

„in akustische Max wellgleichungen“ (inhomogen)

$$\| \nabla \cdot \underline{\underline{E}} \rangle = \frac{p_{\text{meh}}}{\epsilon_0} - \frac{\nabla \cdot \langle \underline{P} \rangle}{\epsilon_0}$$

Analog f. Magnetfeld, Spektroskopische Methoden Bsp. circular Dichroismus

$$\| \nabla \times \langle \underline{B} \rangle = \mu_0 \langle \underline{j}_{\text{meh}} \rangle + \mu_0 \partial_t \langle \underline{P} \rangle + \mu_0 \nabla \times \langle \underline{M} \rangle + \frac{1}{c^2} \partial_t \langle \underline{E} \rangle$$

Wellengleichung herleitung

Startpunkt

$$\nabla \times \underline{E} = - \frac{\partial \underline{B}}{\partial t} \quad | \nabla \times$$

$$\nabla \times \nabla \times \underline{E} = - \frac{\partial}{\partial t} \nabla \times \underline{B} = - \mu_0 \partial_t \langle \underline{j}_{\text{meh}} \rangle$$

$$\nabla \times \nabla \times \underline{E} + \frac{1}{c^2} \partial_t^2 \underline{E} = - \mu_0 \partial_t \langle \underline{j}_{\text{meh}} \rangle - \mu_0 \partial_t^2 \underline{P}$$

Verwend  $\nabla \times (\nabla \times \underline{a}) = \nabla (\nabla \cdot \underline{a}) - \Delta \underline{a}$

$$- \nabla (\nabla \cdot \underline{E}) + \Delta \underline{E} - \frac{1}{c^2} \partial_t^2 \underline{E} = \mu_0 \partial_t \langle \underline{j} \rangle + \mu_0 \partial_t^2 \langle \underline{P} \rangle$$

$$\frac{p_{\text{meh}}}{\epsilon_0} \sim \frac{\nabla \cdot \underline{P}}{\epsilon_0}$$

$$\Delta \underline{E} - \frac{1}{c^2} \partial_t^2 \underline{E} = \mu_0 \partial_t \underline{j} - \mu_0 \partial_t^2 \underline{P} + \nabla \left( \frac{\underline{D}}{\epsilon_0} \right)$$

$$+ \frac{\nabla(\rho_{ext})}{\epsilon_0}$$

$$\underline{P} = \underline{e}_k \cdot e^{i \underline{k} \cdot \underline{r}} \quad \underline{k} \cdot \underline{e}_k \approx 0$$

I. go:

$$\| \Delta \underline{E} - \frac{1}{c^2} \partial_t^2 \underline{E} = \mu_0 \partial_t \underline{j} + \mu_0 \partial_t^2 \underline{P} \|$$

I.1. b) Lsg der Wellengleichung für homogenes Medium

$$\Delta \underline{E}(\omega, \underline{r}) + \frac{1}{c^2} \omega^2 \underline{E}(\omega, \underline{r}) = -\mu_0 \omega^2 \underline{P}(\omega, \underline{r})$$

$$\underline{P}(\omega, \underline{r}) = \epsilon_0 \chi(\omega) \underline{E}(\omega, \underline{r})$$

$$\Delta \underline{E}(\omega, \underline{r}) + \frac{1}{c^2} \omega^2 \underline{E}(\omega, \underline{r}) = -\frac{\omega^2}{c^2} \chi(\omega) \underline{E}(\omega, \underline{r})$$

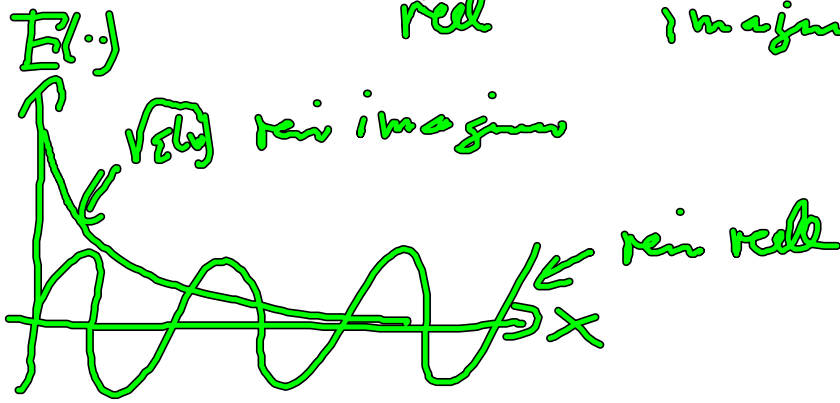
$$\Delta \underline{E}(\omega, \underline{r}) + \frac{1}{c^2} \epsilon(\omega) \omega^2 \underline{E}(\omega, \underline{r}) = 0$$

$$\epsilon(\omega) = 1 + \chi(\omega)$$

$$\underline{E}(\omega, \underline{r}) = \int d\Omega (c_1(\omega, \Omega) e^{i \underline{k}(\omega, \Omega) \cdot \underline{r} - i \omega t} + c_2(\omega, \Omega) e^{-i \underline{k}(\omega, \Omega) \cdot \underline{r} + i \omega t})$$

$$\underline{k}(\omega, \Omega) = \underline{e}_r(\Omega) \frac{\omega n}{c} \quad n = \sqrt{\epsilon(\omega)}$$

$$\sqrt{\epsilon(\omega)} = \underbrace{\tilde{n}(\omega)}_{\text{real}} + i \underbrace{\kappa(\omega)}_{\text{imaginär}}$$



## II 1c) Absorbtionspektrum

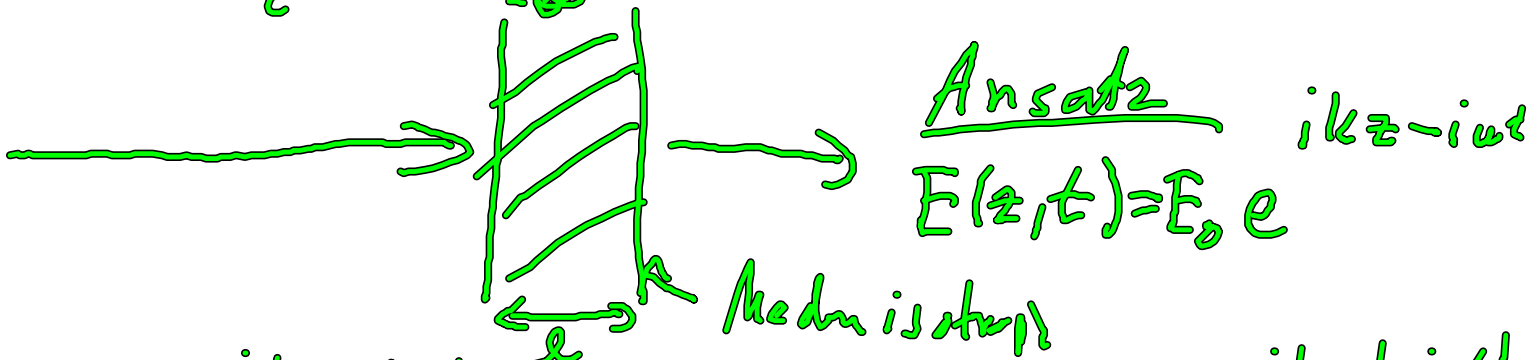
Wir wissen die Polarisierung mit E-Zusammenhang:

$$P^{(1)}(\underline{r}, t) = \int d\underline{r}_1 \int_{-\infty}^t dt_1 \chi^{(1)}(\underline{r}-\underline{r}_1, t-t_1) \underline{E}(\underline{r}_1, t_1)$$

Wie sieht  $\epsilon$  dann aus?

$$\epsilon(\underline{r}-\underline{r}_1, t-t_1) = \delta(t-t_1)\delta(\underline{r}-\underline{r}_1) + \epsilon_0 \chi^{(1)}(\underline{r}-\underline{r}_1, t-t_1)$$

$$\Delta \underline{E} - \frac{1}{c^2} \partial_t^2 \int d\underline{r}' \int_{-\infty}^t dt' (\epsilon(\underline{r}-\underline{r}', t-t') \underline{E}(\underline{r}', t')) = 0$$



$$\partial_z^2 E_0 e^{ikz - i\omega t} - \frac{1}{c^2} \partial_t^2 \int d\underline{r}' \int_{-\infty}^t dt' \epsilon(\underline{r}-\underline{r}', t-t') e^{ikz' - i\omega t'}$$

Definition

$$\underline{\epsilon}(k, \omega) = \int d\underline{r} \int_0^{\infty} dt_1 \underline{\epsilon}(\underline{r} - \underline{r}', t_1) e^{-i\underline{k} \cdot (\underline{r} - \underline{r}') + i\omega t_1}$$

$$-i\underline{k} \cdot \underline{r} + i\underline{k} \cdot \underline{r}' + i\omega t - i\omega t = 0$$

Damit ist es das gleiche

$$t_0 = t - t'$$

$$\underline{r} \rightarrow \underline{r} - \underline{r}'$$

$$-k^2 + \frac{\omega^2}{c^2} \underline{\epsilon}(k, \omega) = 0$$

$$\left\| \frac{k^2 c^2}{\omega^2} = \underline{\epsilon}(k, \omega) \right\|$$

$$\underline{\epsilon}(k, \omega) = 1 + \epsilon_0 \chi(k, \omega)$$

$$\chi(k, \omega) = \int d\underline{r} \int_0^{\infty} dt \chi^{(1)}(\underline{r}, t) e^{-i\underline{k} \cdot \underline{r} + i\omega t}$$

Nur homogene Medien

$$\frac{k c}{\omega} = \sqrt{\underline{\epsilon}(k, \omega)} = \tilde{n}(\omega) + i\kappa(\omega)$$

$$E(\underline{z}, t) = E_0 \exp(i\underline{k} \cdot \underline{z} - i\omega t) = E_0 \exp(i\tilde{n}(\omega) \frac{\omega}{c} \cdot \underline{z} - \kappa(\omega) \frac{\omega}{c} \cdot \underline{z} - i\omega t)$$

$$k' = \frac{\omega \tilde{n}(\omega)}{c} = E_0 \exp(i k' \cdot \underline{z} - \kappa'_a(\omega) \frac{z}{c})$$

$$\kappa'_a(\omega) = \frac{2 \omega \kappa(\omega)}{c}$$

$$I(z) = |E(z, t)|^2 = I_0 \exp(-\kappa'_a(\omega) z)$$

$$\sqrt{\underline{\epsilon}(k, \omega)} = \tilde{n}(\omega) + i\kappa(\omega)$$

||

$$\sqrt{1 + \epsilon_0 \chi'(k, \omega) + i \epsilon_0 \chi''(k, \omega)}$$

$$1 + \varepsilon_0 \chi'(k, \omega) + i \varepsilon_0 \chi''(k, \omega) = (\tilde{n}(\omega) + i \chi''(\omega))^2 \\ \Rightarrow \tilde{n}^2(\omega) + 2i \tilde{n}(\omega) \chi''(\omega) - \chi''^2(\omega)$$

$$1 + \varepsilon_0 \chi'(k, \omega) = \tilde{n}^2(k, \omega) - \chi''^2(k, \omega)$$

$$\varepsilon_0 \chi''(k, \omega) = 2 \tilde{n}(k, \omega) \chi''(k, \omega)$$

$$\chi''(\omega) = \varepsilon_0 \frac{\chi''(k, \omega)}{2 \tilde{n}(k, \omega)}$$

$$1 + \varepsilon_0 \chi'(k, \omega) = \tilde{n}^2(\omega) - \varepsilon_0^2 \frac{\chi''(k, \omega)^2}{4 \tilde{n}^2(\omega)} \quad | \cdot \tilde{n}^4(\omega) / 4$$

$$4 \tilde{n}^4(\omega) - \tilde{n}^2(k, \omega) 4 (1 + \varepsilon_0 \chi'(k, \omega)) + \varepsilon_0^2 \chi''(k, \omega)^2 = 0$$

$$2 \tilde{n}^2(\omega) = 1 + \varepsilon_0 \chi'(k, \omega) + \left( \underbrace{1 + \varepsilon_0 \chi'(k, \omega)}^2 - \underbrace{(\varepsilon_0 \chi''(k, \omega))^2}^{\varepsilon_0^2} \right)^{1/2}$$

$$\underbrace{\varepsilon_0 \chi''}_{\varepsilon_0 \chi''} \ll \underbrace{1 + \varepsilon_0 \chi'}_{1 + \varepsilon_0 \chi'} \quad \sqrt{1+x}$$

$$2 \tilde{n}^2(\omega) = 1 + \varepsilon_0 \chi'(k, \omega) + 1 + \varepsilon_0 \chi'(k, \omega)$$

$$\tilde{n}(\omega) = \sqrt{1 + \varepsilon_0 \chi'}$$

$$\alpha(\omega) = \chi''_a(\omega) = \frac{2\omega}{c} \frac{\chi''(\omega)}{2 \tilde{n}(\omega)} \varepsilon_0 = \varepsilon_0 \frac{\omega}{\tilde{n}(\omega) c} \chi''(\omega)$$

$$I(z) = I_0 \exp(-\alpha_a(\omega) z)$$

$$= I_0 \exp(-\alpha(\omega) z)$$

$$\| \alpha(\omega) = \varepsilon_0 \frac{\omega}{\tilde{n}(\omega) c} \operatorname{Im} \chi(\omega) \|$$

$\rho = \chi E \Leftarrow$  linear Spektrum

z.B. Rayleigh Streuung