

English Summary:

Conditions for nonclassical light

$$g^{(2)}(\tau) \leq g^{(2)}(0), \quad g^{(2)}(0) < 1 \text{ (sub-Poissonian correlation)}$$



4.6 Quantum Master equations

$$\rho_{SR}(t) = \rho_S(t) \otimes \rho_R(0) + \rho_c(t), \quad \rho_S(t) = \text{tr}_R \rho_{SR} \text{ reduced density matrix}$$

system reservoir coupling

$$\dot{\rho}_S(t) = -\frac{i}{\hbar} \text{tr}_R [V(t), \rho_S(t) \otimes \rho_R(0)] - \frac{1}{\hbar^2} \text{tr}_R \int_0^t dt' [V(t), [V(t'), \rho_S(t') \otimes \rho_R(0)]]$$

Master eq. in interaction picture; interaction Ham. V ; Born-Markov approx.

oBdA $\text{tr}_R [V, \rho_R(0)] = 0$ ($\langle \text{reservoir op.} \rangle_R = 0 \Rightarrow 1.$ Term LS. fällt weg)

$$\dot{\rho}_S(t) = -\frac{1}{\hbar^2} \text{tr}_R \int_0^t dt' [V(t), [V(t'), \rho_S(t') \otimes \rho_R(0)]] \quad \text{Quanten-Master-qt.}$$

Beispiel : gedämpfte harmon. Oszillatoren

$$\text{Ham. op. des System } S \otimes R: H = \underbrace{H_S + H_R}_{H_0} + \underbrace{H_{SR}}_V$$

$$H_S = \hbar \omega_0 a^\dagger a \quad \text{harm. Osz.} \quad [a, a^\dagger] = 1$$

$$H_R = \sum_j \hbar \omega_j r_j^\dagger r_j \quad \text{bosonische Resonanzmoden} \quad [r_j, r_k^\dagger] = \delta_{jk}$$

$$\text{in harm. QP. } \mathcal{S}_R = \prod_j e^{-\frac{\hbar \omega_j r_j^\dagger r_j}{kT}} (1 - e^{-\frac{\hbar \omega_j}{kT}})$$

(z.B. Vakuum-Strahlungsfeld oder Phonon-Moden in Festk.)

↑
Zerfall einer Mode ω_j
einer opt. Kavität

↑
Zerfall eines angeregten Atoms
durch Spontane Emission

$$H_{SR} = \sum_j \hbar (\kappa_j^* a r_j^\dagger + \kappa_j a^\dagger r_j) \quad \kappa_j \in \mathbb{C} \text{ damping rate}$$

$$= \hbar (a \Gamma^\dagger + a^\dagger \Gamma) \quad \Gamma, \Gamma^\dagger \text{ coupling op.}$$

im WW-Bild:

$$a_w(t) = e^{i\omega_0 a^\dagger a t} a e^{-i\omega_0 a^\dagger a t} \oplus e^{-i\omega_0 t} = a e^{-i\omega_0 t}$$

$$a_w^\dagger(t) = e^{i\omega_0 a^\dagger a t} a^\dagger e^{-i\omega_0 a^\dagger a t} = a^\dagger e^{i\omega_0 t}$$

$$\Gamma_w^\dagger(t) = e^{i\sum_n \omega_n r_n^\dagger r_n t} \sum_j \kappa_j^* r_j^\dagger e^{-i\sum_n \omega_n r_n^\dagger r_n t}$$

$$= \sum_j \kappa_j^* r_j^\dagger e^{i\omega_j t} \Rightarrow \langle \Gamma_w^\dagger(t) \rangle_R = 0$$

$$\Gamma_w(t) = e^{i\sum_n \omega_n r_n^\dagger r_n t} \sum_j \kappa_j r_j e^{-i\sum_n \omega_n r_n^\dagger r_n t}$$

$$= \sum_j \kappa_j r_j e^{-i\omega_j t} \Rightarrow \langle \Gamma_w(t) \rangle_R = 0$$

⊗ denn $\dot{a} = -i\omega_0 [a, a^\dagger a] = -i\omega_0 (\underbrace{a a^\dagger a - a^\dagger a a}) = -i\omega_0 a$
(Heisenberg-Gln.) $[a, a^\dagger] = 1$

Mastgl. (Born-Markov) in WW-Bild

$$\begin{aligned}
 \dot{g}_S = & - \int_0^t dt' \{ (a a g_S(t) - a g_S(t) a) e^{-i\omega_0(t+t')} \underbrace{\langle \Gamma_W^+(t) \Gamma_W^+(t') \rangle_R}_{\substack{0 \\ \text{in Fock-Basis nachrechnen}}} \\
 & + (a^\dagger a g_S(t) - a g_S(t) a^\dagger) e^{i\omega_0(t+t')} \underbrace{\langle \Gamma_W(t) \Gamma_W(t') \rangle_R}_{0} \\
 & + (a^\dagger g_S(t) - a g_S(t) a) e^{-i\omega_0(t-t')} \underbrace{\langle \Gamma_W^+(t) \Gamma_W(t') \rangle_R}_{\substack{0 \\ \sum_j |k_j|^2 e^{i\omega_j(t-t')} \bar{n}(\omega_j, T)}} \\
 & + (a a g_S(t) - a g_S(t) a^\dagger) e^{i\omega_0(t-t')} \underbrace{\langle \Gamma_W(t) \Gamma_W^+(t') \rangle_R}_{\substack{0 \\ \sum_j |k_j|^2 e^{-i\omega_j(t-t')} (\bar{n}(\omega_j, T) + 1)}} \\
 & + \text{h.c.} \}
 \end{aligned}$$

mit $\bar{n}(\omega_j, T) = \text{tr}_R (\rho_R r_j^\dagger r_j) = \frac{e^{-\frac{\hbar\omega_j}{kT}}}{1 - e^{-\frac{\hbar\omega_j}{kT}}}$ Bose-Verteilung

$\tau = t - t'$

$$\begin{aligned}
 \dot{g}_S = & - (a a^\dagger g_S - a^\dagger g_S a) \int_0^t d\tau e^{-i\omega_0\tau} \langle \Gamma_W^+(t) \Gamma_W(t-\tau) \rangle_R + \text{h.c.} \\
 & - (a^\dagger a g_S - a g_S a^\dagger) \int_0^t d\tau e^{i\omega_0\tau} \langle \Gamma_W(t) \Gamma_W^+(t-\tau) \rangle_R + \text{h.c.}
 \end{aligned}$$

Kontinuumslimit der Reservoir - Korrelationsfkt. es:

$$\langle \Gamma_W^+(t) \Gamma_W(t-\tau) \rangle_R = \int_0^\infty d\omega e^{i\omega\tau} \underbrace{g(\omega) |k(\omega)|^2 \bar{n}(\omega, T)}_{\substack{\leftarrow \text{Zustandsdichte des Reservoirs}}} \approx \delta(\tau)$$

$$\langle \Gamma_W(t) \Gamma_W^+(t-\tau) \rangle_R = \int_0^\infty d\omega e^{-i\omega\tau} \underbrace{g(\omega) |k(\omega)|^2 [\bar{n}(\omega, T) + 1]}_{\substack{\text{langsam variabel}}} \approx \delta(\tau)$$

$\underbrace{\int_0^\infty d\omega}_{\substack{\text{schnell} \\ \text{oszill.}}} \underbrace{g(\omega) |k(\omega)|^2 [\bar{n}(\omega, T) + 1]}_{\substack{\text{langsam variabel}}} \\
 \int_0^\infty d\omega \text{ für } \tau \gg \tau_R = \frac{\hbar}{kT} \text{ Reservoir-Korrelationszeit} \\
 \Leftrightarrow \omega\tau = \frac{\hbar\omega}{kT} \gg 1$

Def. Zerfallskriterien:

$$\gamma_1 := \int_0^t dt \int_0^\infty d\omega e^{-i(\omega - \omega_0)t} g(\omega) |K(\omega)|^2$$

$$\gamma_2 := \int_0^t dt \int_0^\infty d\omega e^{-i(\omega - \omega_0)t} g(\omega) |K(\omega)|^2 \bar{n}(\omega, T)$$

Hauptbeitrag der τ -Integration: kleiner $\tau \Rightarrow$ ersetze $\int_0^t \rightarrow \int_0^\infty$

$$\lim_{t \rightarrow \infty} \int_0^t dt e^{-i(\omega - \omega_0)t} = \pi \delta(\omega - \omega_0) + i \frac{P}{\omega_0 - \omega}$$

Cauchy-Hauptwert

$$\Rightarrow \gamma_1 = \frac{\gamma}{2} + i\Delta$$

$$\gamma_2 = \frac{\gamma}{2} \bar{n} + i\Delta'$$

mit $\gamma := 2\pi g(\omega_0) |K(\omega_0)|^2$

$$\Delta := P \int_0^\infty d\omega \frac{g(\omega) |K(\omega)|^2}{\omega_0 - \omega}$$

$$\Delta' := P \int_0^\infty d\omega \frac{g(\omega) |K(\omega)|^2}{\omega_0 - \omega} \bar{n}$$

Mastergl.:

$$\dot{\rho}_S = (a^\dagger \rho_S a - a \rho_S a^\dagger) \gamma_1^\dagger + \overbrace{(a^\dagger \rho_S a - \rho_S a a^\dagger)}^{h.c. \rho_S = \rho_S^\dagger} \gamma_2$$

$$+ (a \rho_S a^\dagger - a^\dagger \rho_S) (\gamma_1 + \gamma_2) + \underbrace{(a \rho_S a^\dagger - \rho_S a a^\dagger)}_{h.c.} (\gamma_1^\dagger + \gamma_2^\dagger)$$

$$= a \rho_S a^\dagger \gamma \bar{n} - (a a^\dagger \rho_S + \rho_S a a^\dagger) \frac{\gamma}{2} \bar{n} + \underbrace{(a a^\dagger \rho_S - \rho_S a a^\dagger)}_{a^\dagger a \rho_S - \rho_S a^\dagger a} i \Delta' \quad [a, a^\dagger] = 1$$

$$+ a \rho_S a^\dagger \gamma (1 + \bar{n}) - (a^\dagger a \rho_S + \rho_S a^\dagger a) \frac{\gamma}{2} (1 + \bar{n}) - (a^\dagger a \rho_S - \rho_S a^\dagger a) i (\Delta + \Delta')$$

$$= -i\Delta [a^\dagger, \rho_S] + \frac{\gamma}{2} (2a \rho_S a^\dagger - a^\dagger a \rho_S - \rho_S a^\dagger a)$$

$$+ \gamma \bar{n} (a \rho_S a^\dagger + a^\dagger \rho_S a - a^\dagger a \rho_S - \rho_S a a^\dagger)$$

Rücktransform. WW-Bild \rightarrow Schrödinger-Bild $\rho_S^{(S)}$

$$\rho_S^{(S)} = -\frac{i}{\hbar} [H_S, \rho_S^{(S)}] + e^{-\frac{i}{\hbar} H_S t} \rho_S e^{\frac{i}{\hbar} H_S t}$$

$$e^{-i\omega_0 a^\dagger a} a g_s a^\dagger e^{i\omega_0 a^\dagger a} = \underbrace{a g_s a^\dagger}_{\text{Schrod. Bild}} = \underbrace{a g_s^{(s)} a^\dagger}_{\text{WV-Bild}} \quad H_S = \hbar \omega_0 a^\dagger a$$

Mastergl. für gedämpften harmon. Osc.:
(Schrodinger-Bild)

$$\dot{g}_s^{(s)} = -i\omega_0' [a^\dagger a, g_s^{(s)}] + \frac{\chi}{2} (2a g_s^{(s)} a^\dagger - a^\dagger a g_s^{(s)} - g_s^{(s)} a^\dagger a) + \gamma \bar{n} (a g_s^{(s)} a^\dagger + a^\dagger a g_s^{(s)} - a^\dagger a g_s^{(s)} - g_s^{(s)} a^\dagger a)$$

$\omega_0 + \Delta$
renormierte
Frequenz
durch Ankoppl.
an Bad

dissipative Dynamik

Dämpfung durch

- Spontane Em.
- stim. Em./Abs. ($\sim \bar{n}$)
von Bad-Bozonen
(z.B. Photonen, Phononen)

$$\Leftrightarrow \dot{g}_s^{(s)} = -i\omega_0' [a^\dagger a, g_s^{(s)}] + \frac{\chi}{2} ([a, g_s^{(s)} a^\dagger] + [a g_s^{(s)}, a^\dagger]) + \frac{\chi}{2} \bar{n} ([a g_s^{(s)}, a^\dagger] + [a^\dagger, g_s^{(s)} a])$$

$$\Leftrightarrow \dot{g}_s^{(s)} = -i\omega_0' [a^\dagger a, g_s^{(s)}] + \frac{\chi}{2} (\bar{n}+1) (2a g_s^{(s)} a^\dagger - \{a^\dagger a, g_s^{(s)}\}) + \frac{\chi}{2} \bar{n} (2a^\dagger g_s^{(s)} a - \{a a^\dagger, g_s^{(s)}\})$$

$n+1 \rightarrow n$ Anti-Kommutator $n \rightarrow n-1$
 $n-1 \rightarrow n$ $n \rightarrow n+1$

Lindblad-Form