9.2.2 Elektrischer Hertzscher Dipol

- $$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\,\frac{e^{ikr}}{r}\int \mathbf{j}(\mathbf{r}')\,d^3r' \qquad (9.18)$$

mit $$\boxed{\int \mathbf{j}(\mathbf{r})\,d^3r = -i\omega\,\mathbf{p} = -i\omega \int \mathbf{r}\,\rho(\mathbf{r})\,d^3r} \qquad (9.19)$$

$$\longrightarrow \quad \boxed{\mathbf{A}(\mathbf{r}) = -i\omega\,\frac{\mu_0}{4\pi}\,\mathbf{p}\,\frac{e^{ikr}}{r}} \qquad (9.20)$$

- em-Feld:

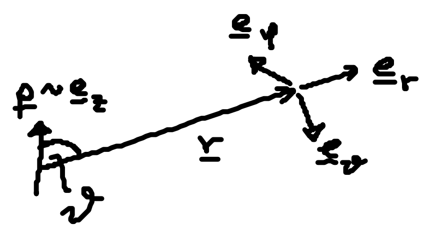
$$\mathbf{H} = \frac{1}{\mu_0}\nabla\times\mathbf{A} \longrightarrow \quad \mathbf{H}(\mathbf{r}) = \frac{c}{4\pi}\,k^2\frac{e^{ikr}}{r}\left(1-\frac{1}{ikr}\right)\mathbf{e}_r\times\mathbf{p}$$

$$\mathbf{E} = i\frac{c}{k}\nabla\times\mu_0\mathbf{H} \longrightarrow \quad \mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\,\frac{e^{ikr}}{r}\left[k^2(\mathbf{e}_r\times\mathbf{p})\times\mathbf{e}_r + \frac{1}{r^2}(1-ikr)\big(3(\mathbf{p}\cdot\mathbf{e}_r)\mathbf{e}_r-\mathbf{p}\big)\right] \qquad (9.21)$$

Diskussion:

(i) $\mathbf{H} \sim \mathbf{e}_r\times\mathbf{p} \perp$ Ebene $(\mathbf{p},\mathbf{e}_r)$

$=$ Azimuthalrichtung $\mathbf{e}_\varphi$



$\mathbf{E}$ in Ebene $(\mathbf{p},\mathbf{e}_r)$

(ii) Nahzone: $r \ll \lambda \rightarrow kr \ll 1 \longrightarrow e^{ikr}\approx 1$

$$\longrightarrow \quad \mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\,\frac{1}{r^3}\left[3(\mathbf{p}\cdot\mathbf{e}_r)\mathbf{e}_r-\mathbf{p}\right]$$

$$\longrightarrow \quad \mathbf{H}(\mathbf{r}) = i\,\frac{c}{4\pi}\,\frac{kr}{r^3}\,\mathbf{e}_r\times\mathbf{p} \qquad (9.22)$$

NB: (1) $|\underline{H}| \sim kr|\underline{E}| \ll |\underline{E}|$
im Nahfeld dominiert das „statische" elektr. Dipolfeld

(2) Faktor $i \longrightarrow$ Phasenverschiebung von $\frac{\pi}{2}$ von $\underline{H}$ gegen $\underline{E}$

(iii) Fernzone: $\frac{1}{r}$ dominiert

$$\longrightarrow \boxed{\begin{aligned} \underline{H}(\underline{r}) &= \frac{c}{4\pi} k^2 \frac{e^{ikr}}{r} \underline{e}_r \times \underline{p} \sim \underline{e}_\varphi \\ \underline{E}(\underline{r}) &= \sqrt{\frac{\mu_0}{\varepsilon_0}}\, \underline{H}(\underline{r}) \times \underline{e}_r \end{aligned}} \quad (9.33)$$

NB: (1) $\underline{H}, \underline{E} \sim \frac{e^{ikr}}{r} \longleftrightarrow$ auslaufende Kugelwelle

(2) $\underline{H}, \underline{E} \sim \frac{1}{r} \longleftrightarrow$ Abstrahlung von Energie!

(3) $\underline{H}, \underline{E}$ in Phase!

(iv) Skizze: $\underline{E}_R = Re(\underline{E}\, e^{-i\omega t}) \sim \cos(kr - \omega t)$ ... Fernfeld

$$\underline{H}_R = Re(\underline{H}\, e^{-i\omega t}) \sim \begin{cases} \sin \omega t & \text{... Nahfeld} \\ \cos(kr - \omega t) & \text{... Fernfeld} \end{cases}$$

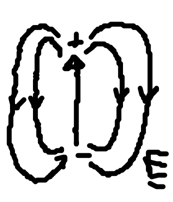
$\underline{p}_t = \underline{p} \cos \omega t \longleftrightarrow$ Strom. $I_t = -I \sin \omega t$

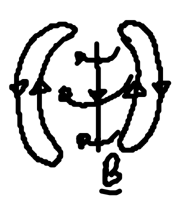
$\overset{+}{\underset{-}{\uparrow}}\, \underline{p} \longleftrightarrow I_t = 0$

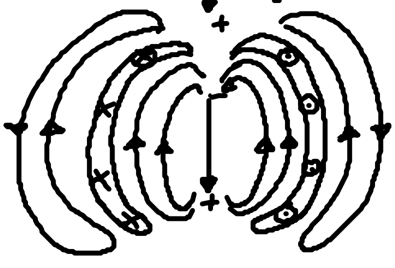
Periode: $T = \frac{2\pi}{\omega}$

$t = 0$ | $t = \frac{T}{4}$ | $t = \frac{T}{2}$ | $t = \frac{3}{4} T$

$\underline{p}_t \overset{+}{\underset{-}{\uparrow}}, I_t = 0$ | $\underline{p}_t = 0, I_t = -I$ | $\underline{p}_t \overset{-}{\underset{+}{\downarrow}}\, I_t = 0$ | $\underline{p}_t = 0, I_t \neq 0$







...

„statisches" Dipolfeld | $\underline{E}$-Feld-Wulst löst sich ab | $\underline{H}$-Feld-Wulst löst sich ab

s. Animation [Wikipedia, Hertzscher Dipol] auslaufende $\underline{E}$-Feld-Welle

Poyntingvektor $\sim \underline{e}_r$

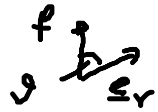
- Strahlungsleistung: für Fernfeld!

(i) Poyntingvektor:

$$\boxed{\underline{S}(\underline{r},t) = \underline{E}_R \times \underline{H}_R = \frac{c^2}{(4\pi)^2} \overset{\sqrt{\frac{\mu_0}{\varepsilon_0}}}{} p^2 k^4 \sin^2\vartheta \; \frac{\cos^2(kr-\omega t)}{r^2} \; \underline{e}_r} \quad (9.34)$$

$$\sqrt{\frac{\mu_0}{\varepsilon_0}} \underbrace{(\underline{H}_R \times \underline{e}_r)}_{\sim \underline{E}_R} \times \underline{H}_R = \sqrt{\frac{\mu_0}{\varepsilon_0}} |H_R|^2 \underline{e}_r \quad \& \quad |\underline{e}_r \times \underline{p}| = p \sin\vartheta$$

$$\underline{e}_r \cdot \underline{H}_R = 0$$

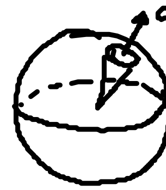


NB: $\underline{S} \sim \underline{e}_r$!

(ii) abgestrahlte mittlere Leistung pro Raumwinkelelement $d\Omega$:

$$\boxed{\frac{dP}{d\Omega} = \frac{1}{T d\Omega} \int_0^T |\underline{S} \cdot \underbrace{d\underline{f}}_{r^2 d\Omega}| \, dt = \frac{c^2}{32\pi^2} \sqrt{\frac{\mu_0}{\varepsilon_0}} \, p^2 k^4 \sin^2\vartheta} \quad (9.35)$$

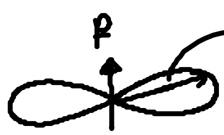
$$\frac{1}{T}\int \cos^2 \ldots = \frac{1}{2}$$



NB: (1) $\frac{dP}{d\Omega}$ unabhängig von r

$\longrightarrow$ Abstrahlung ins Unendliche!

(2) Abstrahlcharakteristik:



Polardiagramm

$$\frac{dP}{d\Omega} = 0, \quad \underline{e}_r \parallel \underline{p}$$

$$\frac{dP}{d\Omega} = \max, \quad \underline{e}_r \perp \underline{p}$$

(3) $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$ ... Wellenwiderstand des Vakuums

(iii) gesamte abgestrahlte Leistung:

$$\boxed{P = \int \frac{dP}{d\Omega}\, d\Omega = \frac{c^2}{12\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\; p^2 k^4} \qquad (9.36)$$

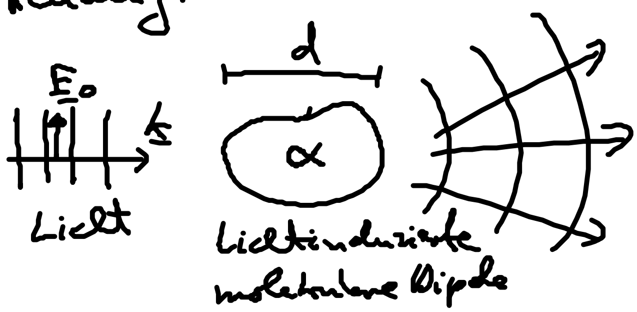
$$\int d\Omega \underbrace{\sin^2\vartheta}_{1-\cos^2\vartheta} = \frac{8\pi}{3}$$

NB: $P \sim p^2 k^4 \sim p^2 \omega^4$

$\longrightarrow$ Dipole strahlen stärker im Blauen!

800 nm $\longrightarrow$ 400 nm $\hat{=}$ Faktor 16
IR $\qquad$ UV

• Rayleighstreuung:



Abstrahlung als Hertzsche Dipole = Lichtstreuung

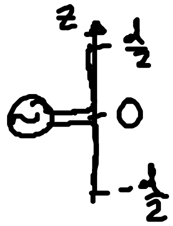
$$\underline{p} = \alpha \underline{E} = \alpha \underline{E}_0 \cos(\underline{k}\underline{r} - \omega t)$$

$$\boxed{\frac{dP}{d\Omega} \sim \alpha^2 \underline{E}_0^2\, \omega^4 \sin^2\vartheta} \qquad (9.37)$$

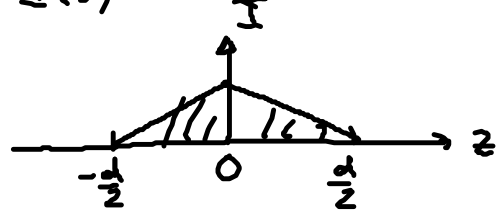
(i) Sonnenstrahlen streuen an Luftmolekülen
$\longrightarrow$ blauer Himmel

(ii) Abendrot: „durchgehendes Licht hat an „blauer" Intensität verloren"

• kurze Stabantenne:



(1) $$\underline{j}(\underline{r}) = \underbrace{I_0\left(1 - \frac{2|z|}{d}\right)}_{I(z)} \delta(x)\,\delta(y)\,\Theta\!\left(\frac{d}{2} - |z|\right) \underline{e}_z$$



(2) (9.19)

$$-i\omega \underline{p} = \int d^3 \underline{j}(\underline{r}) = \underline{e}_z \int_{-\frac{d}{2}}^{\frac{d}{2}} I(z)\,dz = \frac{I_0 d}{2}\,\underline{e}_z$$

$$\xrightarrow{\omega = ck} \quad \boxed{\underline{p} = i\,\frac{I_0 d}{2kc}\,\underline{e}_z} \quad (9.38)$$

(3) gesamte abgestrahlte Leistung: $kd \ll 1$!

$$(9.36) \text{ mit } \underline{p} \longrightarrow \boxed{P = \frac{1}{48\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\, I_0^2 (kd)^2} \quad (9.39)$$

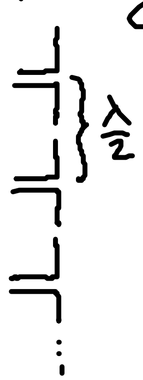
(4) Strahlungswiderstand:

$$P = \frac{I_0^2}{2} R_s \quad \text{mit} \quad R_s = \frac{1}{24\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\,(kd)^2$$

R_s ... Widerstand, der abgestrahlter Leistung („Verlust") entspricht

(5) höhere Multipole: wenn $kd \sim O(1)$

(6) Anordnung von Antennen.



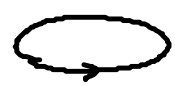
$\xRightarrow{em}$ Fokussierung der Abstrahlung $\perp$ Kette

• magnetischer Hertzscher Dipol:

$$\boxed{\underline{m} = \frac{1}{2}\int \underline{r} \times \underline{j}(\underline{r})\, d^3 r}$$

... magnet. Moment

Bsp.:



$I(t) = I_0 e^{-i\omega t}$

$\underline{E}$, $\underline{H}$ folgen aus den Feldern des elektr. Dipols durch folgende Ersetzung in (9.21):

$$\begin{aligned} \underline{p} &\longrightarrow \frac{\underline{m}}{c} \\ \underline{E} &\longrightarrow Z_0\, \underline{H} \\ Z_0\, \underline{H} &\longrightarrow -\underline{E} \quad , \quad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \end{aligned} \tag{9.41}$$

ohne Beweis!