

DPG - Jahrestagung an der TU Berlin 25.-30.3.2012

→ größte Physiktagung in Europa (6.000 Teiln.)

mitorganisiert von der jungen DPG, z.B. Einstein-Sem,

Do 29.3.12 20:00 Uhr via berlin.jdpg.de

* es werden noch Stamme gesucht *
Meldungen bis 20.2.12 !

7.3 Synchronisation und Desynchronisation von neuronalen Netzwerken

Lehnert, Dahms, Hövel, Schöll: Europhys. Lett. 96, 60013 (2011)

Netzwerk von gekoppelten FitzHugh-Nagumo (FHN)-Systemen:

$$\begin{aligned} \epsilon \dot{u}_i &= u_i - \frac{u_i^3}{3} - v_i + C \sum_{j=1}^N G_{ij} [u_j(t-\tau) - u_i] & \epsilon &= 0.01 \\ \dot{v}_i &= u_i + a & a &= 1.3 \end{aligned}$$

↑
Kopplungsmatrix ($\sum_j G_{ij} = 1$) $i=1, \dots, N$

synchronisierte Lösung: $(u_i(t), v_i(t)) = (u_s(t), v_s(t)) \equiv x_s(t)$
(zero-lag) $i=1, \dots, N$

$$\Rightarrow \dot{x}_s = F(x_s) + C H [x_s(t-\tau) - x_s(t)] \quad H = \begin{pmatrix} 1/\epsilon & 0 \\ 0 & 0 \end{pmatrix}$$

Stab. der synchronis. Lös.: Mode-Stability Function (MSF)

$$\dot{\zeta}(t) = [DF(x_s(t)) - CH] \zeta(t) + \underbrace{(\alpha + i\beta) H}_{\text{kontinuierliche Parameterisierung}} \zeta(t-\tau)$$

→ MSF = größter Lyapunov-exp.
 $\wedge (\alpha + i\beta)$ vgl. § 5

kontinuierliche Parameterisierung
der Eigenwerte $C\alpha_i$ von $C G_{ij}^0$
($i=1, \dots, N$)

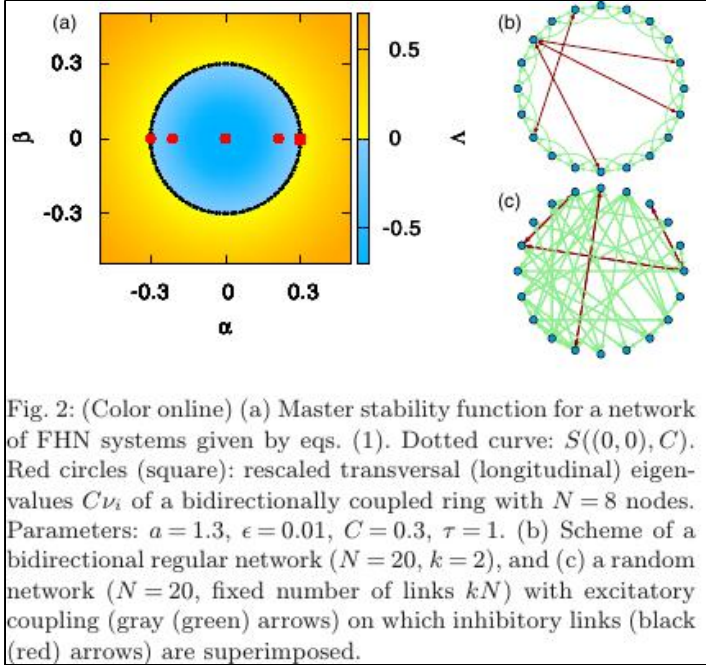
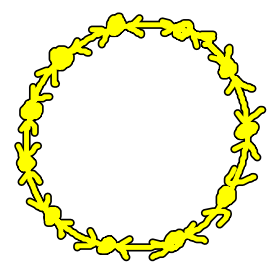


Fig. 2: (Color online) (a) Master stability function for a network of FHN systems given by eqs. (1). Dotted curve: $S((0,0), C)$. Red circles (square): rescaled transversal (longitudinal) eigenvalues $C\nu_i$ of a bidirectionally coupled ring with $N = 8$ nodes. Parameters: $a = 1.3$, $\epsilon = 0.01$, $C = 0.3$, $\tau = 1$. (b) Scheme of a bidirectional regular network ($N = 20$, $k = 2$), and (c) a random network ($N = 20$, fixed number of links kN) with excitatory coupling (gray (green) arrows) on which inhibitory links (black (red) arrows) are superimposed.

Lehnert, EPL



bidirektional gek.
Ring mit $N=8$
exzitar. Koppl.

stets Eigenwert $\nu_i = 1$ (long. EW)
 \hat{z} Eigenvektor $(1, 1, 1, \dots, 1)$
Synchron. Lös.

$N-1$ transversale EW
 $\hat{=}$ transversale Stab.
 \perp Synchronis. mannigf.

- Für große τ ist MSF immer rotations-symm.,
unabhängig von der Dynamik
 \Rightarrow Klassif. der Stab. der Synchron. durch Topologie
(Friedrich, Yendrik, Dahms, Schöll : PRL 105, 254101(2010))

Exzitatorische Kopplung: $G_{ij} \geq 0$

\Rightarrow alle Eigenwerte von G liegen innerhalb der
stab. Region der MSF (Einheitskreis)
(Gershgorin-Theorem)

\Rightarrow stets stabile Synchron., unabhängig von
Netzwerktopologie, Koppl.stärke C , delay τ

Inhibitorische Koppl. : $G_{ij} < 0$

Betrachte exz. gekoppeltes Netzwerk (Ring mit k nächsten Nachbarn)
 + einige wenige inhibitorische Koppl.
 (Wahrsch. p ; small-world Netz.)

⇒ Desynchronisations-Phasenübergang

Bruchteil der desynch. Netzwerke

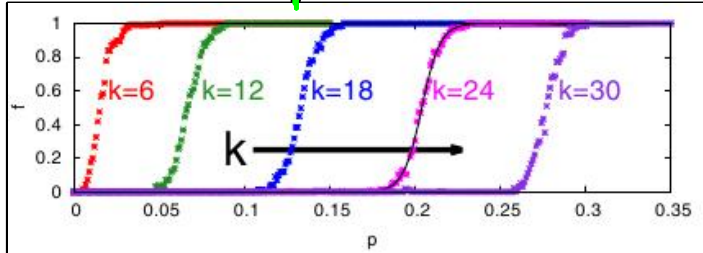


Fig. 3: (Color online) Fraction of desynchronized networks f vs. the probability of additional inhibitory links p for $N = 100$. k varies from 6 to 30. Thin black curve: example fit to $f(p)$ ($p_c = 0.20387$, $b = 186$) for $k = 24$. Number of realizations: 500 for each value of k . Parameters as in fig. 1.

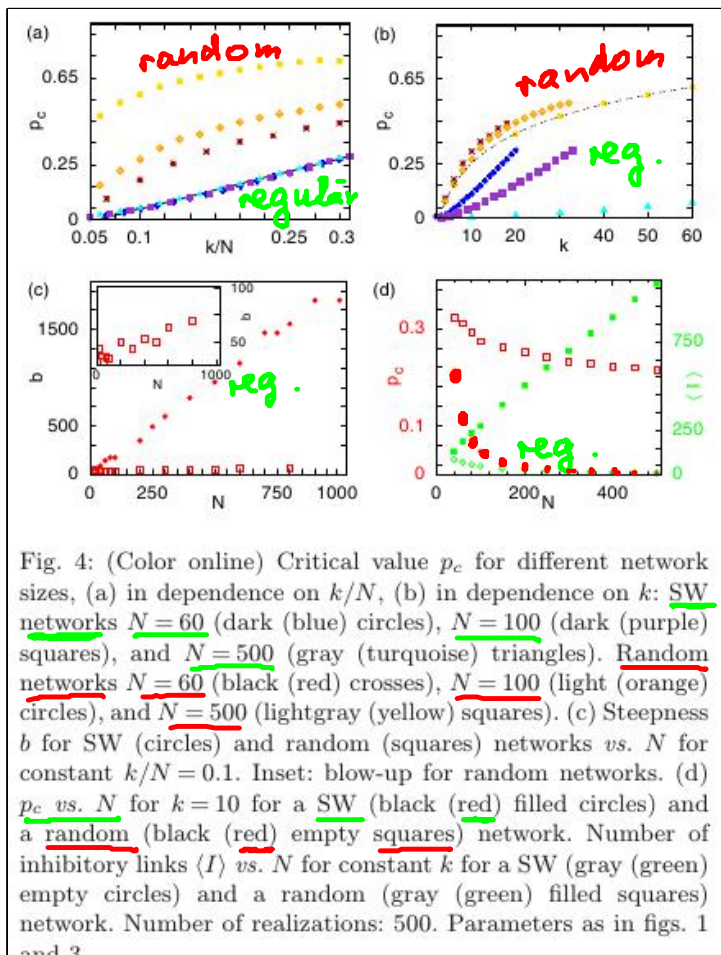


Fig. 4: (Color online) Critical value p_c for different network sizes, (a) in dependence on k/N , (b) in dependence on k : SW networks $N = 60$ (dark (blue) circles), $N = 100$ (dark (purple) squares), and $N = 500$ (gray (turquoise) triangles). Random networks $N = 60$ (black (red) crosses), $N = 100$ (light (orange) circles), and $N = 500$ (lightgray (yellow) squares). (c) Steepness b for SW (circles) and random (squares) networks vs. N for constant $k/N = 0.1$. Inset: blow-up for random networks. (d) p_c vs. N for $k = 10$ for a SW (black (red) filled circles) and a random (black (red) empty squares) network. Number of inhibitory links $\langle I \rangle$ vs. N for constant k for a SW (gray (green) empty circles) and a random (gray (green) filled squares) network. Number of realizations: 500. Parameters as in figs. 1 and 3.

Vergleich:
 reguläres exz. Netzwerk
 + einige inh. links
 exz. Zufallstwert
 + einige inh. links

⇒ small world (SW, regulär)
 desynchronisiert
 leichter als Zufalls-
 netzwerk (krit. Wahrs-
 cheinl. für inhib.
 links $p_c \rightarrow 0$
 für $N \rightarrow \infty$)

$$p_c \sim \frac{k}{N} \quad (\text{regulär exz.})$$

7.4 Chimera-Zustände

Schimäre = Fabelwesen: halb Mensch, halb Tier

Chimera-Zustände: teilweise räumlich kohärent (= synchron.),
teilweise inkohärent (räumlich-chaotisch)

Kuramoto, Battogtokh: *Nonlin. Phen. in Compl. Syst.* 5, 380 (2002)

Abrams & Strogatz: *PRL* 93, 174102 (2004)

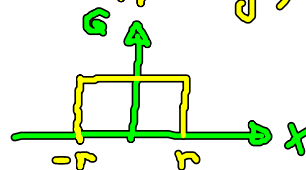
Phasenzittern: τ

$$\dot{\varphi}(x,t) = \omega - \int_{-\pi}^{\pi} G(x-x') \sin[\varphi(x,t) - \varphi(x',t) + \alpha] dx'$$

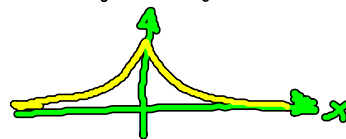
x : Position auf dem Intervall $[0, 2\pi]$

Kopplungsfkt. G (nichtlokale Kopplung):

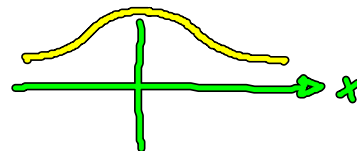
$$(i) G(x) = \begin{cases} \frac{1}{2r} & |x| < r \\ 0 & \text{sonst} \end{cases}$$



$$(ii) G_k(x) = \frac{ke^{-k|x|}}{2(1-e^{-k})}$$



$$(iii) G_A(x) = \frac{1 + A \cos \frac{x}{2}}{2\pi}$$



Omelchenko, Moisevich, Hövel, Schöll: *PRL* 106, 234102 (2011)

Omelchenko, Riemenschneider, Hövel, Maistrenko, Schöll: *PRE* (Feb. 2012),
in print

gekoppelte chaotische Abb.:

$$z_i^{t+1} = f(z_i^t) + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} [f(z_j^t) - f(z_i^t)] \quad i=1, \dots, N$$

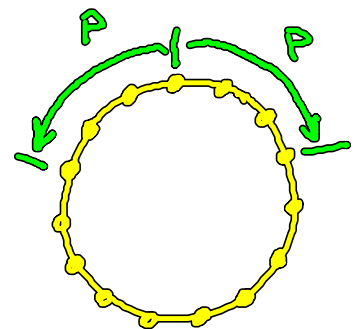
Reichweite der nichtlokalen Koppl. P

normiert $r = \frac{P}{N}$

Koppl. -stärke σ

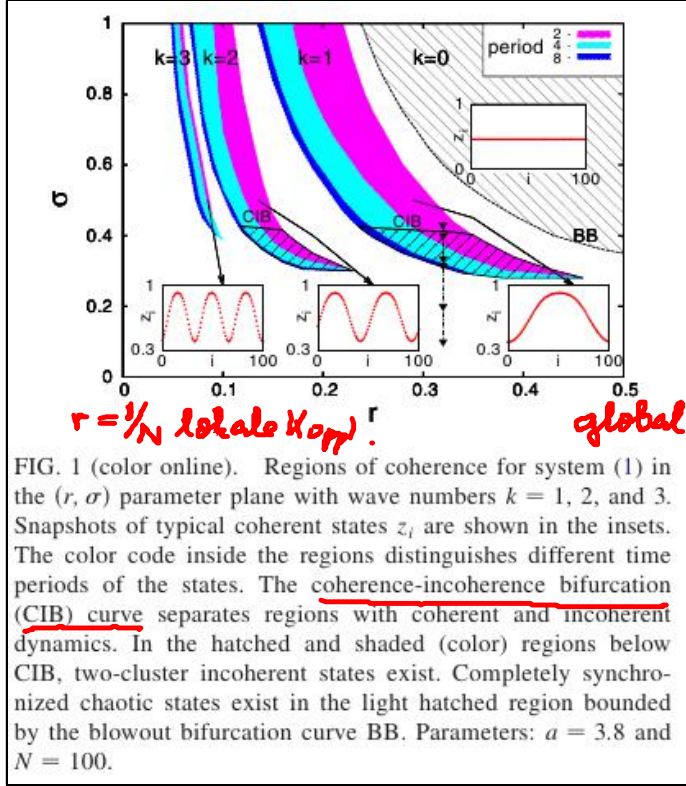
logist. Abb. $f(z) = az(1-z)$

$a=3.8$: chaot. (Lyapunovexp. $\lambda = 0.431$)



Ring

Kontrollpar. ebene (σ, r) :



$k=0$: zeitl. chaotisch, räuml. homogen (chaot. Synchron.)

Kohärenzungen $k=1, 2, 3, \dots$

zeitl. period. (Per 2, 4, 8, ...)

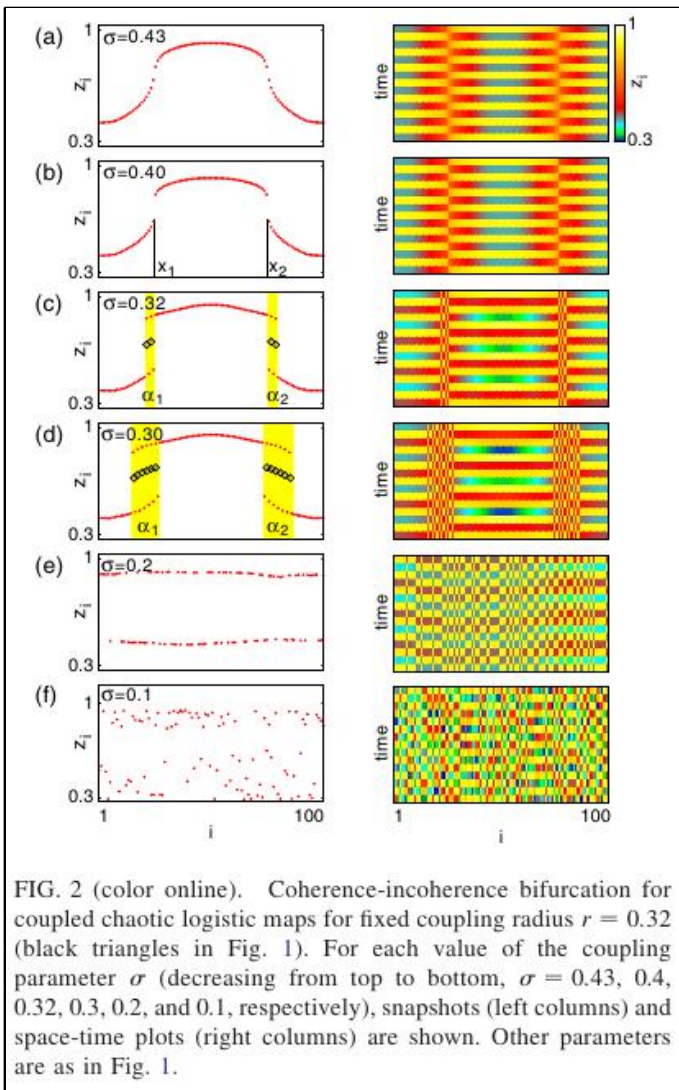
räuml. kohärent

(glattes Profil in z_i -Abt.)

$$z_i^{t+1}(k) = f(z_i^t(k))$$

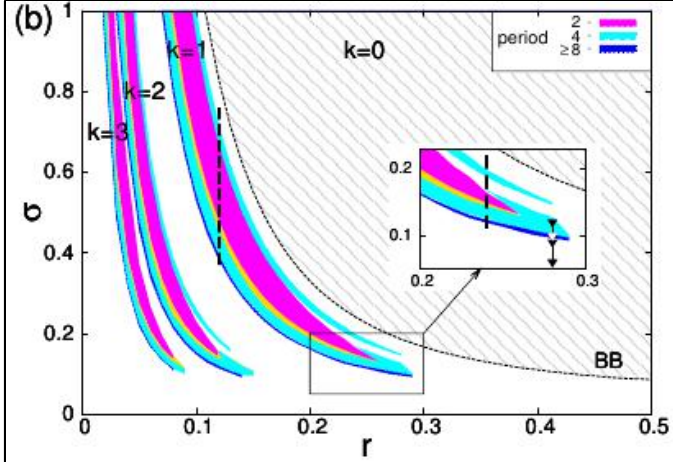
$$+ \frac{\sigma}{2r} \int [f(z_i^t(y)) - f(z_i^t(x))] dy$$

CIB: Coherence - incoherence bifurcation



chaot. logist. Abb.

Universalität: period-/chaot. log. Abb., Rössler - Attraktor



Rössler

FIG. 2. (Color online) Coherence regions in the (r, σ) parameter plane for $N = 100$ logistic maps (a) and Rössler systems (b), labeled by the wave number k . Gray scale (color code) inside the coherence regions distinguishes different periods and the coherence-incoherence bifurcation (CIB) curve separates regimes with coherent and incoherent dynamics. The light hatched region bounded by the blowout bifurcation curve BB refers to completely synchronized chaotic states. The insets in panel (a) display snapshots of typical coherent states. System parameters: panel (a) $a = 3.8$; panel (b) $a = 0.42$, $b = 2$, and $c = 4.0$. The vertical dashed lines refer to values of σ in Fig. 3. Triangles denote parameter values used to describe the bifurcation scenario in Figures 4 and 5.