

# Kontrolle rauschinduzierter Oszillationen im Van-der-Pol-Modell

## Analyt. Näherung der spektralen Leistungsdichte:

Fourier-Transformierte der mean-field-Gln

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= \tilde{\epsilon} y - \omega_0^2 x + D \xi(t) \\ &\quad + K [y(t-\tau) - y(t)] \end{aligned}$$

$$x(t) = \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \hat{x}(\omega)$$

$$-i\omega \hat{x}(\omega) = \hat{y}(\omega)$$

$$-i\omega \hat{y}(\omega) = \tilde{\epsilon} \hat{y}(\omega) - \omega_0^2 \hat{x}(\omega) + D \hat{\xi}(\omega) + K \hat{y}(\omega) (e^{i\omega\tau} - 1)$$

Elim. von  $\hat{x}(\omega) = \frac{1}{\omega} \hat{y}(\omega)$ :

$$-i\omega \hat{y} - \tilde{\epsilon} \hat{y} + i\frac{\omega_0^2}{\omega} \hat{y} - K(e^{i\omega\tau} - 1) \hat{y} = D \hat{\xi} \quad | \cdot i\omega$$

$$\hat{y}(\omega) = \frac{i\omega D \hat{\xi}(\omega)}{\omega^2 - \omega_0^2 - i\omega \tilde{\epsilon} - i\omega K(e^{i\omega\tau} - 1)}$$

$$\langle \hat{y}(\omega) \hat{y}^*(\omega') \rangle = \frac{(i\omega D)(-i\omega' D) \langle \hat{\xi}(\omega) \hat{\xi}^*(\omega') \rangle}{(\omega^2 - \omega_0^2 - i\omega [\tilde{\epsilon} + K(e^{i\omega\tau} - 1)]) (\omega'^2 - \omega_0^2 + i\omega' [\tilde{\epsilon} + K(e^{-i\omega'\tau} - 1)])}$$

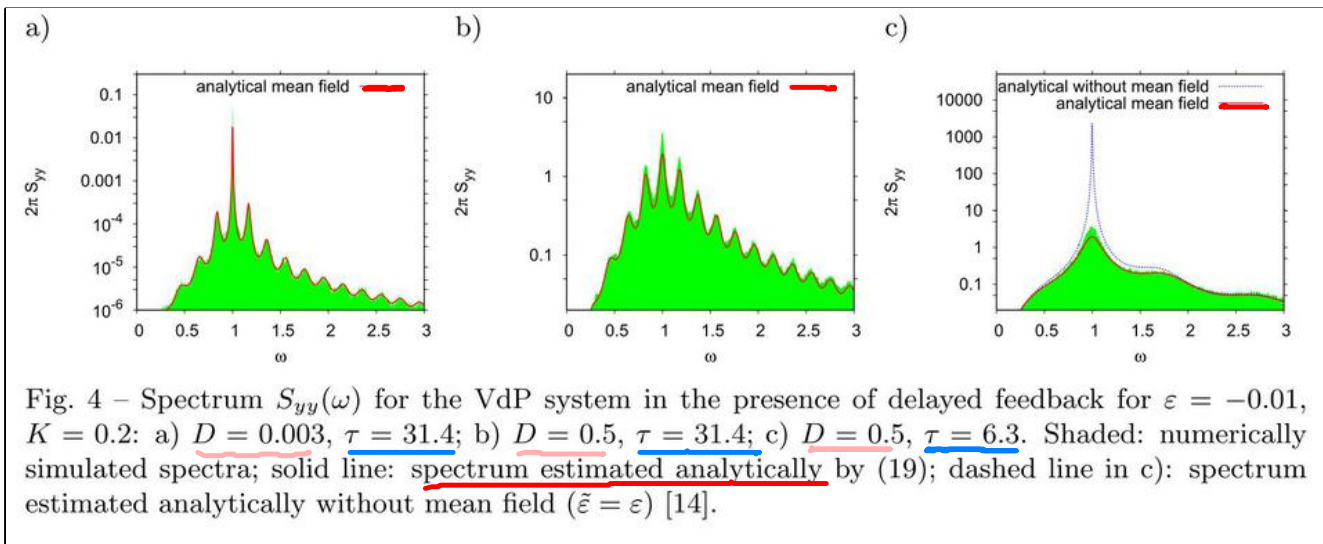
$$\begin{aligned} \langle \hat{\xi}(\omega) \hat{\xi}^*(\omega') \rangle &= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dt' e^{-i\omega' t'} \underbrace{\langle \xi(t) \xi(t') \rangle}_{\delta(t-t')} \\ &= \frac{1}{2\pi} \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{i(\omega - \omega')t} \\ &\quad \underbrace{\hspace{10em}}_{\delta(\omega - \omega')} \end{aligned}$$

$$\omega = \omega' \Rightarrow \langle \hat{y}(\omega) \hat{y}^*(\omega) \rangle = \frac{D^2}{2\pi} \frac{\omega^2 \delta(\omega - \omega')}{(\omega^2 - \omega_0^2 + \omega K \sin \omega \tau)^2 + \omega^2 (\tilde{\epsilon} - K(1 - \cos \omega \tau))^2}$$

Wiener-Khinchin-Theorem:

$$\begin{aligned} \langle \hat{y}(\omega) \hat{y}^*(\omega') \rangle &= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dt' e^{-i\omega' t'} \langle y(t) y(t') \rangle \\ &= \underbrace{\frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{i(\omega - \omega')t}}_{\delta(\omega - \omega')} \underbrace{\frac{1}{2\pi} \int_{-\infty}^{\infty} ds e^{i\omega'(t-t')} \langle y(t) y(t+s) \rangle}_{S(\omega')} \end{aligned}$$

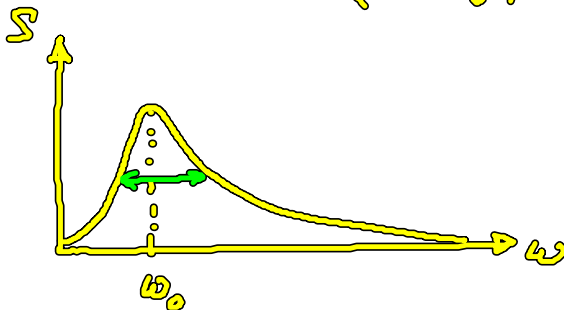
$$\Rightarrow S_{yy}(\omega) = \frac{D^2}{2\pi} \frac{\omega^2}{(\omega^2 - \omega_0^2 + D K \sin \omega \tau)^2 + \omega^2 (\tilde{\epsilon} - K(1 - \cos \omega \tau))^2}$$



Pomplun et al., Europhys. Lett. 2005

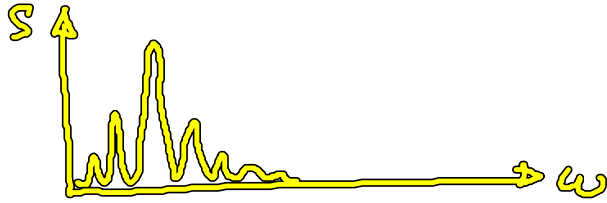
herausragende Übereinstimmung der mean-field-Näherung mit der vollen nichtlin. Simul. der Dgl., sogar für große D

$$K=0: S(\omega) = \frac{D^2}{2\pi} \frac{\omega^2}{(\omega^2 - \omega_0^2)^2 + \omega^2 \tilde{\epsilon}^2}$$



Lorentzkurve mit Max. bei  $\omega_0$  u. Halbwertsbreite  $\approx \frac{|\tilde{\epsilon}|}{2} \approx \frac{|\tilde{\epsilon}|}{2} = \frac{2}{\pi \tau \epsilon_{\text{car}}}$

$K \neq 0$ : immer mehr Nebenmax. mit zunehmendem  $\tau$



## Grenzisches Modell für Anregbarkeit Typ I

(knapp unterhalb der SNIPER-Bifurkation)

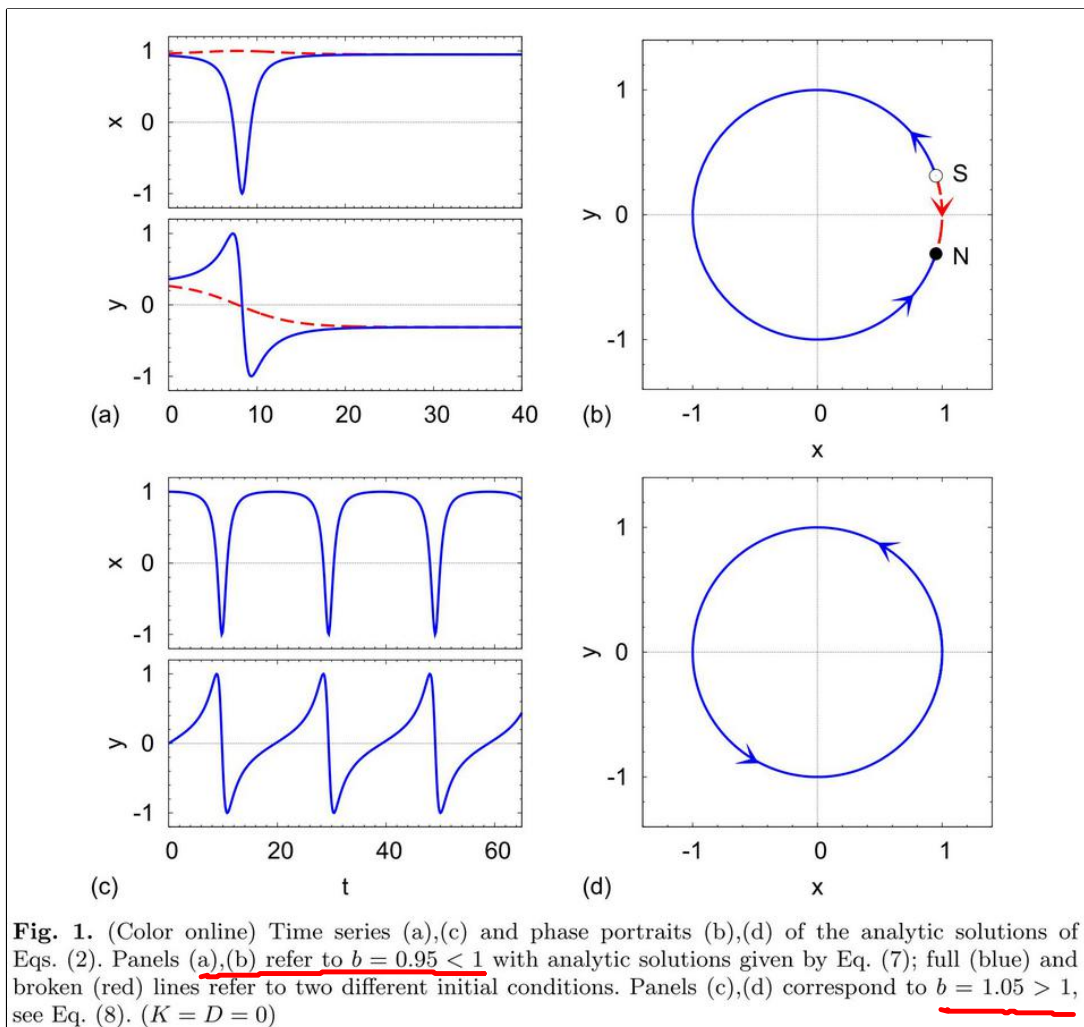
Aust, Hövel, Hizanidis, Schöll, Eur. Phys. J - ST 187, 77 (2010)

$$\begin{aligned} \dot{x} &= x(1-x^2-y^2) + y(x-b) + \mathcal{D}\mathcal{F}(t) + K[x(t-\tau) - x(t)] \\ \dot{y} &= y(1-x^2-y^2) - x(x-b) + \mathcal{D}\mathcal{F}(t) + K[y(t-\tau) - y(t)] \end{aligned}$$

$$K=D=0$$

$b < 1$ : stab. Fixp. (Knoten), Anregbares Regime

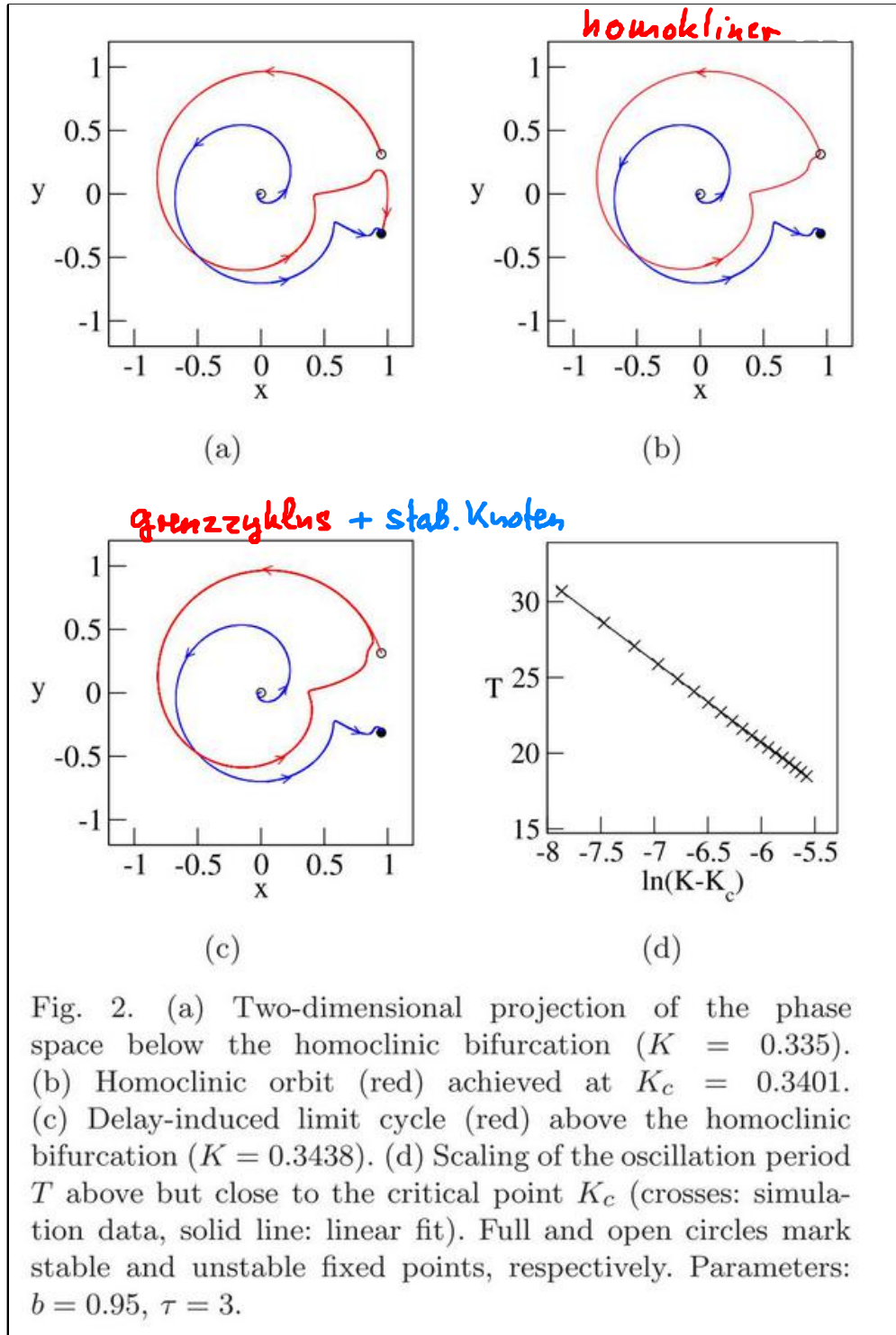
$b = 1$ : Sattel-Knoten-Bif. auf Grenzzyklus  $r=1$ ,  $T \rightarrow \infty$



$\mathcal{D} = 0$  : Delay-induzierte globale (homokline) Bifurk.

Fixpt. N (node) bleibt immer stabil, aber homokline Bif.

 einen stab. Grenzzyklus,  $r > 0, T \rightarrow \infty$



$T \sim \ln |K - K_c|$

Fig. 2. (a) Two-dimensional projection of the phase space below the homoclinic bifurcation ( $K = 0.335$ ). (b) Homoclinic orbit (red) achieved at  $K_c = 0.3401$ . (c) Delay-induced limit cycle (red) above the homoclinic bifurcation ( $K = 0.3438$ ). (d) Scaling of the oscillation period  $T$  above but close to the critical point  $K_c$  (crosses: simulation data, solid line: linear fit). Full and open circles mark stable and unstable fixed points, respectively. Parameters:  $b = 0.95, \tau = 3$ .

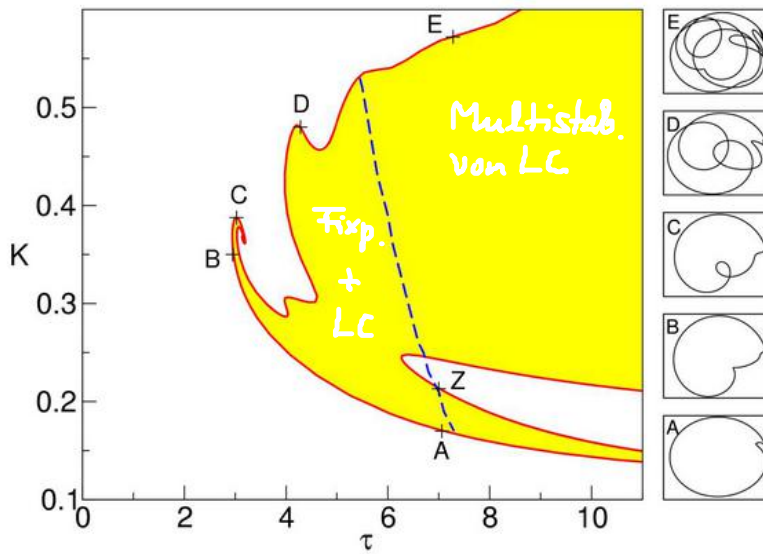


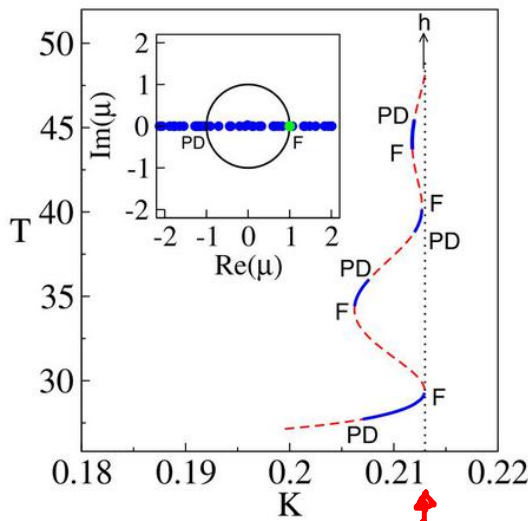
Fig. 3. Curve of homoclinic bifurcations (red) in the  $K$ - $\tau$  plane (left). A-E labels various points with homoclinic orbits, which are shown in the  $x$ - $y$  phase plane in the panel on the right. Delay-induced limit cycles exist, in addition to the stable fixed point, in the yellow area. The blue dashed curve separates the regions  $\sigma_0 < 0$  (left) and  $\sigma_0 > 0$  (right).

Bif. diagramm  
( $D=0$ )

Shilnikov - Theorie

Saddle quantity  
 $\sigma_0 = \text{Re } \lambda_s + \text{Re } \lambda_u$

↑  
 führende stabiler/instabiler  
 Eigenwert des  
 Sattel-Fokus



(b) **homokline B.**

Fig. 5. (a) Period  $T$  of limit cycle born in a homoclinic bifurcation at  $(K, \tau) = (0.17145, 7)$  (point A in Fig. 3,  $\sigma_0 < 0$ ). (b) Period  $T$  of limit cycles in the multistable regime at  $(K, \tau) = (0.213, 7)$  (point Z in Fig. 3,  $\sigma_0 > 0$ ), undergoing infinitely many fold (F) and period-doubling (PD) bifurcations, before ending in a homoclinic orbit  $h$  for  $T \rightarrow \infty$  at  $K = 0.213$ . Solid blue and red dashed lines denote stable and unstable limit cycles, respectively. The insets show the two leading Floquet multipliers of the periodic orbit  $\mu_1 = 1$  (green) and  $\mu_2$  (blue) with  $K$  as a parameter in (a), and  $T$  as a parameter in (b), in the complex plane  $b = 0.95$ .

$D \neq 0$  : Kontrolle rauschinduzierter und

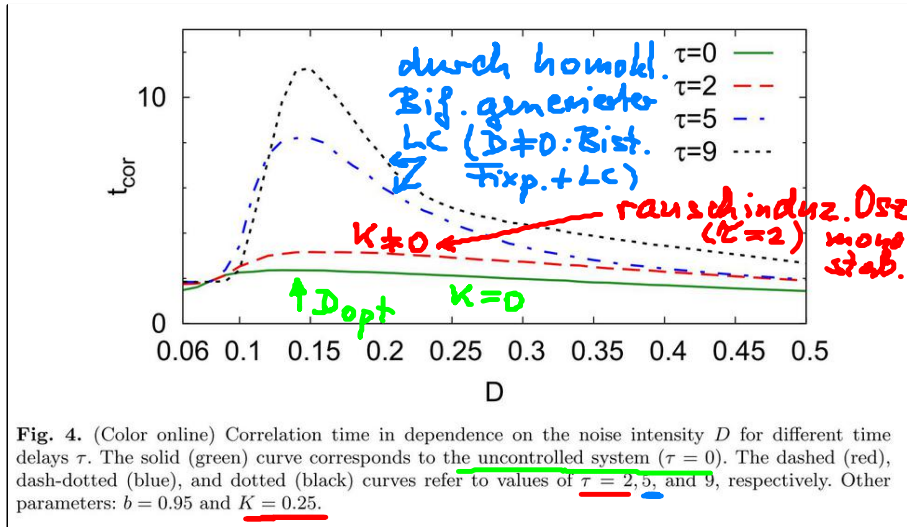


# rausch beeinflusste Oszillationen

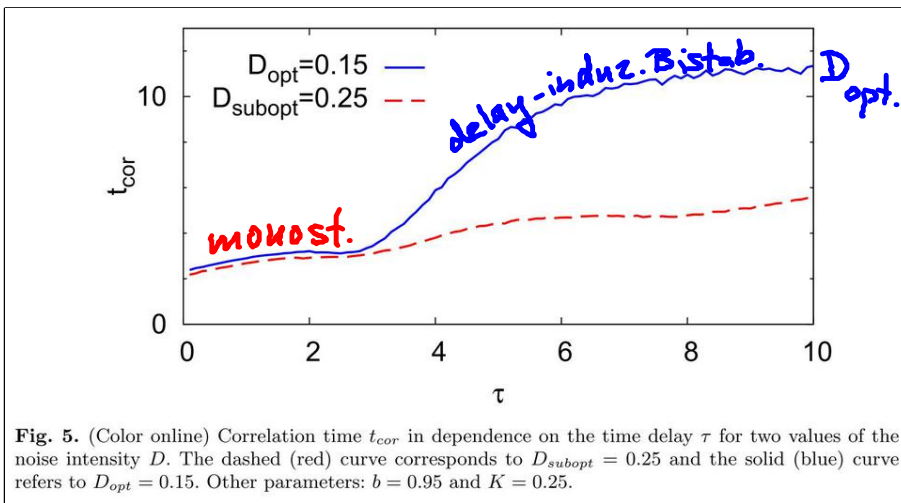


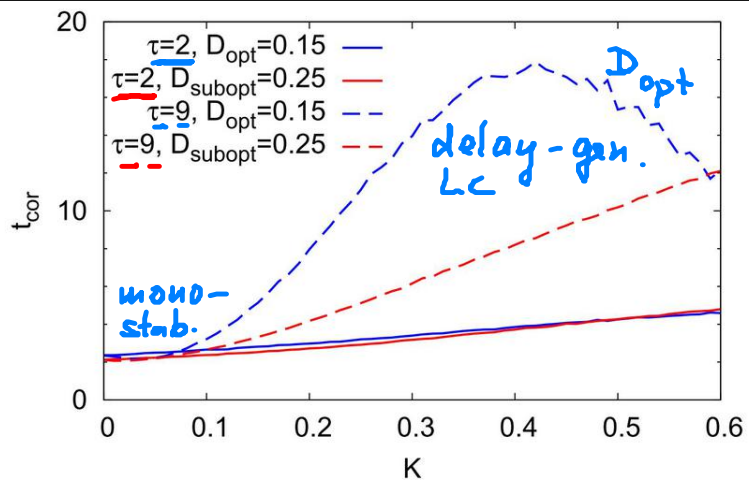
delay induzierte determinist. Grenzzyklen

- Kohärenzresonanz (für  $K=0$ ) wird verstärkt durch zeitverzögerte Rückkopplungskontrolle



Aust et al (2010)





**Fig. 6.** (Color online) Correlation time  $t_{cor}$  in dependence on the control strength  $K$  for two values of the noise intensity  $D$  and two values of the delay time  $\tau$ . The gray (red) and black (blue) curves depict the cases of  $D_{subopt}$  and  $D_{opt}$ , respectively. The solid and dashed lines correspond to  $\tau = 2$  and  $\tau = 9$ , respectively. Other parameter:  $b = 0.95$ .