1. Übung/Projekt zur
Statistischen Physik I, computational methods
Transport in NEMS (nanoelastomechanical systems)

We consider a two-level quantum dot coupled to a single cavity boson mode (photon or phonon),
\[ H_{\text{Rabi}} \equiv \frac{\varepsilon}{2} (|1\rangle\langle 1| - |2\rangle\langle 2|) + g (a + a^\dagger) (|1\rangle\langle 2| + |2\rangle\langle 1|) + \Omega a^\dagger a. \] (1)

Exercise
- Consider the case \( g = 0 \) (no coupling between electrons and bosons) and derive the Master equation for transport through the two dot levels by coupling to a left (\( L \)) and a right (\( R \)) electron reservoir with chemical potentials \( \mu_L = \infty \) and \( \mu_R = -\infty \). Introduce tunnel rates \( \Gamma_{L/R} \) for the two dot levels as in the Lectures.
- Solve the Master equation analytically in the stationary state, i.e. for times \( t \to \infty \). Calculate the occupation of the two levels.
- Solve the Master equation in the stationary state numerically, using MATHEMATICA.
- Solve the Master equation in the stationary state numerically, using Fortran or C++ combined with a suitable LAPACK routine.

Project I
Derive the Master equation for the case \( g > 0 \) in the number state basis and start to write a Fortran or C++ code for its solution.

To follow in this project during this semester as we proceed with the material in the Lectures:
- Stationary electronic current as a function of the coupling strength \( g \).
- How the boson mode can be ‘controlled’ electronically. Wigner function (phase-space) representations.
- Electronic and bosonic quantum noise as a tool to learn more about this coupled electron-boson system.

More detailed instructions will be given on subsequent sheets.

Other Projects to Follow
- One further NEMS project (dot-resonator system), full counting statistics (quantum noise and transport) - this will probably start in one or two weeks.
- One project on the NRG (numerical renormalisation group). This will start in two or three weeks.

NOTE:
Exercises should be done by all participant. Projects are for those who wish to get a ‘Schein’, or for those who wish to apply the material in the Lecture Notes to some interesting problems. These projects are related to some recent research in our group.

Bitte Rückseite beachten!
• Internetseite der Veranstaltung: http://www.itp.tu-berlin.de/stat-i-ss07.html

• Vorlesung: Mittwoch 12:15 - 14:00 Uhr im PN 201 und Donnerstag 14:15 - 16:00 Uhr im PN 731

• Tutorium: Dienstag 8:30 - 10:00 Uhr im PN 731

• Scheinkriterien: Erfolgreiche Teilnahme an den Übungen und erfolgreiche Durchführung eines Projektes

• Sprechstunden:
  – Prof. Dr. Tobias Brandes: Montags, 13:00 - 14:00 Uhr
  – Philipp Zedler: Mittwoch, 11:00 - 12:00 Uhr

• Literatur:
  – Vorlesungsskript (web-page)
  – H. Carmichael, An Open System Approach to Quantum Optics
  – D. F. Walls and G.J. Milburn, Quantum Optics
  – U. Weiss, Quantum Dissipative Systems
  – H. Haug and A. P. Jauho, Quantum Kinetics in Transport and Optics of Semiconductors
  – F. Haake, Quantum Signatures of Chaos
  – Literatur zur NRG wird später angegeben.