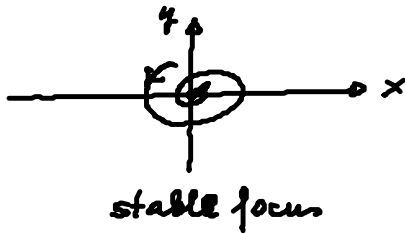


English Summary :

3.2 Noise-induced oscillations

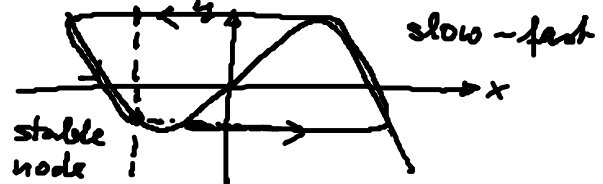
Van der Pol osc. below Hopf bif. ($\epsilon < 0$)

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= (\epsilon - x^2)y - \omega_0^2 x + \sqrt{2D} \xi(t) \end{aligned}$$

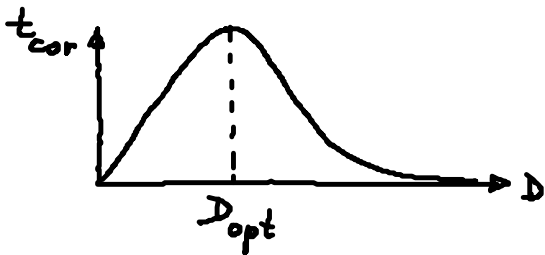


FitzHugh-Nagumo model (exc. type II) ($\alpha > 1$)

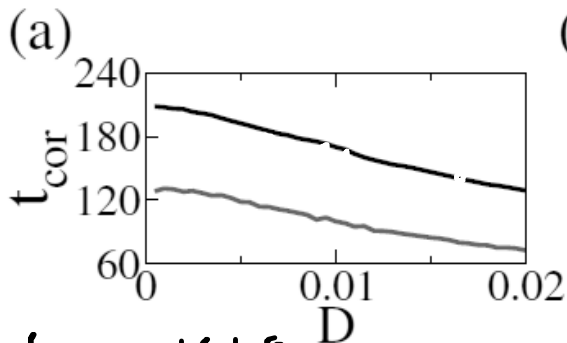
$$\begin{aligned} \epsilon \dot{x} &= x - \frac{x^3}{3} - y \\ \dot{y} &= x + \alpha + \sqrt{2D} \xi(t) \end{aligned}$$



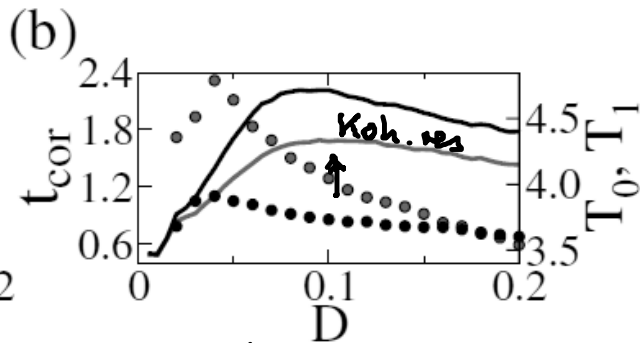
Kohärenzresonanz



Jamov, Balazso, Schöll, PRL (2004)



keine Kohärenzresonanz



Kohärenzresonanz

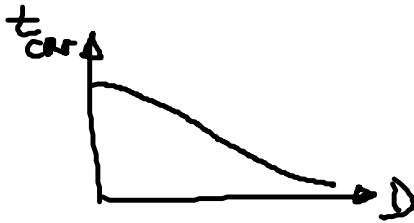
FIG. 2 (color online). Correlation time t_{cor} vs noise intensity D for (a) the Van der Pol oscillator and (b) the FitzHugh-Nagumo system. Grey (green online) lines: $K = 0$; black lines: $K = 0.2$, $\tau = T_0$. (b) Grey (green online) circles: T_0 for $K = 0$; black circles: T_1 for $K = 0.2$, $\tau = T_0$.

Zusammenspiel zweier Zeitskalen (FHN-Modell) :

- Aktivierungszeit (Fixpt. \rightarrow Schwelle) : Irregularität nimmt ab mit D

- Spike-Dauer + Erholungszeit (determ.): Irregularität nimmt zu mit D

NB: Van der Pol-Osz. zeigt keine Kohärenzresonanz



t_{cor} monoton abnehmend
(kein Schwellenverhalten,
nur lokale Eigenschaft des Fixp.)

3.4 Korrelationszeit und spektrale Leistungsdichte

Analyt. Näherung für $t_{cor}(D)$ für Van der Pol-Osz.

(selbstkons. mean-field-Näherung, Pomplun et al., Europhys. Lett 71, 366 (2005))

Näherung für $D \ll |\epsilon|/\omega_0$: $\langle x^2 \rangle$ klein

$$\epsilon - x^2 \approx \epsilon - \langle x^2 \rangle = \tilde{\epsilon} < 0$$

$$\Rightarrow \begin{cases} \dot{x} = y \\ \dot{y} = \tilde{\epsilon} y - \omega_0^2 x + D \xi(t) \end{cases}$$

lin. stoch. Dgl. (multivariater Ornstein-Uhlenbeck-Prozess)
stationär $\langle x_s \rangle = 0$

$$d\underline{x}_s = -\underline{A} \underline{x}_s dt + \underline{B} d\underline{W}(t) \quad \underline{x}_s = \begin{pmatrix} x \\ y \end{pmatrix} \quad \underline{A} = \begin{pmatrix} 0 & -1 \\ \omega_0^2 & -\tilde{\epsilon} \end{pmatrix}$$

$$\underline{B} = \begin{pmatrix} 0 & 0 \\ 0 & D \end{pmatrix}$$

Varianz-Matrix

$$\underline{\sigma} = \langle \underline{x}_s(t) \otimes \underline{x}_s(t) \rangle = \frac{D^2}{-2\tilde{\epsilon}} \begin{pmatrix} \frac{1}{\omega_0^2} & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \langle x^2 \rangle = \frac{D^2}{-2\tilde{\epsilon}\omega_0^2} = \frac{D^2}{-2(\epsilon - \langle x^2 \rangle)\omega_0^2}$$

selbstkonsistente Lösung von $\langle x^2 \rangle$ in Abhäng. von D

$$\langle x^2 \rangle^2 - \epsilon \langle x^2 \rangle - \frac{D^2}{2\omega_0^2} = 0$$

$$\langle x^2 \rangle^2 = \frac{\epsilon}{2} \left(\pm \right) \frac{\epsilon}{2} \sqrt{1 + \frac{2D^2}{\epsilon^2 \omega_0^2}}$$

(+) unphys. Lös.,
da $\langle x^2 \rangle > 0$

$$\Rightarrow \tilde{\epsilon} = \epsilon - \langle x^2 \rangle = \frac{\epsilon}{2} \left(1 + \sqrt{1 + \frac{2D^2}{\epsilon^2 \omega_0^2}} \right)$$

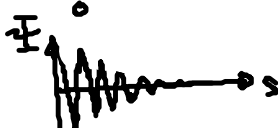
Dämpfung $\tilde{\epsilon} =$ Abstand zur Hopf-Bif. wächst mit Rauschintensität D !

Autokorrel. fkt. für linearen stoch. Prozess

$$\bar{\Phi}_{yy}(s) = \langle y(t+s)y(t) \rangle \approx \bar{\Phi}_{yy}(0) e^{ps} \cos \tilde{\omega}s$$

wobei $\lambda_{1,2} = p \pm i\tilde{\omega} = \frac{\tilde{\epsilon}}{2} \pm i\sqrt{\omega_0^2 - \frac{\tilde{\epsilon}^2}{4}}$ die Eigenwerte von A sind.

$$\Rightarrow t_{cor} = \frac{1}{\tilde{\Phi}(0)} \int_0^\infty |\bar{\Phi}(s)| ds \approx \frac{2}{\pi} \int_0^\infty e^{ps} ds = -\frac{2}{\pi p}$$

$\tilde{\Phi}$  s
 ↑
 Fullfaktor

$$\Rightarrow t_{cor}(D) \approx \frac{4}{\pi |\tilde{\epsilon}(D)|}$$

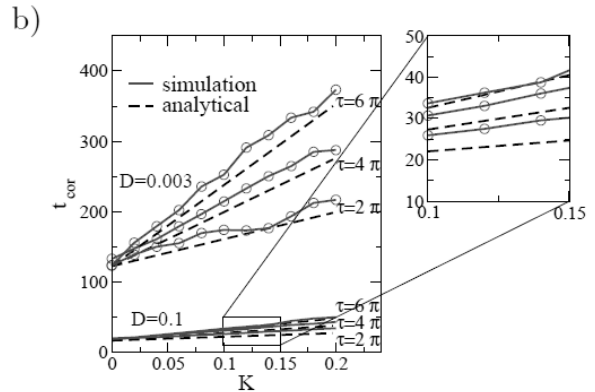
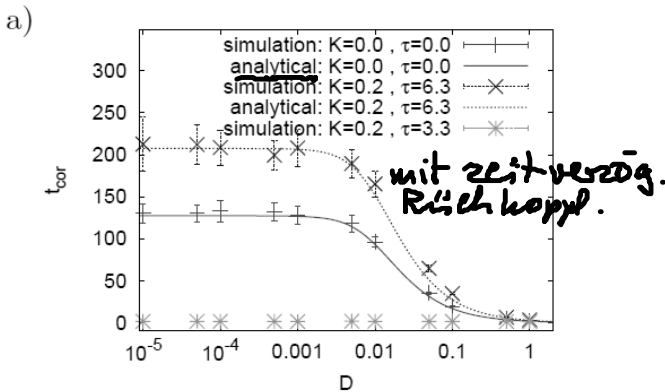


Fig. 2 – Correlation time t_{cor} in the VdP system for $\epsilon = -0.01$ a) vs. noise intensity D for different values of τ and K (symbols: numerical solution; solid line: analytical mean-field estimate from eq. (12) for $K = 0$, and from eq. (18) for $K = 0.2, \tau = 6.3$); b) vs. feedback strength K for three different values of τ ; analytical: from eq. (18).

Pomplun et al (2005)

Spektrale Leistungsdichte

Fourier-Transf. der mean-field-Gl. : $\begin{cases} \dot{x} = y \\ \dot{y} = \tilde{\epsilon} y - \omega_0^2 x + D \zeta(t) \end{cases}$

$$(x(t) = \int_{-\infty}^{\infty} d\omega e^{-i\omega t} \hat{x}(\omega))$$

$$-i\omega \hat{x}(\omega) = \hat{y}(\omega)$$

$$-i\omega \hat{y}(\omega) = \tilde{\epsilon} \hat{y}(\omega) - \omega_0^2 \hat{x}(\omega) + D \hat{\zeta}(\omega)$$

$$\text{Elim. von } \hat{x}(\omega) = \frac{i}{\omega} \hat{y}(\omega) :$$

$$-i\omega \hat{y} - \tilde{\epsilon} \hat{y} + i \frac{\omega_0^2}{\omega} \hat{y} = D \hat{\xi} \quad | i\omega$$

$$\hat{y}(\omega) = \frac{i\omega D \hat{\xi}(\omega)}{\omega^2 - \omega_0^2 - i\omega \tilde{\epsilon}}$$

$$\langle \hat{y}(\omega) \hat{y}^*(\omega') \rangle = \frac{(i\omega D)(-i\omega' D) \langle \hat{\xi}(\omega) \hat{\xi}^*(\omega') \rangle}{(\omega^2 - \omega_0^2 - i\omega \tilde{\epsilon})(\omega'^2 - \omega_0^2 + i\omega' \tilde{\epsilon})}$$

$$\begin{aligned} \langle \hat{\xi}(\omega) \hat{\xi}^*(\omega') \rangle &= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dt' e^{-i\omega' t'} \underbrace{\langle \xi(t) \xi(t') \rangle}_{\delta(t-t')} \\ &= \frac{1}{2\pi} \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{i(\omega - \omega')t} \\ &\quad \underbrace{\hspace{10em}}_{\delta(\omega - \omega')} \end{aligned}$$

R.S.: $\omega = \omega'$

$$\Rightarrow \langle \hat{y}(\omega) \hat{y}^*(\omega') \rangle = \frac{D^2}{2\pi} \frac{\omega^2 \delta(\omega - \omega')}{(\omega^2 - \omega_0^2)^2 + \omega^2 \tilde{\epsilon}^2}$$

andererseits mit Wiener-Khinchin-Theorem:

$$\begin{aligned} \langle \hat{y}(\omega) \hat{y}^*(\omega') \rangle &= \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} dt e^{i\omega t} \int_{-\infty}^{\infty} dt' e^{-i\omega' t'} \langle y(t) y(t') \rangle \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} dt e^{i(\omega - \omega')t} \underbrace{\frac{1}{2\pi} \int_{-\infty}^{\infty} ds e^{i\omega' s} \langle y(t) y(t+s) \rangle}_{\delta(\omega')} \end{aligned}$$

$$S_{yy}(\omega) = \frac{D^2}{2\pi} \frac{\omega^2}{(\omega^2 - \omega_0^2)^2 + \omega^2 \tilde{\epsilon}^2}$$

Lorentzkurve mit Max. bei ω_0

und Halbwertsbreite $\approx \frac{|\tilde{\epsilon}|}{2} = \frac{2}{\pi \tau_{cor}(D)}$



Analyt. Näherung für FitzHugh-Nagumo-Modell:

Prager, Lerch, Schimansky-Geier, Schöll, J. Phys. A 40, 11045 (2007)
Konvaris, Schimansky-Geier, Schöll: Eur. Phys. J. ST 191, 29 (2010)
2-state Master eq.

gekoppelte FitzHugh-Nagumo-Systeme mit Rauschen

Hanschmidt, Balanov, Janson, Schöll: PRE 74, 051906 (2006)

stochast. Synchronisation

farbiges Rauschen: $\langle \eta(t) \eta(t') \rangle = \sigma^2 e^{-|t-t'|/\tau_c}$

↔ generiert durch Ornstein-Uhlenbeck-Prozess

$$\tau_c \dot{\eta} = -\eta + \sqrt{2\sigma^2 \tau_c} \xi(t) \quad \langle \xi(t) \xi(t') \rangle = \delta(t-t')$$

Brandstätter, Dalen, Schöll: Phil. Trans. R. Soc. A 368, 391 (2010)