VI. Ausblick auf bessere Modellmodelle

Bisher bei Diffusion, exponentieller Zerfall angenommen

\[ p(t) = p(0) e^{-rt} \]

Dass ist das einfachste Modell.

Trotz welches gut ist, ist das einfach falsch.

Zu den besseren Modellen.

\[ \uparrow \quad \downarrow \quad \text{kappt an Reservoir von hom.} \]
\[ \text{Oszillatoren} \]

\[ H = \sum_{1 \leq i < j \leq n} \mathcal{J}_{ij} \mathbf{c}_i^+ \mathbf{c}_j + \sum_{i=1}^n \epsilon_i \mathbf{c}_i^+ \mathbf{c}_i \]

- Eigenzustände
- hom. Osz.
- in Bose-Stats
- keine Kopplung
- zur OSZ. und E1

Somit ist
\[ p(t) = tr \langle u^\dagger(H_1^0 < u^\dagger U(t, t_0) g(t_0) U^\dagger(t, t_0)) u(t_0) \rangle \]

\[ \begin{bmatrix} U(t, t_0) = e^{-\frac{i}{\hbar} H(t-t_0)} \end{bmatrix} \]

Anh. result:

\[ S(t_0) = S_{el} \otimes S_B \]

\[ = tr \langle U^\dagger(t, t_0) u^\dagger u U(t, t_0) \rangle \]

\[ = tr \langle \underbrace{u^\dagger u U(t, t_0)}_{S_{el}} \rangle + \ldots \]

\[ = tr \langle \underbrace{\frac{i}{\hbar} [Q_{el}^2 - 2i \hbar \epsilon_2(t-t_0)]}_{S_{el}} e^{-\frac{i}{\hbar} (\frac{\hbar^2}{2} \tilde{H}_{el} \phi^2 + \hbar \epsilon_0 \phi)(t-t_0)} \rangle \]

\[ \tilde{H}_{el} \phi^2 = \frac{1}{2} \sum_i \delta_{ij} (c_i \frac{\gamma}{\hbar} c_i^\dagger) \]

\[ = e^{\frac{i}{\hbar} (\gamma \cdot \epsilon_2)(t-t_0)} tr \langle \underbrace{\frac{i}{\hbar} H_{el} \phi^2(t-t_0)}_{S_{el}} e^{-\frac{i}{\hbar} (\frac{\hbar^2}{2} \tilde{H}_{el} \phi^2 + \hbar \epsilon_0 \phi)(t-t_0)} \rangle \]

Feynman - Disentanglement Theorem:

\[ V(t) = \frac{T_{\hbar}}{\hbar} \exp\left( \int_{t_0}^{t} dt' \left( A_1(t') + A_2(t') \right) \right) \]

\[ V(t) = V_1(t) V_2(t) \quad V_1(t) = T_{\hbar} \exp\left( \int_{t_0}^{t} dt' A_1(t') \right) \]
\[
V_2(t) = T e^{\int_t^{t_0} dt' V^{-1}(t') A(t') V(t')} = \exp(\int_t^{t_0} dt' e^{-\frac{i}{\hbar} \tilde{H}_{\text{phys}}(t')}) e^{-\frac{i}{\hbar} \tilde{H}_{\text{phys}}(t)}
\]

\[
\text{tr}_B \left( e^{ \frac{i}{\hbar} \tilde{H}_{\text{phys}}(t-t_0)} \right) \exp(\int_{t_0}^{t} dt' e^{-\frac{i}{\hbar} \tilde{H}_{\text{phys}}(t')})^{-1} = e^{\int_{t_0}^{t} dt' \tilde{H}_{\text{phys}}(t')} = 1 d + e^{\frac{i}{\hbar} \tilde{H}_{\text{phys}}(t-t_0)} S_B
\]

\[
\text{tr}_B \left( T e^{\int_{t_0}^{t} dt' e^{-\frac{i}{\hbar} \tilde{H}_{\text{phys}}(t')}} \right) S_B
\]

\[
\exp(\int_t^{t_0} dt' \tilde{H}_{\text{phys}}(t')) = \exp(\int_t^{t_0} dt' \frac{i}{\hbar} \tilde{H}_{\text{phys}}(t')) = \exp(\int_t^{t_0} dt' \frac{i}{\hbar} \sum_{\gamma} \tilde{\omega}_\gamma \gamma \tilde{\gamma} (\gamma t') + \gamma e^{-i\tilde{\omega}_\gamma (t' - t)} + \gamma e^{i\tilde{\omega}_\gamma (t' - t)}) S_B
\]

Muß landet wrong

\[
R(t) = \text{tr}_B \left( T e^{\int_{t_0}^{t} dt' \frac{i}{\hbar} \tilde{\gamma} (\gamma t') \left( \gamma e^{-i\tilde{\omega}_\gamma (t' - t)} + \gamma e^{i\tilde{\omega}_\gamma (t' - t)} \right) S_B \right)
\]

Cann what a wickedly

\[
\text{dec.} \quad \frac{1}{2} \left( \tilde{\omega}_\gamma (t) - \tilde{\omega}_\gamma (t) \right) = - \cdot \cdot \cdot
\]

\[
R(t) = e^{\frac{1}{2} \text{Ord} 2} e^{i \text{Ord} 4} e^{i \text{Ord} 8}
\]

\[
R(0) = \text{tr}_B (S_B) = 1 = 1 e^{-\tilde{\omega}_\gamma (t) - \tilde{\omega}_\gamma (t)}
\]
\[ R(t) = \text{tr}(-i \int_{t_0}^{t} dt' \sum \mathbf{q}_i (c^+ e^{-i \omega_i (t-t')} + c e^{i \omega_i (t-t')} g_{\mathbf{q}_i}) = 0 = \text{e}^{-\mathcal{L}(t)} \]

\[ \text{D} \omega \text{tr}(c^+ g_{\mathbf{q}_i}) = 0 \]

2. Ord.

\[ \text{tr} \left[ -i \int_{t_0}^{t} dt' \sum \mathbf{q}_i (c^+ e^{-i \omega_i (t-t')} + c e^{i \omega_i (t-t')} \int dt'' \sum \mathbf{q}_i (c^+ e^{-i \omega_i (t-t'')} + c e^{i \omega_i (t-t'')} g_{\mathbf{q}_i}) \right] \]

\[ \text{D} \omega \text{tr}(c^+ g_{\mathbf{q}_i}) = 0, \quad \text{tr}(c^+ g_{\mathbf{q}_i}) = 0, \quad \text{tr}(c^+ c g_{\mathbf{q}_i}) = \delta_{ij}, \quad \text{tr}(c^+ c^+ g_{\mathbf{q}_i}) = \delta_{ii}, \quad \eta_j \]

\[ = -\frac{A}{\hbar^2} \sum \frac{g_{\mathbf{q}_i}^2}{\omega_j^2} \left[ \frac{1 - e^{-i \omega_j (t-t_0)}}{\omega_j} \right] \eta_j + \frac{1 - e^{-i \omega_j (t-t_0)}}{\omega_j^2} \left( \frac{1}{\eta_j+1} + \frac{1}{\omega_j} \right) \]

\[ = -\frac{A}{\hbar^2} \sum \frac{g_{\mathbf{q}_i}^2}{\omega_j^2} \left[ \frac{1 - e^{-i \omega_j (t-t_0)}}{\omega_j} \right] \eta_j + \frac{1 - e^{-i \omega_j (t-t_0)}}{\omega_j^2} \left( \frac{1}{\eta_j+1} + \frac{1}{\omega_j} (t-t_0) \right) \]

Müssen wir noch immer nicht kühl sein?

Nein! Wann?