$$

$$[c,c^+] = 1 \quad , \Rightarrow \quad c^+c = cc^+ - 1$$

$$[\hat{A}_k,\hat{\Pi}_e] = \frac{\hbar}{2}\delta_{ke}\,\delta(\vec{r}-\vec{r}')$$

$$\hat{A} = \frac{1}{\sqrt{2}}\sqrt{\frac{\hbar}{\varepsilon\omega}}\frac{\vec{u}}{\sqrt{V}}\left[ac + a^*c^+\right] \; ; \; a = \exp\{i(\vec{q}\cdot\vec{r} - \omega t)\} \; , \vec{u}^2 = 1$$

$$\hat{H} = \varepsilon\hat{\dot{A}}^2 = \frac{1}{2}\frac{\hbar}{\varepsilon\omega}\frac{1}{V}(i\omega)^2\left[(-ac + a^*c^+)(-ac + a^*c^+)\right] \; ; \; aa^* = 1$$

$$= \cdots\cdots\cdots \left[-cc^+ - c^+c\right] \qquad \boxed{cc^+ = c^+c + 1}$$

$$= \frac{1}{2}\frac{\hbar}{\varepsilon\omega}\frac{1}{V}(-\omega^2)\left(-(2c^+c + 1)\right]$$

$$= \frac{1}{2}\frac{\hbar\omega}{\cancel{\varepsilon}}\frac{1}{V}(2c^+c + 1) = \frac{1}{\cancel{\varepsilon}V}\hbar\omega\left(c^+c + \frac{1}{2}\right)$$

$$[c^+c, c] = -c \quad ; \quad [c^+c, c^+] = c^+$$

$$\frac{\partial L}{\partial\dot{\vec{r}}} = m\dot{\vec{r}} + e\vec{A} = \vec{P} \quad , \quad \frac{\partial L}{\partial\vec{r}} = e\frac{\partial}{\partial\vec{r}}\dot{\vec{r}}\cdot\vec{A}$$

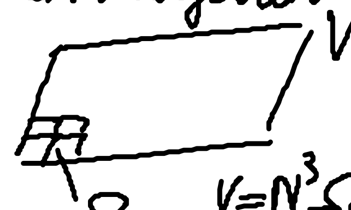
$$\frac{d}{dt}\frac{\partial L}{\partial\dot{\vec{r}}} = m\ddot{\vec{r}} + e\dot{\vec{r}}\cdot\nabla\vec{A} - e\frac{\partial}{\partial\vec{r}}\dot{\vec{r}}\cdot\vec{A} \qquad \Big| \; m\ddot{\vec{r}} = e[\vec{E} + \dot{\vec{r}}\times\vec{B}]$$

$$e\dot{\vec{r}}\times(\nabla\times\vec{A}) = e\nabla\dot{\vec{r}}\cdot\vec{A} - e\dot{\vec{r}}\cdot\nabla\vec{A}$$

$$\int_V \left|\psi_n(\vec{k},\vec{r})\right|^2 d^3r = \frac{1}{N^3} \int_V \left|u_n(\vec{k},\vec{r})\right|^2 d^3r$$

$$= \frac{N^3}{N^3} \int_\Omega \left|u_n(\vec{k},\vec{r})\right|^2 d^3r = 1$$

Grundgebiet



, $V = N^3\Omega$