7.3 Chimera-Zustände

Schimäre = Fabelwesen: Kopf eines Löwen, Körper einer Ziege, Schwanz einer Schlange (griech. Mythologie)

Chimera-Zustände: teilweise räumlich kohärent (= synchronis.)
teilweise inkohärent (räumlich-chaotisch, desynchronisiert)

Motivation: Manche Vögel (Zugvögel) u. Delphine schlafen nur mit einer Hirnhälfte (!), die andere ist wach

Kuramoto, Battogtokh: Nonlin. Phen. in Compl. Syst. $\underline{5}$, 380 (2002)

Abrams & Strogatz: PRL $\underline{93}$, 174102 (2004)

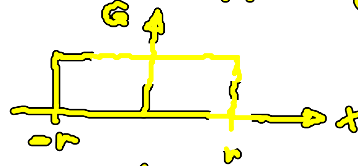
<u>Phasenoszillatoren:</u>

$$\dot{\varphi}(x,t) = \omega - \int_{-\pi}^{\pi} G(x-x') \sin[\varphi(x,t) - \varphi(x',t) + \alpha]\, dx'$$

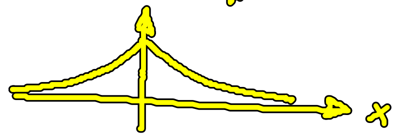
x: Position auf dem Intervall $[0, 2\pi]$

Kopplungsfkt. G (nichtlokale Kopplung):

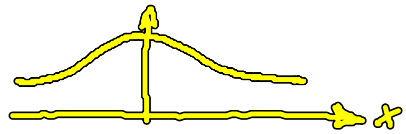
(i) $G(x) = \begin{cases} \frac{1}{2r} & |x| < r \\ 0 & \text{sonst} \end{cases}$



(ii) $G_K(x) = \dfrac{K e^{-K|x|}}{2(1-e^{-K})}$



(iii) $G_A(x) = \dfrac{1 + A\cos\frac{x}{2}}{2\pi}$



Omelchenko, Maistrenko, Hövel, Schöll: PRL $\underline{106}$, 234102 (2011)
Omelchenko, Riemenschneider, Hövel, Maistrenko, Schöll: PRE $\underline{85}$, 026212 (2012)

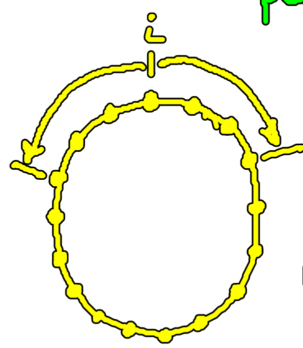
gekoppelte chaot. Abb.:

$$z_i^{t+1} = f(z_i^t) + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} \left[f(z_j^t) - f(z_i^t) \right] \qquad i = 1, \dots, N$$

period. Randb.

Reichweite der <u>nichtlokalen</u> Koppl. P

normiert $r = \dfrac{P}{N}$



Ring-Netzwerk

Koppl.stärke σ

Logist. Abb. $f(z) = az(1-z)$

$a = 3.8$: chaot. (Lyapunovexp. $\lambda = 0.431$)

<u>Kontrollpar.ebene</u> (σ, r):

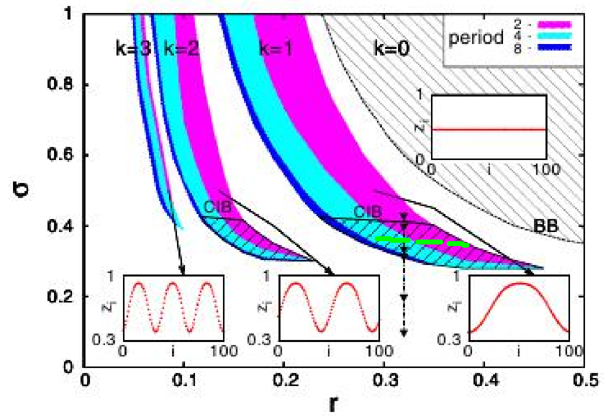


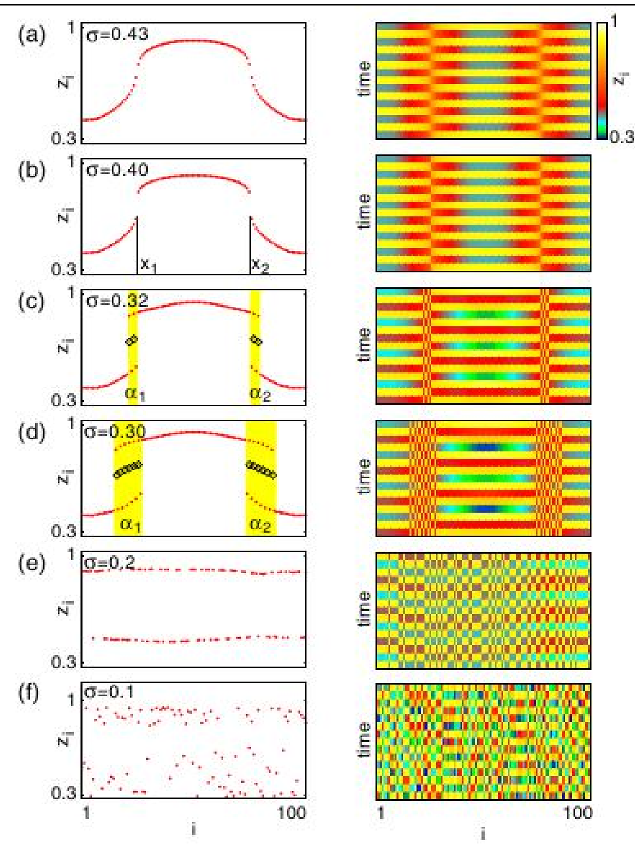
FIG. 1 (color online). Regions of coherence for system (1) in the (r, σ) parameter plane with wave numbers $k = 1$, 2, and 3. Snapshots of typical coherent states z_i are shown in the insets. The color code inside the regions distinguishes different time periods of the states. The coherence-incoherence bifurcation (CIB) curve separates regions with coherent and incoherent dynamics. In the hatched and shaded (color) regions below CIB, two-cluster incoherent states exist. Completely synchronized chaotic states exist in the light hatched region bounded by the blowout bifurcation curve BB. Parameters: $a = 3.8$ and $N = 100$.

$k=0$: zeitl. chaotisch, räuml. homog. (chaot. synchr.)

Kohärenzzungen $k = 1, 2, 3, \ldots$ zeitl. period. (Per. 2, 4, 8, ..) räumlich kohärent (glattes Profil im Limes $N \to \infty$)

$$z^{t+1}(x) = f(z^t(x)) + \frac{\sigma}{2r} \int_{x-r}^{x+r} [f(z^t(y)) - f(z^t(x))] dy$$

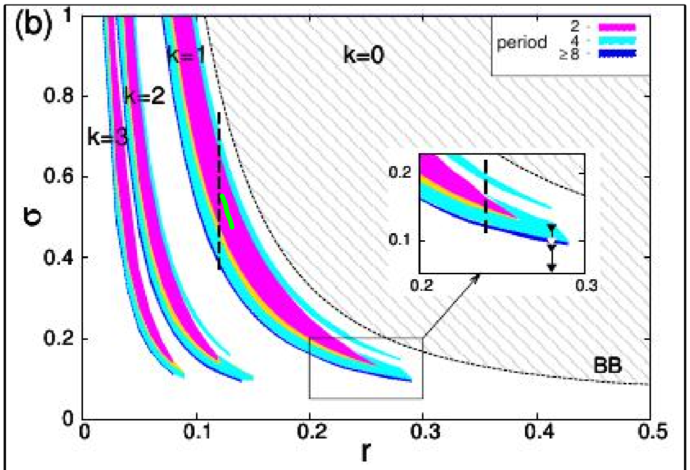
CIB : Coherence-incoherence bifurcation



chaot. logist. Abb.

FIG. 2 (color online). Coherence-incoherence bifurcation for coupled chaotic logistic maps for fixed coupling radius $r = 0.32$ (black triangles in Fig. 1). For each value of the coupling parameter σ (decreasing from top to bottom, $\sigma = 0.43$, 0.4, 0.32, 0.3, 0.2, and 0.1, respectively), snapshots (left columns) and space-time plots (right columns) are shown. Other parameters are as in Fig. 1.

Universalität : period./chaot. logist. Abb., Rössler-Attraktor



Rössler

FIG. 2. (Color online) Coherence regions in the (r, σ) parameter plane for $N = 100$ logistic maps (a) and Rössler systems (b), labeled by the wave number k. Gray scale (color code) inside the coherence regions distinguishes different periods and the coherence-incoherence bifurcation (CIB) curve separates regimes with coherent and incoherent dynamics. The light hatched region bounded by the blowout bifurcation curve BB refers to completely synchronized chaotic states. The insets in panel (a) display snapshots of typical coherent states. System parameters: panel (a) $a = 3.8$; panel (b) $a = 0.42$, $b = 2$, and $c = 4.0$. The vertical dashed lines refer to values of σ in Fig. 3. Triangles denote parameter values used to describe the bifurcation scenario in Figures 4 and 5.

Erster experimenteller Nachweis : 2012

Hagerstrom, Murphy, Roy, Hövel, Omelchenko, Schöll : Nature Phys. 8, 658 (2012)

Optisches Exp. (liquid crystal spatial light modulator)

Tinsley, Nkomo, Showalter : Nature Phys. 8, 662 (2012)

chemischer Osz. (photosensitive Belousov-Zhabotinsky (BZ)-Reaktion)

Hagerstrom-Exp.:

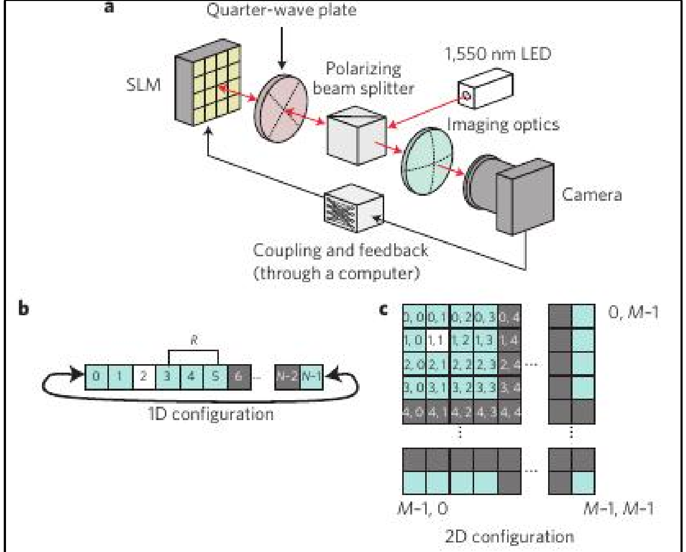


Figure 1 | Experimental apparatus. a, Optical configuration. Polarization optics create a nonlinear relationship between the spatially dependent phase shift applied by the SLM and the intensity of the light falling on the camera. Feedback and coupling are implemented using a computer. **b**,**c**, Schematics of the 1D (**b**) and 2D (**c**) coupling configurations are shown. The site highlighted in white is updated based on the sites indicated in blue. As the elements are coupled diffusively to their neighbours within a range R in either one or two dimensions with periodic boundary conditions, the coupling is identical for all oscillators.

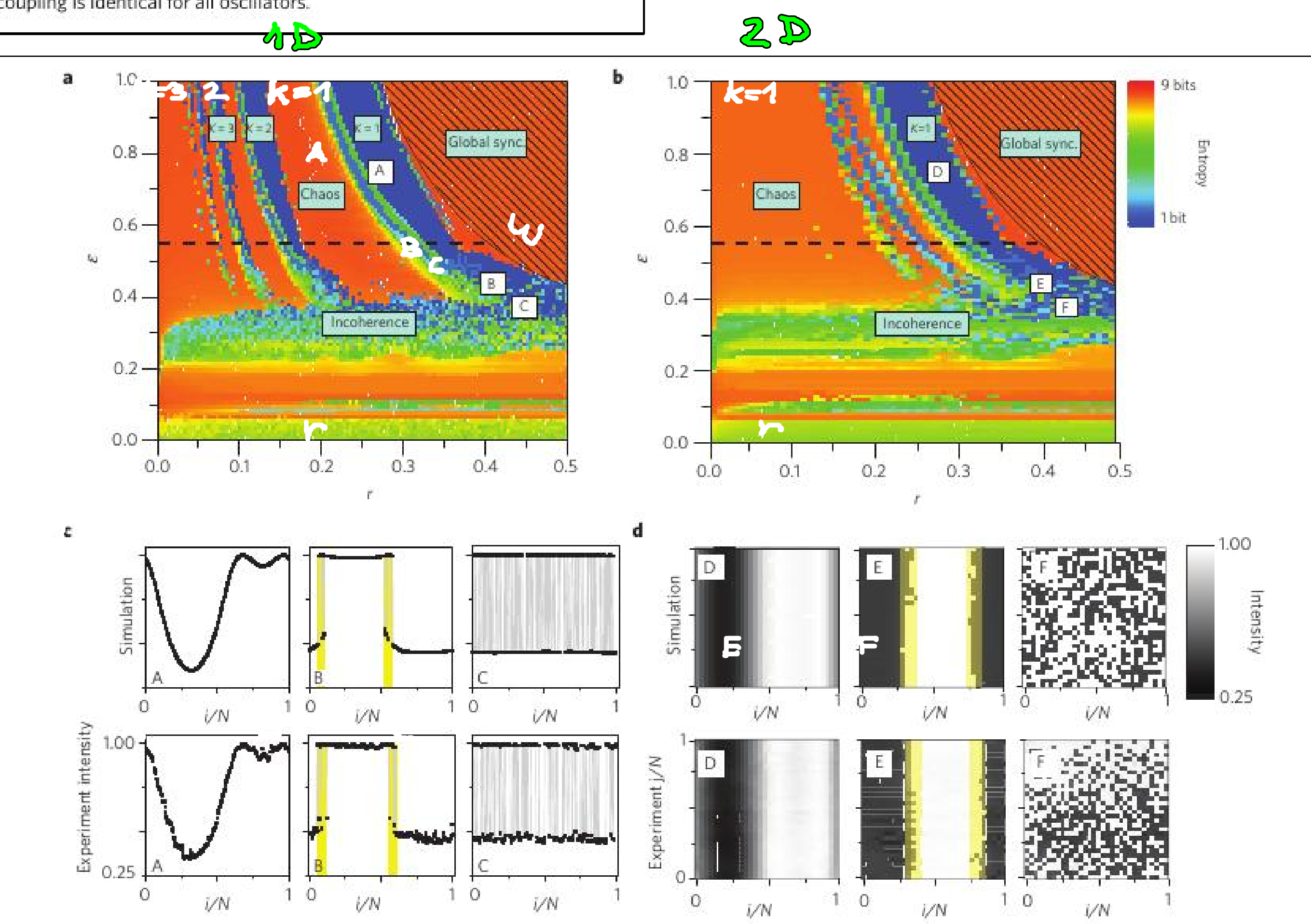


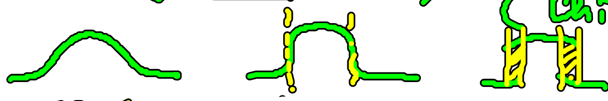
Figure 2 | Parameter space of the 1D and 2D CML systems. a, Parameter space of the 1D configuration ($N = 256$ elements). Blue corresponds to low entropy and periodicity; orange corresponds to high entropy and chaos. The dashed line indicates the critical coupling strength $\epsilon_c = 0.54$. There is a series of tongues containing profiles which are periodic in time and have spatial wavenumbers $K = 1$, $K = 2$ and $K = 3$ as indicated. **b**, Parameter space of the 2D configuration (128 × 128 lattice). There is a prominent $K = 1$ tongue, and two tongues with more complex spatial patterns that are not characterized by a wavenumber. **c**, Experimental and numerical realizations of the 1D system. In B, the incoherent region is highlighted in yellow. Labels A (coherence), B (chimera) and C (incoherence) show positions in the parameter space of **a**. **d**, Experimental and numerical realizations of the 2D system. In E, the incoherent region is highlighted.

Dynamik: $I(\phi) = \frac{1}{2}(1 - \cos\phi)$ opt. Intensität

Phase $\phi_i^{n+1} = 2\pi a \left\{ I(\phi_i^n) + \frac{\epsilon}{2R} \sum_{j=-R}^{R} \left[I(\phi_{i+j}^n) - I(\phi_i^n) \right] \right\}$

$$= f(\phi_i^n) + \frac{\epsilon}{2R}\sum_{j=-R}^{R}\left[f(\phi_{i+j}^n) - f(\phi_i^n)\right] \qquad \text{1D-Ring}$$

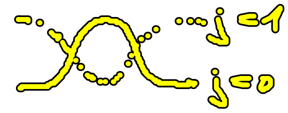
Krit. Kopplungsstärke ϵ_c, wo inkohärente Bereiche auftreten: (Chimera-Zustände)

Profil 

$\epsilon > \epsilon_c$ $\quad$ $\epsilon = \epsilon_c$ $\quad$ $\epsilon < \epsilon_c$

räuml. kontin.: $N \to \infty$, $r = \frac{R}{N}$

$$\phi^{n+1}(x) = f(\phi^n(x)) + \frac{\epsilon}{2r}\int_{x-r}^{x+r}\left[f(\phi^n(y)) - f(\phi^n(x))\right]dy$$

Betrachte $K=1$, Periode-2-Dynamik: 

$$\phi^{1-j}(x) = (1-\epsilon)\, f(\phi^j(x)) + \frac{\epsilon}{2r}\int_{x-r}^{x+r} f(\phi^j(y))\,dy \qquad j=0,1$$

Bilde $\partial_x\phi^{1-j} \approx (1-\epsilon)\, f'(\phi^j)\,\partial_x\phi^j$ $\qquad$ für Wendepkt. mit senkr. Tang. $\frac{\epsilon}{2r}\left[f(\phi^j(x+r)) - f(\phi^j(x-r))\right]$ vernachlässigt

Bilde $\partial_x\phi^0\cdot\partial_x\phi^1 = \left[(1-\epsilon)^2 f'(\phi^0)\, f'(\phi^1)\right]\partial_x\phi^0\,\partial_x\phi^1$

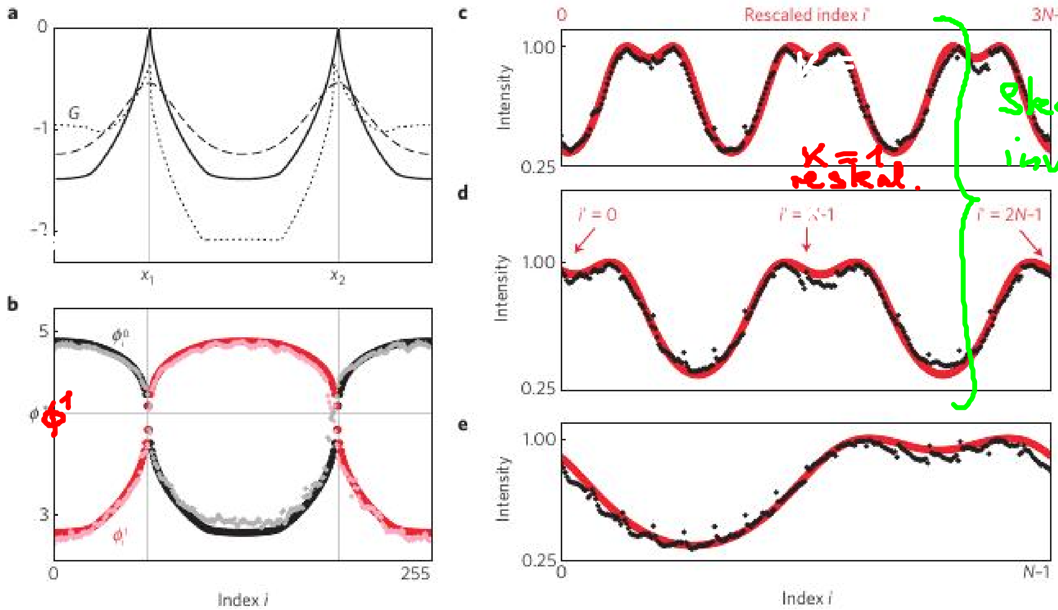
$\Longleftrightarrow \quad 1 = (1-\epsilon)^2 f'(\phi^0(x))\, f'(\phi^1(x)) \qquad f'(\phi) = \pi a \sin\phi$

$\Longleftrightarrow \quad 0 \overset{!}{=} G(x) = (\pi a)^2 (1-\epsilon)^2 \sin\phi^0(x)\,\sin\phi^1(x) - 1$

An den Diskontinuitätspkten x^*: $\phi^0(x) = \phi^1(x) = \phi^*$ Fixp. v. f

$$\Longrightarrow \boxed{\epsilon_c = 1 \,(\mp)\, \frac{1}{\pi a\,|\sin\phi^*|}}$$

Berechn. von ϵ_c

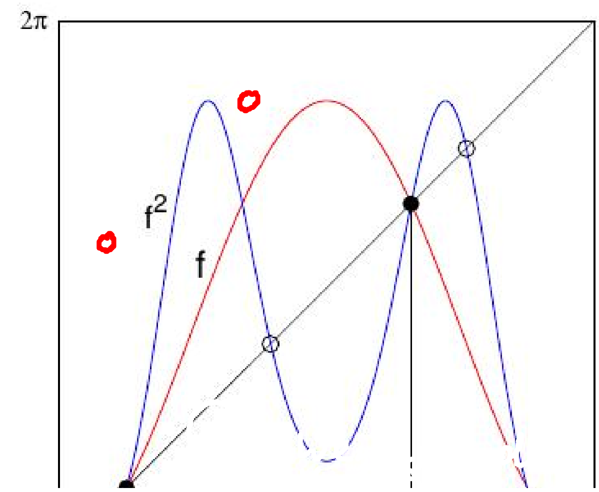


Skalen-invarianz

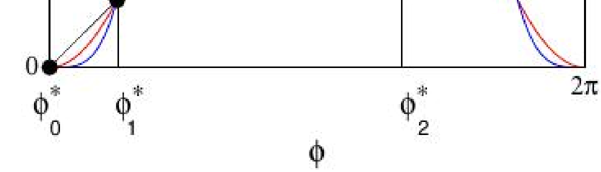
Figure 3 | Critical coupling and scaling of coherent profiles. **a**, Condition G for breaking of the smooth profile. The solid curve corresponds to the critical coupling strength $\epsilon_c = 0.54$. The dashed and dotted curves refer to $\epsilon = 0.675 > \epsilon_c$ and $\epsilon = 0.475 < \epsilon_c$, respectively. **b**, Snapshots of the period-2 solution for even (ϕ_i^0 (black)) and odd (ϕ_i^1 (red)) time steps at the critical coupling strength of **a**. The light red and grey dots refer to the experiment. Points x_1 and x_2 are discontinuity points, where the profile breaks, and ϕ^* marks the fixed point of the local map. Other parameters: $N = 256$, $a = 0.85$ and $r = 88/256 \approx 0.34$. **c**–**e**, Scaling of coherent intensity profiles. The measured coherent profiles with spatial periods $K = 1, 2, 3$ are shown in black for coupling radius $r_3 = 22/256 \approx 0.09$ (**c**), $r_2 = 33/256 \approx 0.13$ (**d**) and $r_1 = 66/256 \approx 0.26$ (**e**). The profile $K = 1$ numerically obtained from equations (1) and (5) and its rescaled profile are shown in red in **c** and **d**,**e**, respectively. Other parameters: $a = 0.85$, $\epsilon = 0.8$ and $N = 256$.

Invarianz der Profile $K = 1, 2, \ldots$: $x \to \frac{x}{K}$ (Wellenzahl) $r \to \frac{r}{K}$

Lokale Abb. $f(\phi)$



Fixpkte von f^2: Periode-2 Orbits



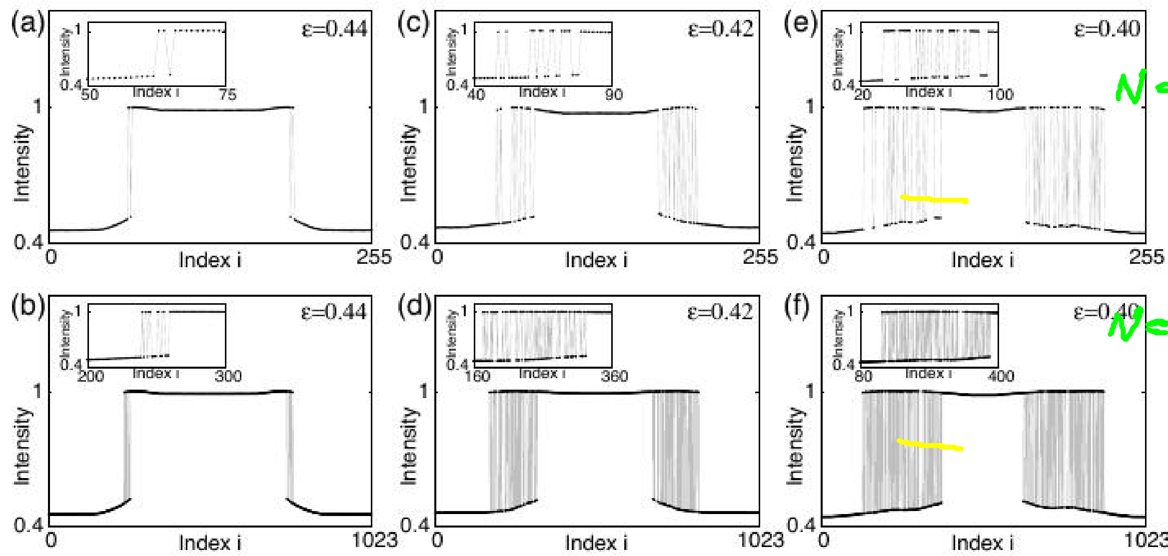


FIG. 4. **Simulations to illustrate scaling with system size of the incoherent regions for the 1D system.** Snapshots for (a),(c),(e) $N = 256$, and (b),(d),(f) $N = 1024$. Decreasing coupling strength is used in panels (a),(b) $\epsilon = 0.44$, (c),(d) $\epsilon = 0.42$, (e),(f) $\epsilon = 0.40$. Other parameters: $a = 0.85$, $r = 0.41$. Transients of 1000 time steps were neglected. The insets show the enlarged left incoherence region for each snapshot.