„makroskopische Maxwellgleichungen" (inhomogen)

$$\nabla \cdot \langle \underline{\underline{E}} \rangle = \frac{\rho_{makr}}{\varepsilon_0} - \frac{\nabla \cdot \langle \underline{P} \rangle}{\varepsilon_0}$$

Analog f. Magnetfeld, Spektroskopische Methoden Bsp. circular Dichroismus

$$\nabla \times \langle \underline{B} \rangle = \mu_0 \langle j_{makr} \rangle + \mu_0 \partial_t \langle \underline{P} \rangle + \mu_0 \nabla \times \langle \underline{M} \rangle + \frac{1}{c^2} \partial_t \langle \underline{E} \rangle$$

Wellengleichung herleitung

Startpunkt

$$\nabla \times \underline{E} = -\frac{\partial B}{\partial t} \qquad | \nabla \times$$

$$\nabla \times \nabla \times \underline{\underline{E}} = -\frac{\partial}{\partial t} \nabla \times \underline{B} = -\mu_0 \partial_t \langle j_{makr} \rangle - \frac{1}{c^2} \partial_t^2 \underline{\underline{E}} - \mu_0 \partial_t^2 \underline{P}$$

$$\nabla \times \nabla \times \underline{\underline{E}} + \frac{1}{c^2} \partial_t^2 \underline{E} = -\mu_0 \partial_t \langle j_{makr} \rangle - \mu_0 \partial_t^2 \underline{P}$$

Verwende $\nabla \times (\nabla \times \underline{a}) = \nabla (\nabla \cdot \underline{a}) - \Delta \underline{a}$

$$-\nabla (\underbrace{\nabla \cdot \underline{E}}_{\frac{\rho_{makr}}{\varepsilon_0} - \frac{\nabla \cdot \underline{P}}{\varepsilon_0}}) + \Delta \underline{E} - \frac{1}{c^2} \partial_t^2 \underline{E} = \mu_0 \partial_t \langle j \rangle + \mu_0 \partial_t^2 \langle \underline{P} \rangle$$

$$\Delta \underline{E} - \frac{1}{c^2}\partial_t^2 \underline{E} = \mu_0 \partial_t \langle j \rangle - \mu_0 \partial_t^2 \langle \underline{P} \rangle + \cancel{\nabla \frac{(\nabla \underline{P})}{\varepsilon_0}} + \cancel{\frac{\nabla (\rho_{ext})}{\varepsilon_0}}$$

$$\underline{P} = \underline{e}_\parallel \cdot e^{i \underline{k} \cdot \underline{r}} \qquad \underline{k} \cdot \underline{e}_\parallel \approx 0$$

Erg.:

$$\Delta E - \frac{1}{c^2}\partial_t^2 \underline{E} = \mu_0 \partial_t j + \mu_0 \partial_t^2 \underline{P}$$

I.1.b) Lsg der Wellengleichung für homogenes Medium

$$\Delta \underline{E}(\omega, \underline{r}) + \frac{1}{c^2}\omega^2 \underline{E}(\omega, \underline{r}) = -\mu_0 \omega^2 \underline{P}(\omega, \underline{r})$$

$$\underline{P}(\omega, \underline{r}) = \varepsilon_0 \chi(\underline{\omega}) \underline{E}(\underline{\omega}, \underline{r})$$

$$\Delta \underline{E}(\omega, \underline{r}) + \frac{1}{c^2}\omega^2 \underline{E}(\omega, \underline{r}) = -\frac{\omega^2}{c^2}\chi(\underline{\omega}) \underline{E}(\underline{\omega}, \underline{r})$$

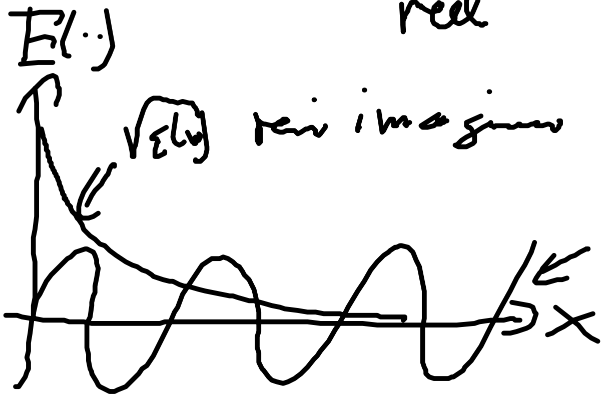
$$\Delta \underline{E}(\omega, \underline{r}) + \frac{1}{c^2}\varepsilon(\omega)\omega^2 E(\underline{\omega}, \underline{r}) = 0$$

$$\varepsilon(\omega) = 1 + \chi(\omega)$$

$$\underline{E}(\omega, \underline{r}) = \int d\Omega \left(c_1(\omega, \Omega) e^{i \underline{k}(\omega, \Omega) \cdot \underline{r} - i\omega t} + c_2(\omega, \Omega) e^{-i \underline{k}(\omega, \Omega) \cdot \underline{r} + i\omega t} \right)$$

$$\underline{k}(\omega, \Omega) = \underline{e}_r(\Omega) \frac{\omega n}{c} \qquad n = \sqrt{\varepsilon(\omega)}$$

$$\sqrt{\varepsilon(\omega)} = \underbrace{\tilde{n}(\omega)}_{\text{reel}} + \underbrace{i\kappa(\omega)}_{\text{imaginär}}$$



II 1c) Absorptionsspektrum

Wir wissen wie Polarisation mit E-Zusammenhängt:

$$\underline{P}^{(1)}(\underline{r},t) = \int d\underline{r}_1 \int_{-\infty}^{t} dt_1\, \chi^{(1)}(\underline{r}-\underline{r}_1, t-t_1)\, \underline{E}(\underline{r}_1, t_1)$$

Wie sieht ε dann aus?

$$\varepsilon(\underline{r}-\underline{r}_1, t-t_1) = \delta(t-t_1)\,\delta(\underline{r}-\underline{r}_1) + \varepsilon_0\, \chi^{(1)}(\underline{r}-\underline{r}_1, t-t_1)$$

$$\Delta \underline{E} - \frac{1}{c^2}\partial_t^2 \int dr' \int_{-\infty}^{t} dt' \left(\varepsilon(\underline{r}-\underline{r}', t-t')\, \underline{E}(\underline{r}', t')\right) = 0$$

Ansatz

$$E(z,t) = E_0\, e^{ikz-i\omega t}$$

$$\partial_z^2 E_0\, e^{ikz-i\omega t} - \frac{1}{c^2}\partial_t^2 \int dr' \int_{-\infty}^{t} dt'\, \varepsilon(r-\underline{r}', t-t')\, e^{ikz'-i\omega t'}$$

$$1 = 0$$

$$e^{-ikz+ikz+i\omega t-i\omega}$$

Definiere $\underline{\varepsilon}(\underline{k},\omega) = \int d\underline{r} \int_0^\infty dt_1\, \varepsilon(\underline{r}-\underline{r}', t_1)\, e^{-i\underline{k}\cdot(\underline{r}-\underline{r}')+i\omega t_1}$

$t_1 = t - t'$

$\underline{r} \to \underline{r} - \underline{r}'$

Damit ist es das Gleiche wie

$$-k^2 + \frac{\omega^2}{c^2}\varepsilon(k\cdot e_z, \omega) = 0$$

$$\left\| \frac{k^2c^2}{\omega^2} = \varepsilon(k,\omega) \right\|$$

$$\varepsilon(k,\omega) = 1 + \varepsilon_0 \chi(\underline{k},\omega)$$

$$\chi(\underline{k},\omega) = \int d\underline{r} \int_0^\infty dt\, \chi^{(1)}(\underline{r},t)\, e^{-i\underline{k}x+i\omega t}$$

Nichtmagnetisches Medium

$$\frac{kc}{\omega} = \sqrt{\varepsilon(k,\omega)} = \tilde{n}(\omega) + i\varkappa(\omega)$$

$$\underline{E}(z,t) = \underline{E}_0 \exp(ikz - i\omega t) = \underline{E}_0 \exp\left(i\tilde{n}(\omega)\frac{\omega}{c}\cdot z - \varkappa(\omega)\frac{\omega}{c}z - i\omega t\right)$$

$$= \underline{E}_0 \exp\left(ik'z - \varkappa'_a(\omega)\frac{z}{2}\right)$$

$$k' = \frac{\omega\tilde{n}(\omega)}{c} \qquad \varkappa'_a(\omega) = \frac{2\omega\varkappa(\omega)}{c}$$

$$I(z) = |E(z,t)|^2 = I_0 \exp(-\varkappa'_a(\omega) z)$$

$$\sqrt{\varepsilon(k,\omega)} = \tilde{n}(\omega) + i\varkappa(\omega)$$

$$\| $$

$$\sqrt{1 + \varepsilon_0\chi'(k,\omega) + i\varepsilon_0\chi''(k,\omega)}$$

$$1+\varepsilon_0\chi'(k,\omega)+i\varepsilon_0\chi''(k,\omega) = (\hat n(\omega)+i\kappa(\omega))^2$$
$$= \hat n^2(\omega)+2i n(\omega)\kappa(\omega)-\kappa^2(\omega)$$

$$1+\varepsilon_0\chi'(k,\omega) = \hat n^2(k,\omega)-\kappa^2(k,\omega)$$

$$\varepsilon_0\chi''(k,\omega) = 2\hat n(k,\omega)\,\kappa(k,\omega)$$

$$\kappa(\omega) = \varepsilon_0\frac{\chi''(k,\omega)}{2\hat n(k,\omega)}$$

$$1+\varepsilon_0\chi'(k,\omega) = \hat n^2(\omega)-\varepsilon_0^2\frac{\chi''(k,\omega)^2}{4\hat n^2(\omega)} \quad \Big|\cdot\hat n^2(\omega)\,4$$

$$4\hat n^4(\omega)-\hat n^2(k,\omega)\,4\,(1+\varepsilon_0\chi(k,\omega))+\varepsilon_0^2\,\chi''(k,\omega)^2 = 0$$

$$2\hat n^2(\omega) = 1+\varepsilon_0\chi'(k,\omega)+\left((1+\varepsilon_0\chi(k,\omega))^2-(\varepsilon_0\chi''(k,\omega))^2\right)^{\frac{1}{2}}$$

$$\varepsilon_0\chi'' \ll 1+\varepsilon_0\chi' \qquad \sqrt{1+x}$$

$$2\hat n^2(\omega) = 1+\varepsilon_0\chi'(k,\omega)+1+\varepsilon_0\chi'(k,\omega)$$

$$\hat n(\omega) = \sqrt{1+\varepsilon_0\chi'}$$

$$\alpha(\omega) = \kappa'_a(\omega) = \frac{2\omega}{c}\frac{\chi''(\omega)}{2\hat n(\omega)}\varepsilon_0 = \varepsilon_0\frac{\omega}{\hat n(\omega)c}\chi'(\omega)$$

$$I(z) = I_0\exp(-\kappa_a(\omega)z))$$
$$= I_0\exp(-\alpha(\omega)z))$$

$$\Big\|\ \alpha(\omega) = \varepsilon_0\frac{\omega}{n(\omega)c}\,\mathrm{Im}\,\chi(\omega)\ \Big\|$$

$P = \chi E \Leftarrow$ linear Spektrum

z.B. Rayleigh Streuung