Stabilisierung von instabilen Fixpunkten

charakt. Gl.: $\boxed{\Lambda + K(1-e^{-\Lambda\tau}) = \lambda \pm i\omega}$

Stabilitätsgrenzen: $\mathrm{Re}\,\Lambda = 0$

Char. Gl. in Re und Im aufgespalten mit $\Lambda = p + iq$:

$$\boxed{\begin{aligned} \lambda &= p + K\left[1 - e^{-p\tau}\cos q\tau\right] \\ \omega &= q + K e^{-p\tau}\sin q\tau \end{aligned}} \qquad \lambda > 0$$

$\mathrm{Re}\,\Lambda = p \overset{!}{=} 0$:

$$\lambda = K(1-\cos q\tau) \qquad (1)$$
$$\omega = q + K\sin q\tau \qquad (2)$$

Systempar. λ, ω geg.

Kontrollpar. $K, \tau > 0$

Kurvenpar. der Stab.grenze im (K,τ)-Raum q

$(1) \Rightarrow 0 \leq 1-\cos q\tau \leq 2 \Rightarrow \frac{\lambda}{K} \leq 2 \Rightarrow \boxed{K \geq \frac{\lambda}{2}}$

notwendige Bed. für Stabilis.
(minimale Rückkoppl.stärke)

$$K_{min} = \frac{\lambda}{2} \overset{(1)}{\Rightarrow} \cos q\tau = -1 \Rightarrow q\tau = (2n+1)\pi \qquad (3)$$
$$n = 0, 1, 2, \ldots$$

$$\Rightarrow \sin q\tau = 0 \overset{(2)}{\Rightarrow} q = \omega$$

$$\Rightarrow \boxed{\tau = \frac{\pi}{\omega}(2n+1) = T_0\,\frac{2n+1}{2}} \qquad n = 0, 1, 2, \ldots$$

Für $\tau = \frac{2\pi n}{\omega} = nT_0$ ist keine Stabilisierung möglich, weil

$$(2) \Rightarrow q = \omega \overset{(1)}{\Rightarrow} \frac{K-\lambda}{K} = \cos(q\tau)_{q\tau = 2\pi n} = 1 \Leftrightarrow 1 - \frac{\lambda}{K} = 1$$

↯

Analyt. Berechnung der Stabilitätsgrenze in der (K,τ)-Ebene:

$$(1), (2) \Rightarrow \left(\frac{K-\lambda}{K}\right)^2 + \left(\frac{\omega-q}{K}\right)^2 = \cos^2 q\tau + \sin^2 q\tau = 1$$

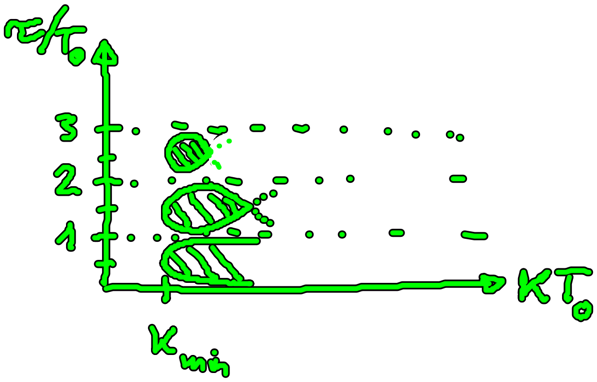
$$\Rightarrow \quad \omega - q = \pm K \sqrt{1 - \left(\frac{K-\lambda}{K}\right)^2}$$

$$q = \omega \mp \sqrt{(2K-\lambda)\lambda}$$ eliminiert q aus (1), (2)

feste Par. λ, ω des unkontrollierten Systems

$\Rightarrow$ Relation zwischen τ und K aus (1): $\tau(K) = \dfrac{\arccos \frac{K-\lambda}{K}}{\omega \mp \sqrt{(2K-\lambda)\lambda}}$

arccos hat mehrere Zweige :



Stabilisierung !

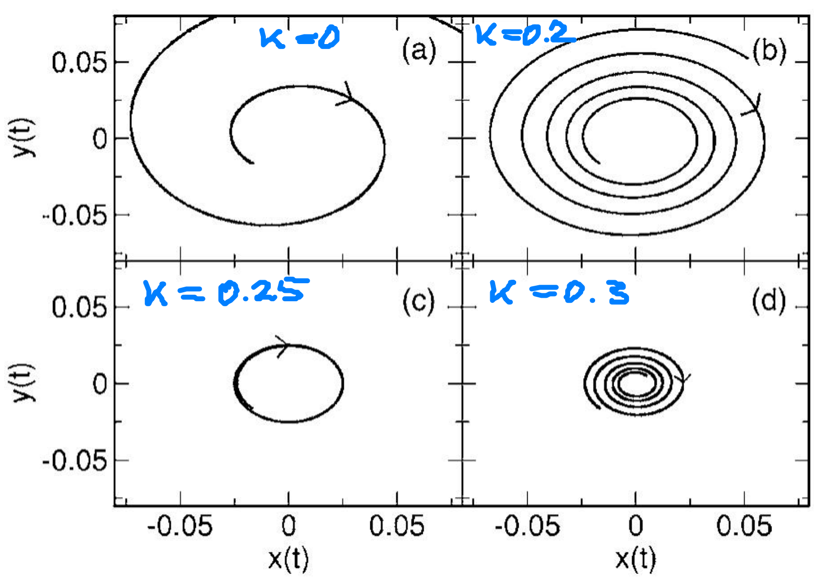


FIG. 1. Control of an unstable focus with $\lambda=0.5$ and $\omega=\pi$ in the configuration space for different values of the feedback gain K. Panels (a)–(d) correspond to $K=0,0.2,0.25$, and 0.3, respectively. The time delay τ of the TDAS control scheme is chosen as 1, corresponding to $\tau=T_0/2=\pi/\omega$.

Hövel u. Schöll: Phys. Rev. E 72, 0620 (2005)

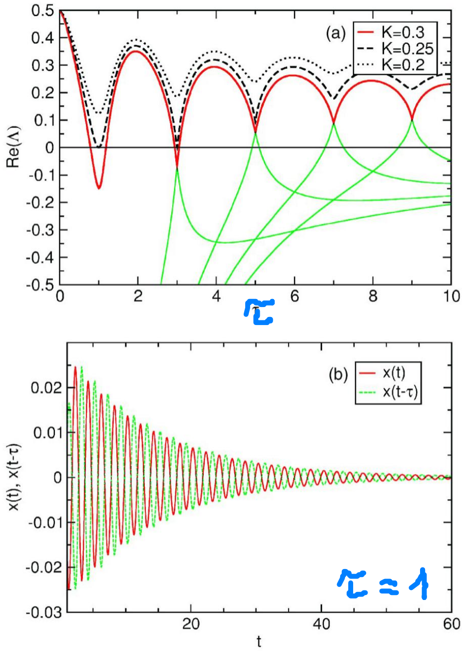


FIG. 2. (Color online) (a) Largest real part of the complex eigenvalues Λ vs τ for $\lambda = 0.5$ and $\omega = \pi$ for different K. Some lower eigenvalues are also displayed for $K = 0.3$ (green online). (b) Time series of the x component of the unstable focus: The solid line (red online) corresponds to $x(t)$, the dashed line (green online) to the delayed x component $x(t-\tau)$ with $\tau = 1$. The parameters of the unstable focus and the control scheme are as in panel (d) of Fig. 1.

Stabilisierung für geeignete τ und K!

$\tau \approx \frac{T_0}{2}, \frac{3}{2} T_0, \ldots$

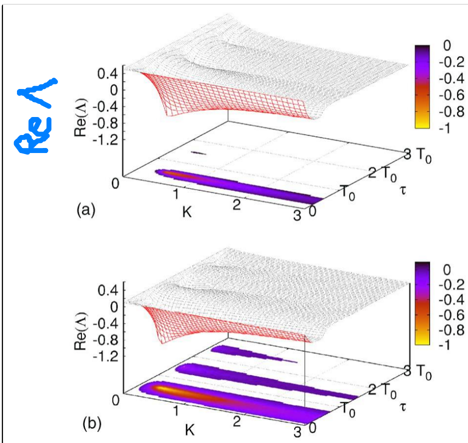


FIG. 3. (Color online) Domain of control in the K-τ plane and largest real part of the complex eigenvalues Λ as a function of K and τ according to Eq. (7). The two-dimensional projection at the bottom shows combinations of τ and K, for which $\mathrm{Re}(\Lambda)$ is negative and thus the control successful [panel (a): $\lambda = 0.5$ and $\omega = \pi$; panel (b): $\lambda = 0.1$ and $\omega = \pi$].

$\lambda = 0.5$

$\lambda = 0.1$

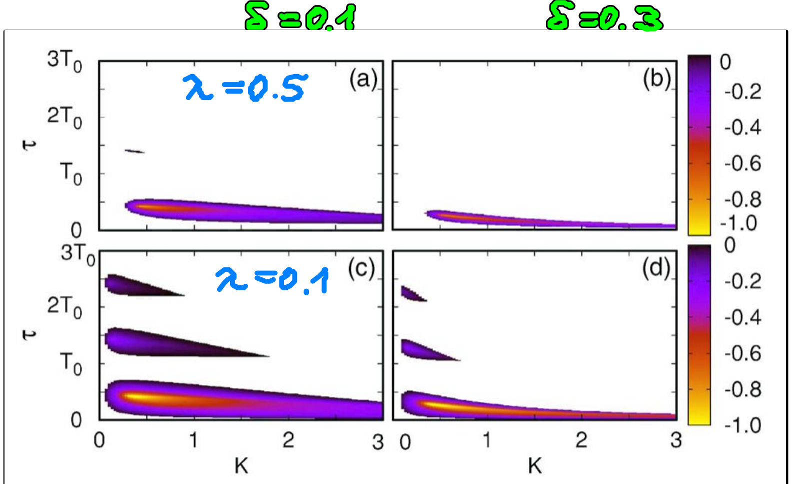


FIG. 6. (Color online) Domain of control in the K-τ plane for different latency times [panels (a) and (c): δ=0.1; panels (b) and (d): δ=0.3]. The shaded areas indicate combinations of τ and K, for which the largest real part of the complex eigenvalues Λ is negative and thus control is successful. The value of Re(Λ) is indicated by the greyscale (color online). The parameters of the unstable focus are chosen as $\omega=\pi$ in all panels and $\lambda=0.5$ in (a) and (b) and $\lambda=0.1$ in (c) and (d).

Latenzzeit δ :

$z(t-\delta) - z(t-\delta-\tau)$

Erweiterung (Socolar, Sukow, Gauthier, PRE 50, 3245 (94))

- multiple-time feedback (ETDAS, extended time-delay autosynchronization)

$$K\sum_{n=0}^{\infty} R^n \left[x(t-n\tau) - x(t-(n+1)\tau)\right]$$ Gedächtnisspan. R ($0 \le R < 1$)

Eigenwertgl.

$$\boxed{\Lambda + K\frac{1-e^{-\Lambda\tau}}{1-Re^{-\Lambda\tau}} = \lambda \pm i\omega}$$ Stab. bereich vergrößert

Dahms, Hövel, Schöll, PRE 76, 056201 (2007)

- Latenz-Effekte

$$K\left[x(t-\delta) - x(t-\delta-\tau)\right]$$ Latenzzeit δ

Stab. bereich verkleinert

- phasen-abhängige Rückkoppl.

$$K\begin{pmatrix}\cos\varphi & -\sin\varphi\\ \sin\varphi & \cos\varphi\end{pmatrix}\begin{pmatrix}x(t)-x(t-\tau)\\ y(t)-y(t-\tau)\end{pmatrix}$$

- asymptot. Skalierungsverhalten für große τ

  Yanchuk, Wolfrum, Hövel, Schöll, PRE $\underline{74}$, 026201 (2006)

  Hövel, PhD Thesis (TU Berlin 2009, Springer 2010)

  Wolfrum et al, EPJ-ST (2010)

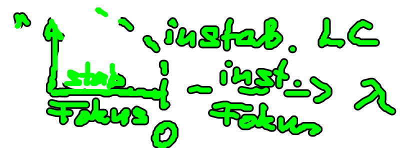
3.2.2 Stabilisierung instabiler periodischer Orbits

Normalform einer subkrit. Hopf-Bif.:

$$\boxed{\dot z = (\lambda + i\omega + (1+i\gamma)|z|^2)\, z + b\,(z(t-\tau) - z(t))}$$

$\lambda < 0$, $\omega = 1$, $\gamma > 0$, $b = b_0 e^{i\beta} \in \mathbb{C}$

ohne Kontrolle:



$z = re^{i\phi}$: $\dot r = (\lambda + r^2)\, r$

$\dot\phi = \omega + \gamma r^2$

LC: $r^2 = -\lambda$ ex. für $\lambda < 0$

$\dot\phi = \omega - \gamma\lambda \Rightarrow T = \frac{2\pi}{\omega - \gamma\lambda}$ Periode des UPO (unstable periodic orbit)

nichtinvasive Kontrolle (oBdA. $\omega = 1$)

wähle $\tau \overset{!}{=} nT = \frac{2\pi n}{1-\gamma\lambda}$, $n \in \mathbb{N}$

($\underline{\text{Pyragas-Kurve}}$ in der (τ, λ)-Ebene)