

C) Kohärenter Laserstrahl

Eigenwertglg von c : $c|\alpha\rangle = \alpha|\alpha\rangle$

α : Eigenwert von c , komplex $\alpha = |\alpha| \exp\{i\varphi\}$

$$\alpha = \langle \alpha | c | \alpha \rangle = \langle c^\dagger \alpha | \alpha \rangle = \langle \alpha | c^\dagger | \alpha \rangle^*$$

$$\Rightarrow c^\dagger | \alpha \rangle = \alpha^* | \alpha \rangle$$

$$\hat{E} = i \sqrt{\frac{\hbar\omega}{2\varepsilon V}} \vec{u} \left[\exp\{i(\vec{q}\cdot\vec{r}-\omega t)\} c - \exp\{-i(\vec{q}\cdot\vec{r}-\omega t)\} c^\dagger \right]$$

$$\begin{aligned} \langle \alpha | \hat{E} | \alpha \rangle &= i \sqrt{\frac{\hbar\omega}{2\varepsilon V}} \vec{u} \left[\exp\{i(\vec{q}\cdot\vec{r}-\omega t)\} \alpha - \exp\{-i(\vec{q}\cdot\vec{r}-\omega t)\} \alpha^* \right] \\ &= i \sqrt{\frac{\hbar\omega}{2\varepsilon V}} \hbar \vec{u} \left[\exp\{i(\vec{q}\cdot\vec{r}-\omega t+\varphi)\} - \exp\{-i(\vec{q}\cdot\vec{r}-\omega t+\varphi)\} \right] \\ &= - \sqrt{\frac{2\hbar\omega}{\varepsilon V}} |\alpha| \vec{u} \sin\{\vec{q}\cdot\vec{r}-\omega t+\varphi\} \end{aligned}$$

$$[c, c^\dagger] = 1 \quad \text{oder} \quad c c^\dagger = c^\dagger c + 1$$

$$\begin{aligned} [c, c^{\dagger n}] &= c c^{\dagger n} - c^{\dagger n} c \\ &= c^\dagger c c^{\dagger(n-1)} + c^{\dagger(n-1)} - c^{\dagger n} c \\ &= c^{\dagger 2} c c^{\dagger(n-2)} + c^{\dagger(n-1)} + c^{\dagger(n-1)} - c^{\dagger n} c \\ &= c^{\dagger n} c + n c^{\dagger(n-1)} - c^{\dagger n} c = n c^{\dagger(n-1)} \end{aligned}$$

Beweis von $\exp\{a+b\} = \exp\{a\} \exp\{b\} \exp\{-\frac{x}{2}\}$

mit $[a, b] = x$, $[a, x] = 0 = [b, x]$, $[a, b^n] = n b^{n-1} x$; $[a^n, b] = n a^{n-1} x$

$$1) \exp\{a+b\} = \sum_{n=0}^{\infty} \frac{1}{n!} (a+b)^n$$

$$= 1 + (a+b) + \frac{1}{2} (a+b)^2 + \frac{1}{3!} (a+b)^3 + \frac{1}{4!} (a+b)^4 + \frac{1}{5!} (a+b)^5$$

$$2) \exp\{a\} \exp\{b\} = \left(1 + a + \frac{1}{2} a^2 + \frac{1}{3!} a^3 + \dots\right) \left(1 + b + \frac{1}{2} b^2 + \frac{1}{3!} b^3 + \dots\right)$$

$$(a+b)_g^2 = a^2 + 2ab + b^2$$

$$(a+b)_g^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)_g^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$(a+b)_g^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

$$3) (a+b)^2 = a^2 + ab + ba + b^2 = (a+b)_g^2 - x$$

$$(a+b)^3 = a^3 + a^2b + aba + ab^2 + ba^2 + bab + b^2a + b^3$$
$$= (a+b)_g^3 - 3x(a+b)$$

$$(a+b)^4 = (a+b)_g^4 - 6x(a+b)_g^2 + 3x^2$$

$$(a+b)^5 = (a+b)_g^5 - 10(a+b)_g^3 x + 15(a+b)x^2$$

$$4) \exp\{a+b\} = 1 + (a+b) + \frac{1}{2} (a+b)^2 + \frac{1}{3!} (a+b)^3 + \dots$$

$$= \exp\{a\} \exp\{b\} \exp\{-\frac{x}{2}\}$$