English summary

2 Phenomenological models

few equations, simple nonlinearities, feasible for bifurcation analysis
qualitative agreement of time series

2.1 FitzHugh–Nagumo model

$$\varepsilon \dot{x} = x - \frac{x^3}{3} - y$$
$$\dot{y} = x + a$$

x: activator, a: bifurcation parameter
y: inhibitor, ε: timescale separation
$\varepsilon \ll 1 \Rightarrow x$ fast, y slow

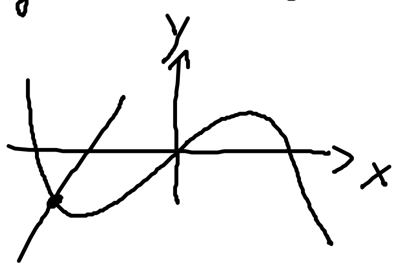
$|a|<1$: oscillatory (limit cycle)
$|a|>1$: excitable (fixed point)

$\xrightarrow{\text{linear stability analysis}}$ fixed point ($\dot{x}=0, \dot{y}=0$)
$x^* = -a$
$y^* = -a + \frac{a^3}{3}$

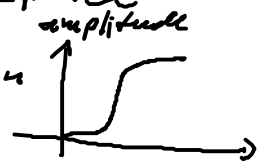
→ unstable for $|a|<1$
→ stable for $|a|>1$

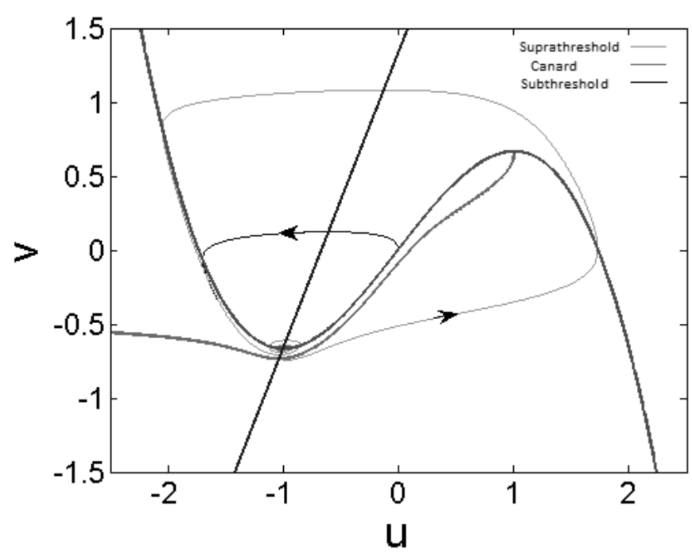
(extended) FitzHugh–Nagumo model $\hat{=}$ Bonhoeffer–van der Pol model

$$\varepsilon \dot{x} = x - \frac{x^3}{3} - y$$
$$\dot{y} = x + a - \gamma y$$



Canard trajectory:
separates subthreshold oscillations from large excursions in phase space
⇒ explosion





excitability type II: at bifurcation point: limit cycle with amplitude 0
and finite frequency $\operatorname{Im} \Lambda \big|_{|a|=1} = \frac{1}{\sqrt{\varepsilon}}$

2.2 SNIPER model

Saddle-Node Infinite PERiod bifurcation

auch als SNIC: Saddle-Node on an Invariant Cycle bekannt.

$$\left.\begin{aligned}\dot{x} &= x(1-x^2-y^2)+y(x+b)\\ \dot{y} &= y(1-x^2-y^2)-x(x+b)\end{aligned}\right\} \overset{x=r\cos\varphi}{\underset{y=r\sin\varphi}{\Longleftrightarrow}} \quad \begin{aligned}\dot{r} &= r(1-r^2)\\ \dot{\varphi} &= b-r\cos\varphi\end{aligned}$$

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Resonancelike responses of autonomous nonlinear systems to white noise

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Stochastic Resonance without External Periodic Force

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(Received 21 December 1992)

Bestimmung des Fixpunkts:

$$0 = x(1-x^2-y^2)+y(x-b)$$

$$0 = y(1-x^2-y^2)-x(x-b)$$

trivialer Fixpunkt am Ursprung $(x_A^*, y_A^*) = (0,0)$

weitere Fixpunkte für $x-b=0 \Rightarrow x^*=b$

und $1-x^2-y^2=0 \Rightarrow y^2 = 1-b^2 \Rightarrow y^* = \pm\sqrt{1-b^2}$

$$\left.\begin{aligned}\Rightarrow (x_B^*, y_B^*) &= (b, \sqrt{1-b^2})\\ (x_C^*, y_C^*) &= (b, -\sqrt{1-b^2})\end{aligned}\right\} \text{ existieren nur für } |b|<1 \quad \left|\ \begin{aligned}(r_B^*, \varphi_B^*) &= (1, \arccos b)\\ (r_C^*, \varphi_C^*) &= (1, -\arccos b)\end{aligned}\right.$$

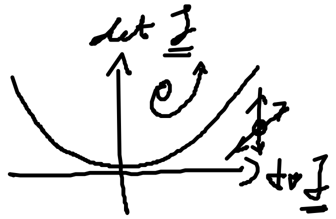
Lineare Stabilitätsanalyse:

Jacobi-Matrix: $\underline{\underline{J}} = \begin{pmatrix} 1-3x^2-y^2+y & -2xy+x-b \\ -2xy-2x+b & 1-x^2-3y^2 \end{pmatrix}$

Eigenwerte: $\Lambda_{1,2} = \frac{\mathrm{tr}\,\underline{\underline{J}} \pm \sqrt{(\mathrm{tr}\,\underline{\underline{J}})^2 - 4\det \underline{\underline{J}}}}{2}$

1. Fall: $(x_A^*, y_A^*) = (0,0) \Rightarrow \underline{\underline{J}}_A = \begin{pmatrix} 1 & -b \\ b & 1 \end{pmatrix} \Rightarrow \Lambda_{1,2} = \frac{2 \pm \sqrt{4-4(1+b^2)}}{2}$

$= 1 \pm ib$



(x_A^*, y_A^*) ist ein instabiler Fokus

($\mathrm{Re}\,\Lambda > 0$, $\mathrm{Im}\,\Lambda \neq 0$ für $b \neq 0$)

2. Fall: $(x_B^*, y_B^*) = (b, +\sqrt{1-b^2})$

$\underline{\underline{J}}_B = \begin{pmatrix} -2b^2+\sqrt{1-b^2} & -2b\sqrt{1-b^2} \\ -2b\sqrt{1-b^2}-b & -2+2b^2 \end{pmatrix}$

Eigenwerte als Lösung der charakteristischen Gleichung:

$\det(\underline{\underline{J}}_B - \Lambda \mathbb{1}) = \ldots = (\Lambda+2)(\Lambda - \sqrt{1-b^2}) = 0$

Eigenwerte $\Lambda_1 = -2$, $\Lambda_2 = \sqrt{1-b^2} > 0$ für $|b| < 1$

$\Rightarrow$ 1 stabile, 1 instabile Richtung $\Rightarrow$ Sattelpunkt

3. Fall: $(x_C^*, y_C^*) = (b, -\sqrt{1-b^2})$

$\det(\underline{\underline{J}}_C - \Lambda \mathbb{1}) = \ldots = (\Lambda+2)(\Lambda + \sqrt{1-b^2}) = 0$

$\Rightarrow$ Eigenwerte $\Lambda_1 = -2$, $\Lambda_2 = -\sqrt{1-b^2} < 0$ für $|b|<1$

$\Rightarrow$ 2 stabile Richtungen $\Rightarrow$ Knoten

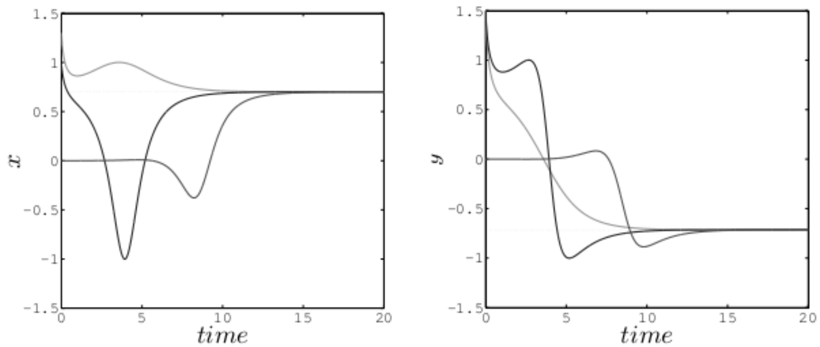
Bei $|b| = 1$: $(x_B^*, y_B^*) = (1, 0) = (x_C^*, y_C^*)$

⇒ Fixpunkte kollidieren mit $\mathrm{Im}\,\Lambda = 0$

⇒ intrinsische Zeit $T \sim \frac{1}{\mathrm{Im}\,\Lambda} \to \infty$

Dynamische Szenarien:

(i) unterhalb der Bifurkation $|b| < 1$, z.B. $b = 0.7$

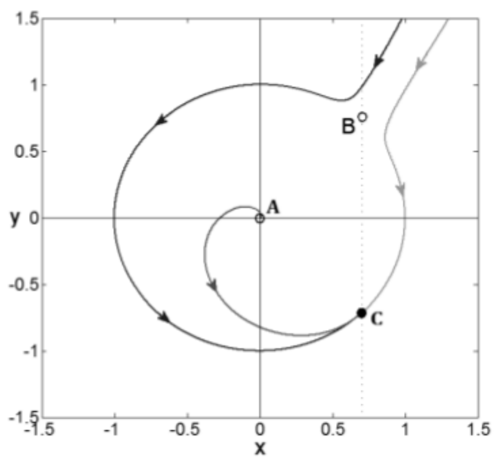


3 Fixpunkte

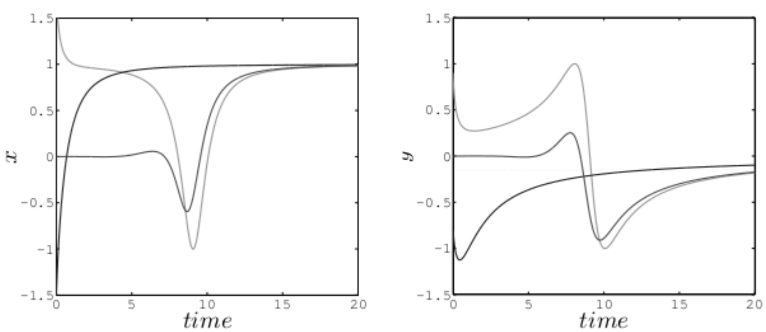
Fokus A

Sattelpunkt B

Stabiler Knoten C

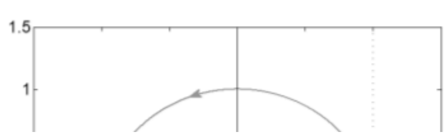


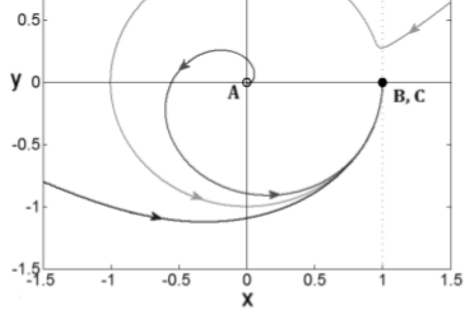
(ii) an der Bifurkation $|b| = 1$, z.B. $b = 1$



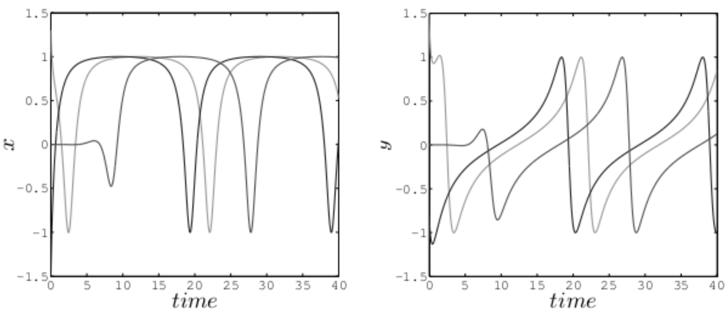
A: Fokus

Fixpunkte B & C kollidieren





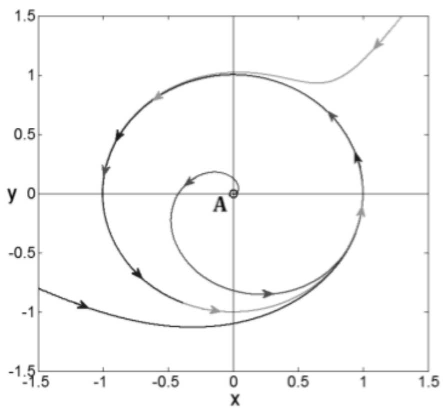
(iii) oberhalb der Bifurkation $|b|>1$, z.B. $b=1.05$



Fokus A

Grenzzyklus mit Radius 1

langsame Dynamik in der Nähe von $(1,0)$ $\Rightarrow$ "Geist"



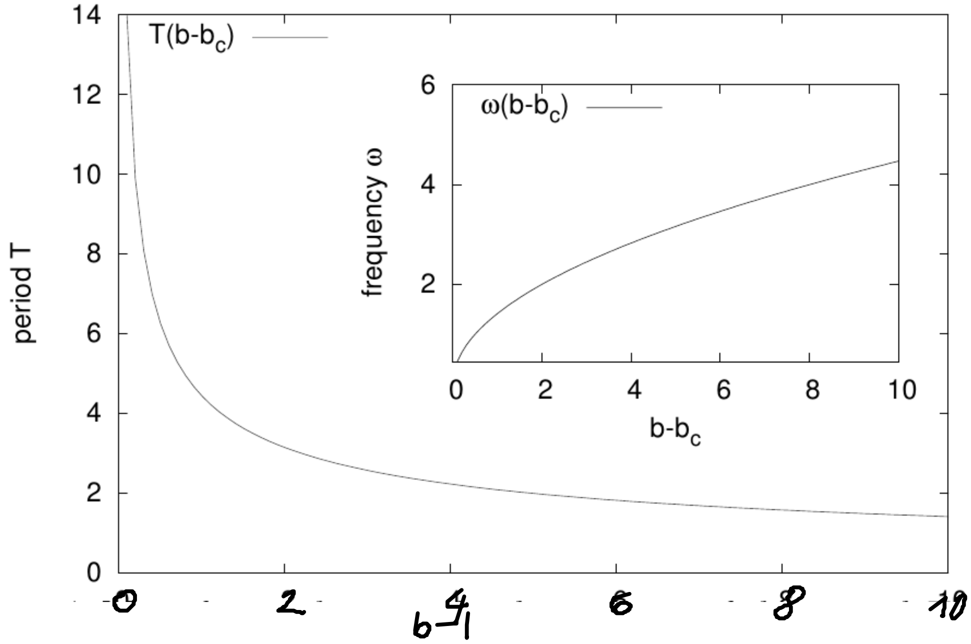
Berechnung der Periode des Grenzzyklus $(b>1)$

$$\dot{\varphi} = b - \cos\varphi \Rightarrow d\varphi = (b-\cos\varphi)\,dt$$

$$\Rightarrow dt = \frac{d\varphi}{b-\cos\varphi}$$

$$\Rightarrow \text{Periode } T = \int_0^{2\pi} \frac{d\varphi}{b-\cos\varphi} = \ldots = \frac{2\pi}{\sqrt{b^2-1}}$$

$\Rightarrow$ Periode T divergiert für $b \to 1$



Am Bifurkationspunkt:
unendliche Periode
endliche Amplitude
⇒ Anregbarkeit Typ I

2.3 Hindmarsh-Rose-Modell (2D)

$$\dot{x} = c\left(x - \frac{x^3}{3} - y + z\right)$$

$$\dot{y} = \frac{1}{c}\left(x^2 + dx - by + a\right)$$

Kubische und quadratische Nullkline

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A model of the nerve impulse using two first-order differential equations

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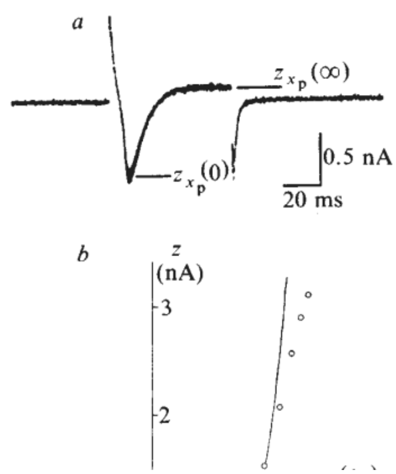

Thus the assumed form for our equations is

$$\dot{x} = -a(f(x) - y - z) \tag{7}$$

$$\dot{y} = b(f(x) - q\, e^{rx} + s - y) \tag{8}$$

where $f(x) = cx^3 + dx^2 + ex + h$, and a–h, q, r and s are constants.

After measuring a and b, and fitting cubic and exponential functions to the $z_{x_p}(0)$ and $z_{x_p}(\infty)$ data of Fig. 1b, the solutions of equations (7) and (8) were obtained by numerical integration.

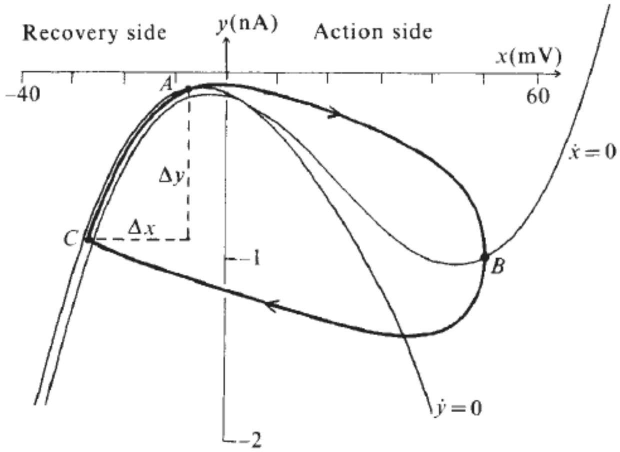

Fig. 3 Phase plane representation of the limit cycle solution to equations (7) and (8). The values of the constants are the same

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BIFURCATIONS IN TWO-DIMENSIONAL HINDMARSH–ROSE TYPE MODEL

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Bestimmung der FP: $\dot{x} = 0$, $\dot{y} = 0$

⇒ Lösen der Gleichung: $\alpha x^3 + \beta x^2 + \gamma x + \delta = 0$

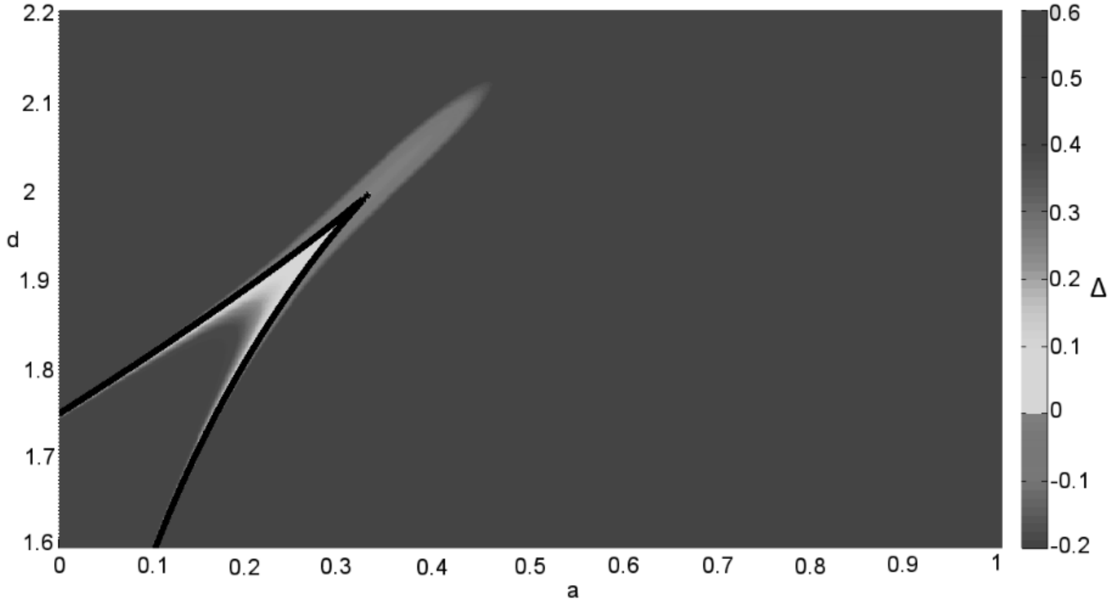
hier (für $b=1$, $c=3$, $z=0$): $\alpha = 1$, $\beta = 3$, $\gamma = 3(d-1) + 3a$

⇒ Berechne Diskriminante: $\Delta = \beta^2\gamma^2 - 4\alpha\gamma^3 - 4\beta^3\delta - 27\alpha^2\delta^2 + 18\alpha\beta\gamma\delta$

(a) $\Delta > 0$: 3 reelle Lösungen

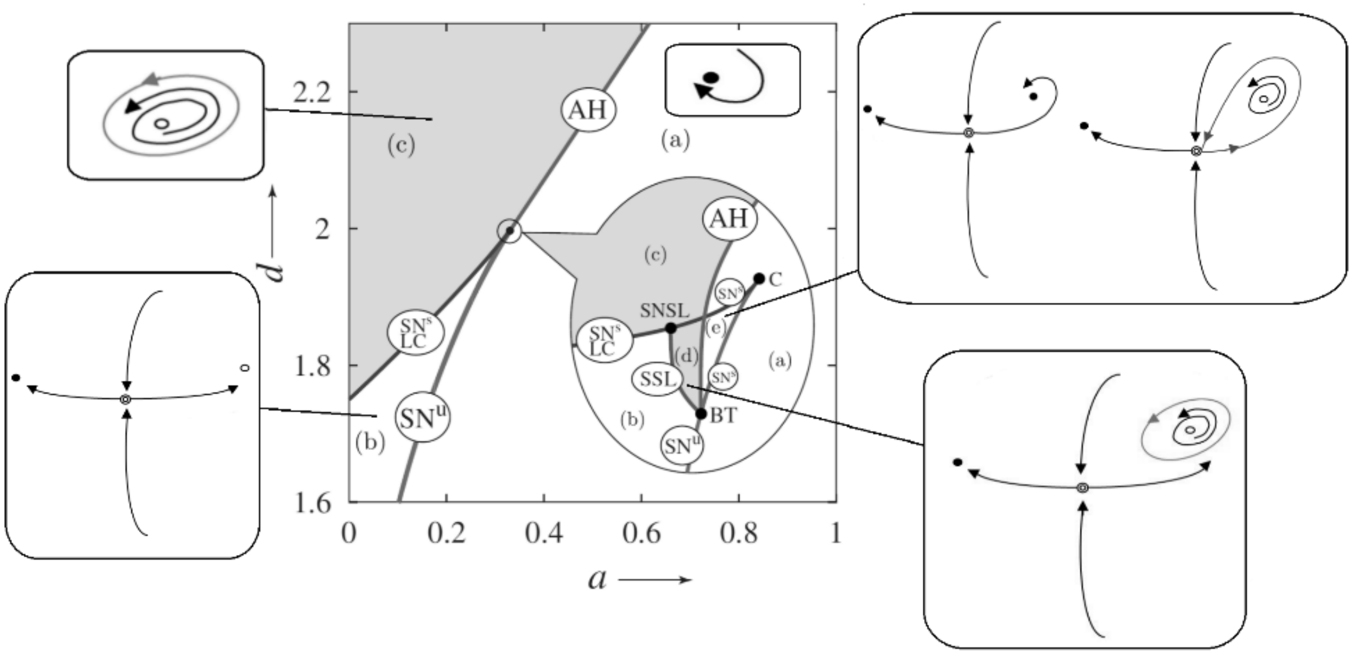
(b) $\Delta = 0$: 1 Lösung mit Vielfachheit 2, 1 weitere Lösung

(c) $\Delta < 0$: 1 reelle Lösung, 2 komplex konjugierte Lösungen



Eigenwerte per linearer Stabilitätsanalyse

$$\underline{\underline{J}} = \begin{pmatrix} c - c(x^*)^2 & -c \\ \frac{1}{c}(2x^*+d) & -\frac{b}{c} \end{pmatrix}$$



Label	Description
(a)	1 stable fixed point
(b)	1 saddle, 1 stable, 1 unstable fixed point
(c)	1 unstable fixed point, 1 stable limit cycle
(d)	1 saddle, 1 stable fixed points, 1 unstable fixed point, 1 stable limit cycle
(e)	1 saddle, 2 stable fixed points
AH	(Andronov-)Hopf bifurcation
SNLC	Saddle-node bifurcation on a limit cycle
SN	Saddle-node bifurcation (of equilibria)
BT	Bogdanov-Takens bifurcation
C	Cusp bifurcation
SSN	Saddle-separatrix loop bifurcation
SNSL	Saddle-node on separatrix loop bifurcation

Fortsetzung folgt...