DPG-Jahrestagung an der TU Berlin 25.–30.3.2012

→ größte Physiktagung in Europa (6.000 Teiln.)

mitorganisiert von der jungen DPG, z.B. Einstein Slam,

Do 29.3.12 20:00 Urania berlin.jdpg.de

* es werden noch Slammer gesucht *

Meldungen bis 20.2.12 !

## 7.3 Synchronisation und Desynchronisation von neuronalen Netzwerken

Lehnert, Dahms, Hövel, Schöll: Europhys. Lett. 96, 60013 (2011)

Netzwerk von gekoppelten FitzHugh-Nagumo (FHN)-Systemen:

$$\epsilon \dot{u}_i = u_i - \frac{u_i^3}{3} - v_i + C \sum_{j=1}^{N} \underbrace{G_{ij}}_{\text{Kopplungsmatrix } (\sum_j G_{ij} = 1)} \left[ u_j(t-\tau) - u_i \right] \qquad \epsilon = 0.01, \quad a = 1.3$$

$$\dot{v}_i = u_i + a \qquad i = 1, \dots, N$$

synchronisierte Lösung (zero-lag): $(u_i(t), v_i(t)) = (u_s(t), v_s(t)) \equiv x_s(t)$, $i = 1, \dots, N$

$$\Rightarrow \dot{x}_s = F(x_s) + C H \left[ x_s(t-\tau) - x_s(t) \right] \qquad H = \begin{pmatrix} 1/\epsilon & 0 \\ 0 & 0 \end{pmatrix}$$

Stab. der synchronis. Lös.: Master-Stability Function (MSF)

$$\dot{\zeta}(t) = \left[ DF(x_s(t)) - CH \right] \zeta(t) + \underbrace{(\alpha + i\beta)} H \zeta(t-\tau)$$

kontinuierliche Parametrisierung der Eigenwerte $C\nu_i$ von $CG_{ij}$ $(i = 1, \dots, N)$

$\Rightarrow$ $\boxed{\text{MSF = größter Lyapunov-exp. } \Lambda(\alpha + i\beta) \quad \text{vgl. § 5}}$

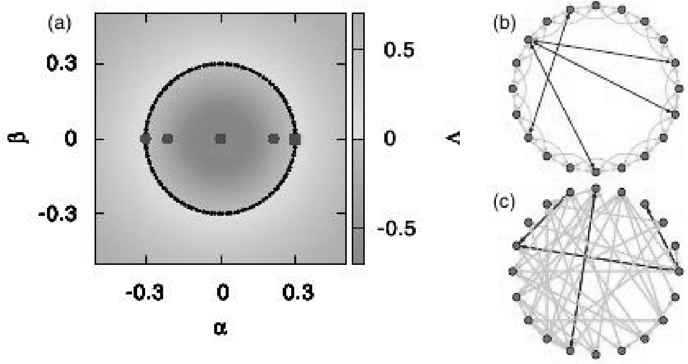
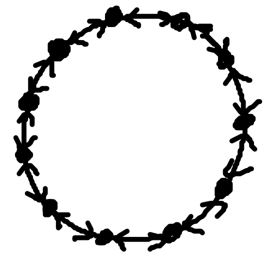



Fig. 2: (Color online) (a) Master stability function for a network of FHN systems given by eqs. (1). Dotted curve: $S((0,0),C)$. Red circles (square): rescaled transversal (longitudinal) eigenvalues $C\nu_i$ of a bidirectionally coupled ring with $N=8$ nodes. Parameters: $a=1.3$, $\epsilon=0.01$, $C=0.3$, $\tau=1$. (b) Scheme of a bidirectional regular network ($N=20$, $k=2$), and (c) a random network ($N=20$, fixed number of links $kN$) with excitatory coupling (gray (green) arrows) on which inhibitory links (black (red) arrows) are superimposed.

Lehnert, EPL



bidirektional gek. Ring mit $N=8$ exzitator. Koppl.

stets Eigenwert $\nu_i = 1$ (long. EW)
$\hat{=}$ Eigenvektor $(1,1,1,\dots,1)$
Synchron. Lös.

$N-1$ transversale EW
$\hat{=}$ transversale Stab.
$\perp$ Synchronis. mannigf.

- Für große $\tau$ ist MSF immer rotationssymm., unabhängig von der Dynamik
$\Rightarrow$ Klassif. der Stab. der Synchron. durch Topologie
(Flunkert, Yanchuk, Dahms, Schöll: PRL 105, 254101 (2010))

Exzitatorische Kopplung: $G_{ij} \geq 0$

$\Rightarrow$ alle Eigenwerte von $G$ liegen innerhalb der stab. Region der MSF (Einheitskreis)
(Gershgorin-Theorem)

$\Rightarrow$ stets stabile Synchron., unabhängig von Netzwerktopologie, Koppl. stärke $C$, delay $\tau$

Inhibitorische Koppl.: $G_{ij} < 0$

Betrachte exz. gekoppeltes Netzwerk (Ring mit $k$ nächsten Nachbarn)

+ einige wenige inhibitorische Koppl.

(Wahrscheinl. $p$; small-world Netzw.

$\Rightarrow$ Desynchronisations-Phasenübergang

Bruchteil der desynch. Netzwerke

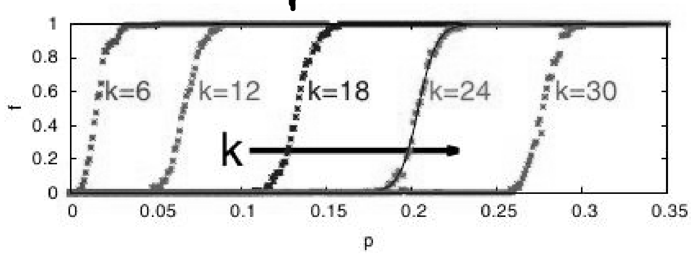



Fig. 3: (Color online) Fraction of desynchronized networks $f$ *vs.* the probability of additional inhibitory links $p$ for $N=100$. $k$ varies from 6 to 30. Thin black curve: example fit to $f(p)$ ($p_c = 0.20387$, $b = 186$) for $k=24$. Number of realizations: 500 for each value of $k$. Parameters as in fig. 1.

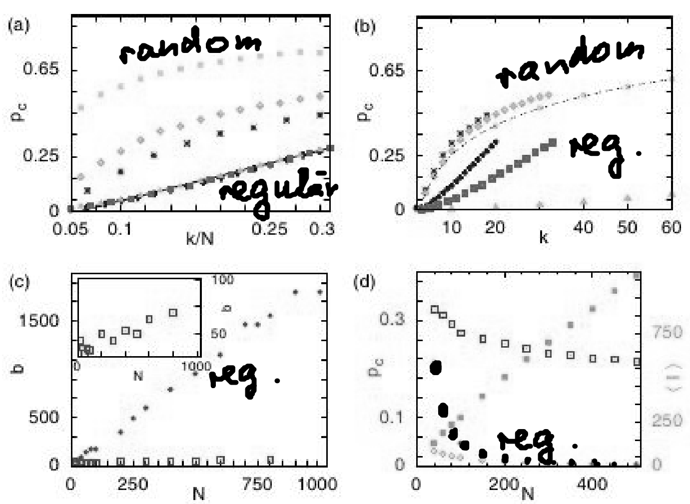



Fig. 4: (Color online) Critical value $p_c$ for different network sizes, (a) in dependence on $k/N$, (b) in dependence on $k$: SW networks $N=60$ (dark (blue) circles), $N=100$ (dark (purple) squares), and $N=500$ (gray (turquoise) triangles). Random networks $N=60$ (black (red) crosses), $N=100$ (light (orange) circles), and $N=500$ (lightgray (yellow) squares). (c) Steepness $b$ for SW (circles) and random (squares) networks *vs.* $N$ for constant $k/N=0.1$. Inset: blow-up for random networks. (d) $p_c$ *vs.* $N$ for $k=10$ for a SW (black (red) filled circles) and a random (black (red) empty squares) network. Number of inhibitory links $\langle I\rangle$ *vs.* $N$ for constant $k$ for a SW (gray (green) empty circles) and a random (gray (green) filled squares) network. Number of realizations: 500. Parameters as in figs. 1 and 3

Vergleich:

reguläres exz. Netzwerk
+ einige inh. Links

exz. Zufallsnetzwerk
+ einige inh. Links

$\Rightarrow$ small world (SW, regulär) desynchronisiert leichter als Zufallsnetzwerk (krit. Wahrscheinl. für inhib. Links $p_c \to 0$ für $N \to \infty$)

$p_c \sim \frac{k}{N}$ (regulär exz.)

## 7.4 Chimera-Zustände

Schimäre = Fabelwesen : halb Mensch, halb Tier

Chimera-Zustände : teilweise räumlich kohärent (= synchr.), teilweise inkohärent (räumlich-chaotisch)

Kuramoto, Battogtokh : Nonlin. Phen. in Compl. Syst. 5, 380 (2002)
Abrams & Strogatz : PRL 93, 174102 (2004)

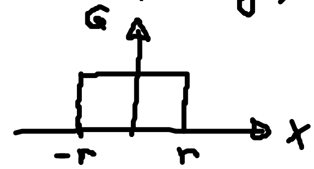
Phasenoszillatoren :

$$\dot{\varphi}(x,t) = \omega - \int_{-\pi}^{\pi} G(x-x') \sin\left[\varphi(x,t) - \varphi(x',t) + \alpha\right] dx'$$

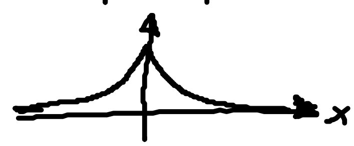
x : Position auf dem Intervall $[0, 2\pi]$

Kopplungsfkt. G (nichtlokale Kopplung) :

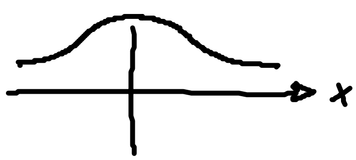
(i) $G(x) = \begin{cases} \frac{1}{2r} & |x| < r \\ 0 & \text{sonst} \end{cases}$




(ii) $G_K(x) = \dfrac{K e^{-K|x|}}{2(1-e^{-K})}$




(iii) $G_A(x) = \dfrac{1 + A\cos\frac{x}{2}}{2\pi}$



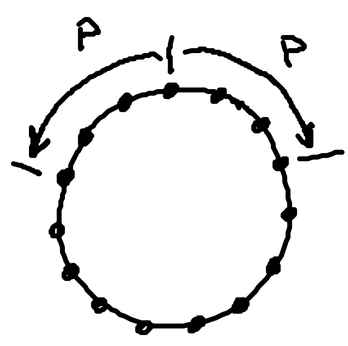

Omelchenko, Maistrenko, Hövel, Schöll : PRL 106, 234102 (2011)
Omelchenko, Riemenschneider, Hövel, Maistrenko, Schöll : PRE (Feb. 2012), in print

gekoppelte chaotische Abb. :

$$z_i^{t+1} = f(z_i^t) + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} \left[ f(z_j^t) - f(z_i^t) \right] \quad i = 1, \dots, N$$

Reichweite der nichtlokalen Koppl. P

normiert $r = \dfrac{P}{N}$



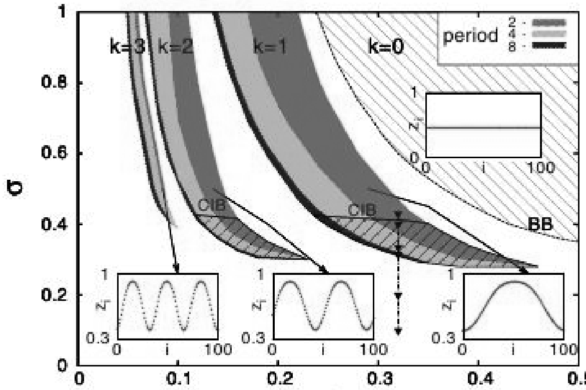

Ring

Koppl. stärke $\sigma$

logist. Abb. $f(z) = az(1-z)$

$a = 3.8$ : chaot. (Lyapunovexp. $\lambda = 0.431$)

Kontrollpar. ebene $(\sigma, r)$ :




$r = 1/N$ lokale Kopp.! global

FIG. 1 (color online). Regions of coherence for system (1) in the $(r, \sigma)$ parameter plane with wave numbers $k = 1$, 2, and 3. Snapshots of typical coherent states $z_i$ are shown in the insets. The color code inside the regions distinguishes different time periods of the states. The coherence-incoherence bifurcation (CIB) curve separates regions with coherent and incoherent dynamics. In the hatched and shaded (color) regions below CIB, two-cluster incoherent states exist. Completely synchronized chaotic states exist in the light hatched region bounded by the blowout bifurcation curve BB. Parameters: $a = 3.8$ and $N = 100$.

$k=0$: zeitl. chaotisch, räuml. homog (chaot. Synchr.)

Kohärenzzungen $k = 1, 2, 3, \ldots$

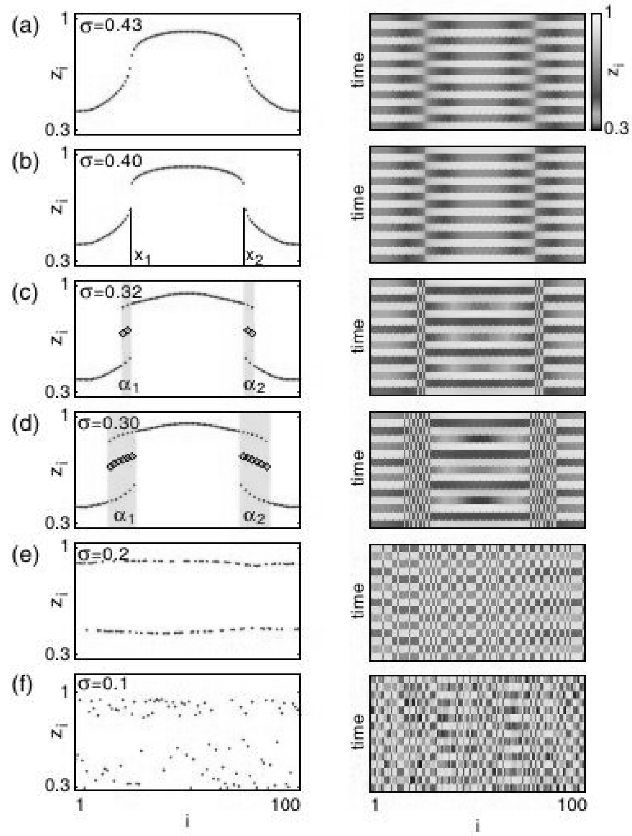
zeitl. period. (Per. 2, 4, 8, ..)

räuml. kohärent

(glattes Profil im Limes $N \to \infty$)

$$z^{t+1}(x) = f(z^t(x)) + \frac{\sigma}{2r}\int [f(z^t(y)) - f(z^t(x))]\,dy$$

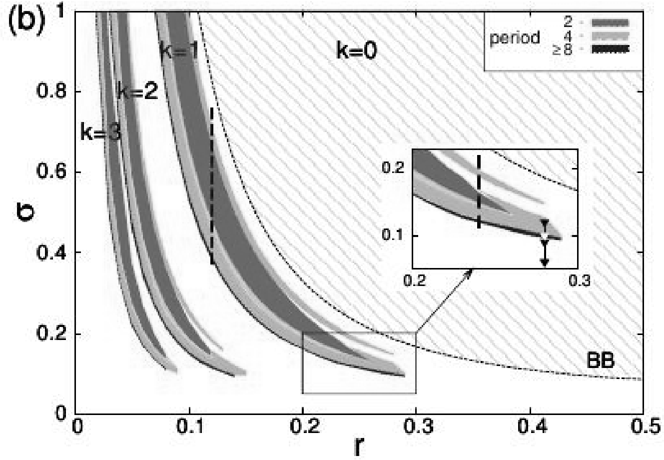
CIB: Coherence-incoherence bifurcation




chaot. logist. Abb.

FIG. 2 (color online). Coherence-incoherence bifurcation for coupled chaotic logistic maps for fixed coupling radius $r = 0.32$ (black triangles in Fig. 1). For each value of the coupling parameter $\sigma$ (decreasing from top to bottom, $\sigma = 0.43$, 0.4, 0.32, 0.3, 0.2, and 0.1, respectively), snapshots (left columns) and space-time plots (right columns) are shown. Other parameters are as in Fig. 1.

Universalität: period-/chaot. log. Abb., Rössler-Attraktor




Rössler

FIG. 2. (Color online) Coherence regions in the $(r, \sigma)$ parameter plane for $N = 100$ logistic maps (a) and Rössler systems (b), labeled by the wave number $k$. Gray scale (color code) inside the coherence regions distinguishes different periods and the coherence-incoherence bifurcation (CIB) curve separates regimes with coherent and incoherent dynamics. The light hatched region bounded by the blowout bifurcation curve BB refers to completely synchronized chaotic states. The insets in panel (a) display snapshots of typical coherent states. System parameters: panel (a) $a = 3.8$; panel (b) $a = 0.42$, $b = 2$, and $c = 4.0$. The vertical dashed lines refer to values of $\sigma$ in Fig. 3. Triangles denote parameter values used to describe the bifurcation scenario in Figures 4 and 5.