

6.4 Kohärente Zustände

$$\hat{E} = \frac{i}{\sqrt{2}} \sqrt{\frac{\hbar\omega}{\epsilon V}} \vec{n} \left[\exp\{i(\vec{q}\cdot\vec{r} - \omega t)\} c - \exp\{-i(\vec{q}\cdot\vec{r} - \omega t)\} c^\dagger \right]$$

$$c|\alpha\rangle = \alpha|\alpha\rangle, \alpha \in \mathbb{C}, \alpha = |\alpha| \exp\{i\varphi\}, \langle\alpha|\alpha\rangle = 1$$

$$\Rightarrow \alpha = \langle\alpha|c|\alpha\rangle = \langle c^\dagger|\alpha\rangle|\alpha\rangle = \langle\alpha|c^\dagger|\alpha\rangle^* \Rightarrow \langle\alpha|c^\dagger|\alpha\rangle = \alpha^*$$

$$c^\dagger|\alpha\rangle = \alpha^*|\alpha\rangle$$

$$\begin{aligned} \Rightarrow \langle\alpha|\hat{E}|\alpha\rangle &= \frac{i}{\sqrt{2}} \sqrt{\frac{\hbar\omega}{\epsilon V}} \vec{n} |\alpha| \left[\exp\{i(\vec{q}\cdot\vec{r} - \omega t + \varphi)\} - \exp\{-i(\vec{q}\cdot\vec{r} - \omega t + \varphi)\} \right] \\ &= -\sqrt{2} \sqrt{\frac{\hbar\omega}{\epsilon V}} \vec{n} |\alpha| \sin(\vec{q}\cdot\vec{r} - \omega t + \varphi) \end{aligned}$$

$$\exp\{a\} \exp\{b\} = \exp\{a+b\} = \exp\{b\} \exp\{a\} \text{ nur bei } [a,b]=0$$

$$\text{Wenn } [a,b]=x \text{ und } [a,x]=0 = [b,x]$$

$$[a, b^n] = n b^{n-1} x$$

vollständige Induktion

$$\begin{aligned} [a, b^{n+1}] &= a b^n b - b^n b a = b^n a b + n b^{n-1} x b - b^n b a \\ &= \cancel{b^n b a} + b^n x + n b^n x - \cancel{b^n b a} \\ &= (n+1) b^n x \end{aligned}$$

$$[a, \exp\{b\}] = x \exp\{b\} \Rightarrow \exp\{b\} a = (a-x) \exp\{b\}$$

$$\Rightarrow \exp\{b\} a^n = (a-x)^n \exp\{b\} \quad \text{v.I.}$$

$$\exp\{b\} a^{n+1} = (a-x) \exp\{b\} a^n = (a-x)(a-x)^n \exp\{b\} = (a-x)^{n+1} \exp\{b\}$$