Prof. Dr. Dr. h.c. Eckehard Schöll, PhD

Projekte zur Nichtlinearen Dynamik und Kontrolle

Durchführung

Die Projekte beinhalten Aufgaben aus verschiedenen Bereichen der nichtlinearen Dynamik und Kontrolle und können nach eigenen Vorstellungen bearbeitet werden (Numerik, Analytik, Zusammenfassung der Literatur, Experimente . . .). Die in jeder Projektbeschreibung aufgeführten Punkte können als Leitfaden dienen, Sie können aber auch in Absprache mit den BetreuerInnen eigene Ideen verfolgen.
Die Projekte sind so konzipiert, dass die Bearbeitung mit der angegebenen Literatur und dem Wissen aus der Vorlesung möglich ist.
Zur vollständigen Bearbeitung gehören folgende Punkte:

1. Bearbeitung des Projekts in Dreiergruppen
2. Präsentation der Ergebnisse in einem 15 minütigen Kurzvortrag (+5 Minuten Diskussion) am 17.7 oder 19.7.18. Wichtig ist hierbei in erster Linie die verständliche Darstellung. Beschränken Sie sich deshalb auf die zum Verständnis wesentlichen Punkte.
3. Abgabe einer schriftlichen Ausarbeitung mit vollständiger Dokumentation der Lösungswege und vollständigen Quellenangaben bis 31.7.18. Auch hier steht die Verständlichkeit und übersichtliche Darstellung im Vordergrund. Der Umfang der Ausarbeitung soll fünf bis zehn Seiten umfassen.

Während der gesamten Bearbeitungszeit stehen Ihnen die BetreuerInnen des jeweiligen Projektes für Fragen zur Verfügung. Bitte machen Sie individuell Termine mit den Betreuern aus.

Projekt 1: *Crowd synchrony on the Millenium Bridge in London*

Betreuer: Eckehard Schöll

On its opening day the London Millennium footbridge experienced unexpected large amplitude wobbling subsequent to the migration of pedestrians onto the bridge [1-3]. Modeling the stepping of the pedestrians on the bridge as phase oscillators allows to obtain a model for the combined dynamics of people and the bridge that is analytically tractable. It provides predictions for the phase dynamics of individual walkers and for the critical number of people for the onset of oscillations. Numerical simulations and analytical estimates reproduce the linear relation between pedestrian force and bridge velocity as observed in experiments. They allow prediction of the amplitude of bridge motion, the rate of relaxation to the synchronized state and the magnitude of the fluctuations due to a finite number of people.

- familirize with the literature
- reproduce the results of [3]

Literatur

[1] https://www.youtube.com/watch?v=gQK21572oSU
[2] Strogatz, S. H., Abraham, D., McRobbie, A. D., Eckhardt, Bruno and Ott, E., *Crowd synchrony on the Millennium Bridge*, Nature **438**, 43 (2005).
[3] Eckhardt, Bruno, Ott, E., Strogatz, S. H. , Abrams, D. M. and McRobie, Allan, *Modeling walker synchronization on the Millennium Bridge*, Phys. Rev. E **75, 2, 021110 (2007)**

Projekt 2: *External synchronization of chimeras in multilayer neural networks*

Betreuer: Anna Zakharova

Chimera states are intriguing spatiotemporal patterns made up of spatially separated domains of synchronized (spatially coherent) and desynchronized (spatially incoherent) behavior that arise surprisingly in networks of identical units and symmetric coupling topologies [1].
Recently, *multilayer networks* have been suggested to offer better representation of the topology and dynamics of real-world systems in comparison with isolated one-layer structures [2]. In multilayer networks the nodes are distributed in different layers according to the type of the relation they share. For example, in the case of a neuronal network the neurons can form different layers depending on their connectivity through a chemical link or by an ionic channel. In brain networks different regions can be seen connected by functional and structural neural networks.
A challenging open question concerning the dynamics of multilayer networks is the synchronization of chimera states. The main goal of this project is to study the *external synchronization* of chimera states in a two-layer network of coupled neural systems. Each layer is represented by a nonlocally coupled ring of FitzHugh-Nagumo (FHN) oscillators. This two-dimensional system is a paradigmatic model for neural excitability. Previously, chimera states have been found in one-layer networks consisting of coupled oscillatory FHN systems [3]. In the present project we also focus on the oscillatory FHN neurons:

$$\begin{aligned}
\varepsilon\frac{du_{1i}}{dt} &= u_{1i} - \frac{u_{1i}^3}{3} - v_{1i} + \frac{\sigma_1}{2R_1}\sum_{j=i-R_1}^{i+R_1}[b_{uu}(u_{1j}-u_{1i})+\\
&\quad +b_{uv}(v_{1j}-v_{1i})],\\
\frac{dv_{1i}}{dt} &= u_{1i} + a_i + \frac{\sigma_1}{2R_1}\sum_{j=i-R_1}^{i+R_1}[b_{vu}(u_{1j}-u_{1i})+\\
&\quad +b_{vv}(v_{1j}-v_{1i})],\\
\varepsilon\frac{du_{2i}}{dt} &= u_{2i} - \frac{u_{2i}^3}{3} - v_{2i} + \frac{\sigma_2}{2R_2}\sum_{j=i-R_2}^{i+R_2}[b_{uu}(u_{2j}-u_{2i})+\\
&\quad +b_{uv}(v_{2j}-v_{2i})] + \sigma_{12}(u_{1i}-u_{2i}),\\
\frac{dv_{2i}}{dt} &= u_{2i} + a_i + \frac{\sigma_2}{2R_2}\sum_{j=i-R_2}^{i+R_2}[b_{vu}(u_{2j}-u_{2i})+\\
&\quad +b_{vv}(v_{2j}-v_{2i})],
\end{aligned} \tag{1}$$

where u_{1i} and v_{1i} are the activator and inhibitor variables in the first(upper) layer, respectively, $i = 1, ..., N$ with N being the total number of elements in the network. All indices are modulo N. In a similar way u_{2i} and v_{2i} represent the activator and inhibitor variables in the second(lower) layer, respectively. The strength of the coupling within the layer (intra-layer coupling) is given by σ_1 for the first layer and σ_2 for the second layer. R_1 and R_2 indicate the number of nearest neighbours in each direction on a ring for the first and second layer, respectively. Since we focus here on the *external synchronization*, the coupling between the layers (inter-layer coupling) is unidirectional. This means that nodes in the first (driver/master) layer are coupled unidirectionally to the nodes in the second (response/slave) layer. Also the considered coupling is diffusive and its strength is characterized by σ_{12}. We introduce coupling range for both layers. It is represented by the normalized number of nearest neighbours for first(upper) layer $r_1 = R_1/N$ and for second(lower) layer $r_2 = R_2/N$. A small parameter responsible for the time scale separation of fast activator and slow inhibitor is given by $\varepsilon > 0$ and a_i defines the excitability threshold. For an individual FHN element it determines whether the system is in excitable ($|a_i| > 1$), or oscillatory ($|a_i| < 1$) regime. In the present study we assume that all elements are in the oscillatory regime ($a_i \equiv a = 0.5$). Eq. (1) contains not only direct, but also cross couplings between activator (u) and inhibitor (v) variables, which is modeled by a rotational coupling matrix [3]:

$$B = \begin{pmatrix} b_{\mathrm{uu}} & b_{\mathrm{uv}} \\ b_{\mathrm{vu}} & b_{\mathrm{vv}} \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}, \tag{2}$$

where $\phi \in [-\pi; \pi)$. Here we fix the parameter $\phi = \pi/2 - 0.1$. Chimera states have been found for this value of ϕ in deterministic oscillatory [3] regime.
The following steps will be taken:

- Familiarize with literature, and reproduce the results of [3] (one-layer network, nonlocal coupling on a ring).
- Consider a two-layer network (nonlocal coupling on a ring in each layer, identical parameters for the layers, both layers in chimera regime). First, analyze the synchronization of chimeras qualitatively. Calculate space-time plots and mean phase velocity profiles of the patterns observed in the two layers for different values of the coupling σ_{12} between them. Are these patterns the same in both layers? Are only the coherent domains of chimeras the same in both layers or do the incoherent domains also coincide?
- Second, to quantify the synchronization of chimera states calculate the synchronization error. Calculate the dependence of the synchronization error on the coupling between the layers σ_{12}. For which values of the coupling between the layers σ_{12} do you observe synchronization of chimera states?
- Contact the group working on mutual synchronization and compare your results.

Literatur

[1] Panaggio, M. J. and Abrams, D. M., *Chimera states: Coexistence of coherence and incoherence in networks of coupled oscillators*, Nonlinearity **28**, R67 (2015).
[2] S. Boccaletti, G. Bianconi, R. Criado, C. I. del Genio, J. Goomez-Gardennes, M. Romance, I. Sendinna Nadal, Z. Wang, and M. Zanin, *The structure and dynamics of multilayer networks*, Physics Reports **544**, 1 (2014).
[3] I. Omelchenko, O. E. Omel'chenko, P. Hövel, and E. Schöll, *When nonlocal coupling between oscillators becomes stronger: patched synchrony or multichimera states*, Phys. Rev. Lett. **110**, 224101 (2013).

Projekt 3: *Mutual synchronization of chimeras in multilayer neural networks*

Betreuer: Anna Zakharova

Chimera states are intriguing spatiotemporal patterns made up of spatially separated domains of synchronized (spatially coherent) and desynchronized (spatially incoherent) behavior that arise surprisingly in networks of identical units and symmetric coupling topologies [1].
Recently, *multilayer networks* have been suggested to offer better representation of the topology and dynamics of real-world systems in comparison with isolated one-layer structures [2]. In multilayer networks the nodes are distributed in different layers according to the type of the relation they share. For example, in the case of a neuronal network the neurons can form different layers depending on their connectivity through a chemical link or by an ionic channel. In brain networks different regions can be seen connected by functional and structural neural networks.
A challenging open question concerning the dynamics of multilayer networks is the synchronization of chimera states. The main goal of this project is to study the *mutual synchronization* of chimera states in a two-layer network of coupled neural systems. Each layer is represented by a nonlocally coupled ring of FitzHugh-Nagumo (FHN) oscillators. This two-dimensional system is a paradigmatic model for neural excitability. Previously, chimera states have been found in one-layer networks consisting of coupled oscillatory FHN systems [3]. In the present project we also focus on the oscillatory FHN neurons:

$$\begin{aligned}
\varepsilon\frac{du_{1i}}{dt} &= u_{1i} - \frac{u_{1i}^3}{3} - v_{1i} + \frac{\sigma_1}{2R_1}\sum_{j=i-R_1}^{i+R_1}[b_{uu}(u_{1j}-u_{1i})+ \\
&\quad +b_{uv}(v_{1j}-v_{1i})] + \sigma_{12}(u_{2i}-u_{1i}), \\
\frac{dv_{1i}}{dt} &= u_{1i} + a_i + \frac{\sigma_1}{2R_1}\sum_{j=i-R_1}^{i+R_1}[b_{vu}(u_{1j}-u_{1i})+ \\
&\quad +b_{vv}(v_{1j}-v_{1i})], \\
\varepsilon\frac{du_{2i}}{dt} &= u_{2i} - \frac{u_{2i}^3}{3} - v_{2i} + \frac{\sigma_2}{2R_2}\sum_{j=i-R_2}^{i+R_2}[b_{uu}(u_{2j}-u_{2i})+ \\
&\quad +b_{uv}(v_{2j}-v_{2i})] + \sigma_{12}(u_{1i}-u_{2i}), \\
\frac{dv_{2i}}{dt} &= u_{2i} + a_i + \frac{\sigma_2}{2R_2}\sum_{j=i-R_2}^{i+R_2}[b_{vu}(u_{2j}-u_{2i})+ \\
&\quad +b_{vv}(v_{2j}-v_{2i})],
\end{aligned} \tag{1}$$

where u_{1i} and v_{1i} are the activator and inhibitor variables in the first(upper) layer, respectively, $i = 1, ..., N$ with N being the total number of elements in the network. All indices are modulo N. In a similar way u_{2i} and v_{2i} represent the activator and inhibitor variables in the second(lower) layer, respectively. The strength of the coupling within the layer (intra-layer coupling) is given by σ_1 for the first layer and σ_2 for the second layer. R_1 and R_2 indicate the number of nearest neighbours in each direction on a ring for the first and second layer, respectively. Since we focus here on the *mutual synchronization*, the coupling between the layers (inter-layer coupling) is bidirectional. Also the considered coupling is diffusive and its strength is characterized by σ_{12}. We introduce coupling range for both layers. It is represented by the normalized number of nearest neighbours for first(upper) layer $r_1 = R_1/N$ and for second(lower) layer $r_2 = R_2/N$. A small parameter responsible for the time scale separation of fast activator and slow inhibitor is given by $\varepsilon > 0$ and a_i defines the excitability threshold. For an individual FHN element it determines whether the system is in excitable ($|a_i| > 1$), or oscillatory ($|a_i| < 1$) regime. In the present study we assume that all elements are in the oscillatory regime ($a_i \equiv a = 0.5$). Eq. (1) contains not only direct, but also cross couplings between activator (u) and inhibitor (v) variables, which is modeled by a rotational coupling matrix [3]:

$$B = \begin{pmatrix} b_{\mathrm{uu}} & b_{\mathrm{uv}} \\ b_{\mathrm{vu}} & b_{\mathrm{vv}} \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}, \quad (2)$$

where $\phi \in [-\pi; \pi)$. Here we fix the parameter $\phi = \pi/2 - 0.1$. Chimera states have been found for this value of ϕ in deterministic oscillatory [3] regime.
The following steps will be taken:

- Familiarize with literature, and reproduce the results of [3] (one-layer network, nonlocal coupling on a ring).
- Consider a two-layer network (nonlocal coupling on a ring in each layer, identical parameters for the layers, both layers in chimera regime). First, analyze the synchronization of chimeras qualitatively. Calculate space-time plots and mean phase velocity profiles of the patterns observed in the two layers for different values of the coupling σ_{12} between them. Are these patterns the same in both layers? Are only the coherent domains of chimeras the same in both layers or do the incoherent domains also coincide?
- Second, to quantify the synchronization of chimera states calculate the synchronization error. Calculate the dependence of the synchronization error on the coupling between the layers σ_{12}. For which values of the coupling between the layers σ_{12} do you observe synchronization of chimera states?
- Contact the group working on external synchronization and compare your results.

Literatur

[1] Panaggio, M. J. and Abrams, D. M., *Chimera states: Coexistence of coherence and incoherence in networks of coupled oscillators*, Nonlinearity **28**, R67 (2015).
[2] S. Boccaletti, G. Bianconi, R. Criado, C. I. del Genio, J. Goomez-Gardennes, M. Romance, I. Sendinna Nadal, Z. Wang, and M. Zanin, *The structure and dynamics of multilayer networks*, Physics Reports **544**, 1 (2014).
[3] I. Omelchenko, O. E. Omel'chenko, P. Hövel, and E. Schöll, *When nonlocal coupling between oscillators becomes stronger: patched synchrony or multichimera states*, Phys. Rev. Lett. **110**, 224101 (2013).

Projekt 4: *Impact of inhomogeneity on synchronization of chimeras in multilayer neural networks*

Betreuer: Anna Zakharova

Chimera states are intriguing spatiotemporal patterns made up of spatially separated domains of synchronized (spatially coherent) and desynchronized (spatially incoherent) behavior that arise surprisingly in networks of identical units and symmetric coupling topologies [1].
Recently, *multilayer networks* have been suggested to offer better representation of the topology and dynamics of real-world systems in comparison with isolated one-layer structures [2]. In multilayer networks the nodes are distributed in different layers according to the type of the relation they share. For example, in the case of a neuronal network the neurons can form different layers depending on their connectivity through a chemical link or by an ionic channel. In brain networks different regions can be seen connected by functional and structural neural networks.
A challenging open question concerning the dynamics of multilayer networks is the synchronization of chimera states. The main goal of this project is to study the impact of inhomogeneities on the *mutual synchronization* of chimera states in a two-layer network of coupled neural systems. Each layer is represented by a nonlocally coupled ring of FitzHugh-Nagumo (FHN) oscillators. This two-dimensional system is a paradigmatic model for neural excitability. Previously, chimera states have been found in one-layer networks consisting of coupled oscillatory FHN systems [3]. In the present project we also focus on the oscillatory FHN neurons:

$$
\begin{aligned}
\varepsilon \frac{du_{1i}}{dt} &= u_{1i} - \frac{u_{1i}^3}{3} - v_{1i} + \frac{\sigma_1}{2R_1} \sum_{j=i-R_1}^{i+R_1} [b_{uu}(u_{1j} - u_{1i}) + \\
&\quad + b_{uv}(v_{1j} - v_{1i})] + \sigma_{12}(u_{2i} - u_{1i}), \\
\frac{dv_{1i}}{dt} &= u_{1i} + a_i + \frac{\sigma_1}{2R_1} \sum_{j=i-R_1}^{i+R_1} [b_{vu}(u_{1j} - u_{1i}) + \\
&\quad + b_{vv}(v_{1j} - v_{1i})], \\
\varepsilon \frac{du_{2i}}{dt} &= u_{2i} - \frac{u_{2i}^3}{3} - v_{2i} + \frac{\sigma_2}{2R_2} \sum_{j=i-R_2}^{i+R_2} [b_{uu}(u_{2j} - u_{2i}) + \\
&\quad + b_{uv}(v_{2j} - v_{2i})] + \sigma_{12}(u_{1i} - u_{2i}), \\
\frac{dv_{2i}}{dt} &= u_{2i} + a_i + \frac{\sigma_2}{2R_2} \sum_{j=i-R_2}^{i+R_2} [b_{vu}(u_{2j} - u_{2i}) + \\
&\quad + b_{vv}(v_{2j} - v_{2i})],
\end{aligned}
\tag{1}
$$

where u_{1i} and v_{1i} are the activator and inhibitor variables in the first(upper) layer, respectively, $i = 1, ..., N$ with N being the total number of elements in the network. All indices are modulo N. In a similar way u_{2i} and v_{2i} represent the activator and inhibitor variables in the second(lower) layer, respectively. The strength of the coupling within the layer (intra-layer coupling) is given by σ_1 for the first layer and σ_2 for the second layer. R_1 and R_2 indicate the number of nearest neighbours in each direction on a ring for the first and second layer, respectively. Since we focus here on the mutual synchronization, the coupling between the layers (inter-layer coupling) is bidirectional. Also the considered coupling is diffusive and its strength is characterized by σ_{12}. We introduce coupling range for both layers. It is represented by the normalized number of nearest neighbours for first(upper) layer $r_1 = R_1/N$ and for second(lower) layer $r_2 = R_2/N$. A small parameter responsible for the time scale separation of fast activator and slow inhibitor is given by $\varepsilon > 0$ and a_i defines the excitability threshold. For an individual FHN element it determines whether the system is in excitable ($|a_i| > 1$), or oscillatory ($|a_i| < 1$) regime. In the present study we assume that all elements are in the oscillatory regime. Eq. (1) contains not only direct, but also cross couplings between activator (u) and inhibitor (v) variables, which is modeled by a rotational coupling matrix [3]:

$$B = \begin{pmatrix} b_{\mathrm{uu}} & b_{\mathrm{uv}} \\ b_{\mathrm{vu}} & b_{\mathrm{vv}} \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}, \tag{2}$$

where $\phi \in [-\pi; \pi)$. Here we fix the parameter $\phi = \pi/2 - 0.1$. Chimera states have been found for this value of ϕ in deterministic oscillatory [3] regime.
The following steps will be taken:

- Familiarize with literature, and reproduce the results of [3] (one-layer network, nonlocal coupling on a ring).
- Consider a two-layer network (nonlocal coupling on a ring in each layer, identical parameters for the layers $a_1 = a_2 = 0.5$, both layers in chimera regime). First, analyze the synchronization of chimeras qualitatively. Calculate space-time plots and mean phase velocity profiles of the patterns observed in the two layers for different values of the coupling σ_{12} between them. Are these patterns the same in both layers? Are only the coherent domains of chimeras the same in both layers or do the incoherent domains also coincide?
- Second, to quantify the synchronization of chimera states calculate the synchronization error. Calculate the dependence of the synchronization error on the mismatch between the layers ($a_1 \neq a_2$). For example, you can fix $a_1 = 0.5$ and tune a_2 (from 0.5 to 0.9). For which values of the coupling between the layers σ_{12} do you observe synchronization of chimera states? How does this value depend on the detuning between the layers?
- Contact the group working on mutual synchronization of identical elements and compare your results.

Literatur

[1] Panaggio, M. J. and Abrams, D. M., *Chimera states: Coexistence of coherence and incoherence in networks of coupled oscillators*, Nonlinearity **28**, R67 (2015).
[2] S. Boccaletti, G. Bianconi, R. Criado, C. I. del Genio, J. Goomez-Gardennes, M. Romance, I. Sendinna Nadal, Z. Wang, and M. Zanin, *The structure and dynamics of multilayer networks*, Physics Reports **544**, 1 (2014).
[3] I. Omelchenko, O. E. Omel'chenko, P. Hövel, and E. Schöll, *When nonlocal coupling between oscillators becomes stronger: patched synchrony or multichimera states*, Phys. Rev. Lett. **110**, 224101 (2013).

Projekt 5: *Role of the nonlinearity of local dynamics in two-layer networks*

Betreuer: Iryna Omelchenko

Complex topologies in networks allow for existence of various nontrivial spatio-temporal patterns. Chimera states are an example of intriguing partial synchronization phenomena, which exhibit a hybrid structure combining coexisting spatial domains of coherent (synchronized) and incoherent (desynchronized) dynamics [1,2].

In many investigations of chimera states, usually the character of the local node dynamics has been considered as fixed. However, the impact of the local dynamics is indeed high. In Ref. [3] the properties of chimera states have been analyzed in the case, when the dynamics of individual oscillators smoothly changes from sinusoidal to nonlinear relaxation oscillations. For this reason, the Van der Pol oscillator is a good choice to describe the dynamics of each node. The Van der Pol oscillator has a long history of being used in both the physical and biological sciences, as a generic model for electrical circuits and action potentials of neurons, respectively.

Networks of Van der Pol oscillators have nonlocal topology, where each node interacts with some fixed rage of its nearest neighbors:

$$\ddot{u}_k = \varepsilon(1 - u_k^2)\dot{u}_k - u_k + \frac{\sigma}{2R}\sum_{j=k-R}^{k+R}\left[b_1(u_j - u_k) + b_2(\dot{u}_j - \dot{u}_k)\right],$$

where $k = 1, ..., N$, all indices are taken mod N, ε defines the local dynamics of each individual unit, σ is the coupling strength, b_1 and b_2 are interaction parameters, and R is the number of coupled neighbors (in both directions).

In [3] we have shown that nonlinearity of local dynamics (defined by parameter ε) induces existence of chimera states with multiple incoherent domains, The aim of this project is to study two-layer networks of nonlocally coupled Van der Pol oscillators, where each layer has nonlocal topology, but nonlinearity of local dynamics differs between the layers.

- Provide a literature study on chimera states
- Reproduce the results from Ref. [3]
- Construct a two-layer network of Van der Pol oscillators with nonlocal topologies within the layer. Choose first identical coupling range and nonlinearity parameters ($\varepsilon_1 = \varepsilon_2$).
- Provide numerical experiments for two-layer networks with ($\varepsilon_1 \neq \varepsilon_2$). Can chimera states still be observed? Construct space-time plots and snapshots of obtained patterns.
- Vary slightly range of the nonlocal coupling in one of the layers. Which qualitative changes in the system's dynamics can be observed?

Literatur

[1] Panaggio, M. J. and Abrams, D. M., Chimera states: Coexistence of coherence and incoherence in networks of coupled oscillators, Nonlinearity **28**, R67 (2015).

[2] Schöll, E. , Synchronization patterns and chimera states in complex networks: interplay of topology and dynamics, Eur. Phys. J. Spec. Top. **225**, 891 (2016).

[3] Omelchenko, I., Zakharova, A. , Hövel, P., Siebert, J. and Schöll, E. , Nonlinearity of local dynamics promotes multi-chimeras, Chaos **25**, 083104 (2015).

Projekt 6: *Complex patterns in two-layer networks of Van der Pol oscillators with fractal connectivity*

Betreuer: Iryna Omelchenko

Complex topologies in networks allow for existence of various nontrivial spatio-temporal patterns. Chimera states are an example of intriguing partial synchronization phenomena, which exhibit a hybrid structure combining coexisting spatial domains of coherent (synchronized) and incoherent (desynchronized) dynamics [1,2].
First, studies of chimera states have been focused on regular nonlocal topologies, however later these patterns have been observed in the variety of more complex topologies. Recent studies in neuroscience have revealed an intricate architecture in the neuron interconnectivity of the human and mammalian brain. The analysis of Diffusion Tensor Magnetic Resonance Imaging (DT-MRI) images has shown that the connectivity of the neuron axons network represents a hierarchical (fractal) structure. The network topology with quasi-fractal connectivity can be constructed using Cantor algorithm, as described in [3].
Within this project networks with quasi-fractal (hierarchical) connectivity of Van der Pol oscillators will be considered:

$$\ddot{u}_k = \varepsilon(1 - u_k^2)\dot{u}_k - u_k + \frac{\sigma}{N}\sum_{j=1}^{N} C_{kj}\left[b_1(u_j - u_k) + b_2(\dot{u}_j - \dot{u}_k)\right],$$

where $k = 1, ..., N$, all indices are taken mod N, ε defines the local dynamics of each individual unit, σ is the coupling strength, $\mathbf{C} = \{C_{kj}\}_{k,j=1,...,N}$ is the coupling matrix, b_1 and b_2 are interaction parameters. The hierarchical coupling matrix is constructed using Cantor algorithm.
The aim of this project is to understand what kind of complex spatio-temporal patterns can be observed in the two-layer network, where each layer is characterized by fractal connectivity.

- Provide a literature study on chimera states
- Reproduce the results from [3]
- Construct a two-layer network of identical Van der Pol oscillators with fractal topologies within the layer. First, consider identical layers. Base patterns and system parameters can be chosen as in [3] in order to obtain chimera states in isolated layers.
- Check how the strength of the coupling between the layers influence patterns in whole network. Can chimera states still be observed in each layer? Are they synchronized? Which qualitative changes can be observed with increasing of the coupling strength between the layers?
- Construct an example of two-layer network, where layers have different fractal topologies. Analyze observed patterns.

Literatur

[1] Panaggio, M. J. and Abrams, D. M., Chimera states: Coexistence of coherence and incoherence in networks of coupled oscillators, Nonlinearity **28**, R67 (2015).

[2] Schöll, E. , Synchronization patterns and chimera states in complex networks: interplay of topology and dynamics, Eur. Phys. J. Spec. Top. **225**, 891 (2016).
[3] Ulonska, S., Omelchenko, I., Zakharova, A. and Schöll, E. , Chimera states in networks of Van der Pol oscillators with hierarchical connectivities, Chaos **26**, 094825 (2016).

Projekt 7: *Watanabe-Strogatz Transformation und das Kuramoto-Model*

Betreuer: Rico Berner

Die Beschreibung kollektiver Phänomene, systemische Zustände bei denen nicht das individuelle Verhalten sondern vielmehr das Verhalten einer Gruppe im Vordergrund steht, ist in den letzten Jahrzehnten immer mehr in den Fokus der Forschung gerückt. Über die Modellierung durch komplexe dynamische Netzwerke konnten eine Vielzahl von Effekten und Vorgängen in der Physik, Biologie, Chemie bis hin zur Ökonomie sowie Soziologie verstanden werden.
In diesem Projekt soll ein System gekoppelter Oszillatoren mittels des sogenannten Kuramoto(-Sakaguchi)-Models

$$\frac{\mathrm{d}\phi_i}{\mathrm{d}t} = -\frac{1}{N}\sum_{j=1}^{N}\sin(\phi_i - \phi_j + \alpha)$$

beschrieben werden [1, 2]. Hierbei sind die N Oszillatoren über eine Phase $\phi_i \in [0, 2\pi)$ dargestellt ($i \in 1, \ldots, N$), welche untereinander über die nichtlineare Kopplungsfunktion $\sin(\Delta\phi + \alpha)$ wechselwirken, wobei $\alpha \in [0, 2\pi)$ einen allgemeinen Phasenversatz ausdrückt. Es is bekannt, dass dieses Modell unter bestimmten Voraussetzungen auf ein System von 3 dynamischen Variablen reduziert werden kann [3, 4].

- Führen Sie eine Literaturrecherche zu diesem Thema durch.
- Wie sieht die Variablentransformation und das dreidimensionale System gewöhnlicher Differentialgleichungen aus.
- Geben Sie eine geometrische Deutung für die Transformation an [5, 6].
- Diskutieren Sie die Einschränkungen für die Watanabe-Strogatz Transformation. Welche Klasse von Differentialgleichungen können mit dieser Methode behandelt werden?
- **Forschungsfrage:** In der Watanabe-Strogatz Transformation werden N feste Phasen eingeführt, welche „gleichmäßig“ auf dem Intervall $[0, 2\pi)$ verteilt sein sollen. Eine ähnliche Bedingung garantiert bei adaptiv gekoppelten Phasen-Oszillatoren die Existenz von sogenannten „phase-lock“ -Zuständen. Kommentieren Sie einen möglichen Zusammenhang.

Literatur

[1] Y. Kuramoto: *Chemical Oscillations, Waves and Turbulence* (Springer-Verlag, Berlin, 1984).

[2] H. Sakaguchi and Y. Kuramoto: *A soluble active rotater model showing phase transitions via mutual entertainment*, Progress of Theoretical Physics **76**, 576 (1986).

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Projekt 8: *Natürliche Uhren in dynamischen Systemen*

Betreuer: Rico Berner

In unserer Umwelt sind „natürliche Uhren"überall dort anzutreffen wo sich Objekte oszillierend bewegen oder Prozesse periodisch ablaufen. Eines der wohl derzeit bekanntesten Bespiele für solche natürlichen Uhren ist der circadiane Rhythmus „Die innere Uhr", für dessen Beschreibung im letzten Jahr der Nobelpreis für Physiologie/Medizin vergeben wurde (https://www.nobelprize.org/-nobel_prizes/medicine/laureates/2017/press.html).

In diesem Projekt wollen wir uns in einem dynamischen Netzwerk gekoppelter Oszillatoren auf die Suche nach „natürlich Uhren"machen. Als grundlegendes Modell soll das sogenannte Kuramoto-Model verwendet werden [1]

$$\frac{\mathrm{d}\phi_i}{\mathrm{d}t} = \omega_i - \frac{1}{N}\sum_{j=1}^{N} a_{ij}\sin(\phi_i - \phi_j)$$

Hierbei sind die N Oszillatoren über eine Phase $\phi_i \in [0, 2\pi)$ dargestellt $(i \in 1, \ldots, N)$, welche untereinander, entsprechend ihrer Kopplung $a_{ij} \in 0, 1$, über die nichtlineare Kopplungsfunktion $\sin(\Delta\phi)$ wechselwirken. Die natürliche Eigendynamik der Oszillatoren ist über die Frequenz ω_i beschrieben. In einer Reihe von Publikationen wurde numerisch die Existenz von (quasi-)periodischen Lösungen („natürliche Uhren") gezeigt und das Verhalten dieser Zustände untersucht [2, 3, 4].

- Führen Sie eine Literaturrecherche zu diesem Thema durch.
- Gehen Sie von dem in [2] beschriebenen System aus und reproduzieren Sie die numerischen Ergebnisse. Schreiben Sie dafür ein Programm, welches die Differentialgleichungen löst (Heun-Methode).
- Verifizieren Sie das numerische Ergebnis durch die Verwendung des Runge-Kutta-Verfahrens 4-ter Ordnung und durch eine zusätzliche Neuskalierung der Zeitvariablen.
- Diskutieren Sie die (quasi-)periodischen Lösungen.
- **Forschungsfrage:** Machen Sie einen Störungsansatz in σ (Halbwertsbreite für die Verteilung der natürlichen Frequenzen ω_i) und bestimmen Sie die Koeffizienten der Störungsentwicklung bis zur zweiten Ordnung. Wie gut werden die numerischen Lösungen durch die Störungsreihe approximiert?

Literatur

[1] Y. Kuramoto: *Chemical Oscillations, Waves and Turbulence* (Springer-Verlag, Berlin, 1984).

[2] D. Labavić and H. Meyer-Ortmanns: *Long-period clocks from short-period oscillators*, Chaos: An Interdisciplinary Journal of Nonlinear Science **27**, 083103 (2017).

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Projekt 9: *Exotische „phase-lock“ Zustände in adaptiven Netzwerken*

Betreuer: Rico Berner

Die Erforschung dynamischer Netzwerke erfreut sich immer größer werdender Beliebtheit, nicht zuletzt da sich hierdurch eine Vielzahl interdisziplinärer Problemstellungen aus der Physik, Klimaforschung, Neurowissenschaft, Biologie, Soziologie, Ökonomie oder auch dem Ingenieurwesen lösen lassen. Vor allem die Erforschung kollektiver Phänomene, systemische Zustände bei denen nicht das individuelle Verhalten sondern vielmehr das Verhalten einer Gruppe im Vordergrund steht, lieferte durch die Arbeiten von Winfree [1] oder auch Kuramoto [2] herausragende Erkenntnisse für die moderne Theorie nichtlinearer Systeme. Adaptive Netzwerke, nicht-statische Netzwerke, sind ein natürlicher Bestandteil unserer Umwelt. In neuronalen Systemen beispielsweise kontrollieren verschiedene Mechanismen die Stärke der synaptische Kopplung (synaptische Plastizität) und somit den „Informationsfluss“ zwischen einzelnen wechselwirkenden Neuronen.
In diesem Projekt soll ein System adaptiv gekoppelter Oszillatoren ausgehend von dem sogenannten Kuramoto(-Sakaguchi)-Model

$$\begin{aligned}\frac{\mathrm{d}\phi_i}{\mathrm{d}t} &= -\frac{1}{N}\sum_{j=1}^{N}\kappa_{ij}\sin(\phi_i-\phi_j+\alpha)\\ \frac{\mathrm{d}\kappa_{ij}}{\mathrm{d}t} &= -\epsilon\left(\kappa_{ij}+\sin(\phi_i-\phi_j+\beta)\right)\end{aligned} \tag{1}$$

untersucht werden [3, 4]. Hierbei sind die N Oszillatoren über eine Phase $\phi_i \in [0, 2\pi)$ dargestellt ($i \in 1, \dots, N$), welche untereinander entsprechend ihrer Kopplungsgewichte $\kappa_{ij} \in [-1, 1]$ über die nichtlineare Kopplungsfunktion $\sin(\Delta\phi + \alpha)$ wechselwirken, wobei $\alpha \in [0, 2\pi)$ einen allgemeinen Phasenversatz ausdrückt. Die Adaption des Netzwerkes an den Zustand des oszillatorischen Systems ist durch eine zusätzliche Differentialgleichung beschrieben, wobei $\beta \in [0, 2\pi)$ als Parameter die Form der (neuronalen) Plastizität bestimmt. Es kann gezeigt werden, dass in einem solchen System (1) die unterschiedlichsten „phase-lock“ Zustände koexistieren können und eine Charakterisierung über den sogenannten Kuramoto-Ordnungsparameter (teilweise) möglich ist.

- Führen Sie eine Literaturrecherche zu diesem Thema durch.
- Finden Sie alle „phase-lock“ Zustände des Systems (1).
- Schlagen Sie unter Verwendung des Kuramoto-Ordnungsparameters eine geeignete Klassifizierung der Zustände vor.
- Veranschaulichen Sie die Zustände über eine Darstellung der Phasen auf dem Einheitskreis.
- **Forschungsfrage:** Führen Sie für die Klasse von „phase-lock“ Zuständen mit glqq variablen“ Ordnungsparameter eine lineare Stabilitätsanalyse durch. Stellen Sie im Parameterraum $\{(\alpha, \beta) : \alpha \in [0, \pi/2), \beta \in [0, 2\pi\}$ die Bereiche für die Stabilität der Lösungen abhängig von dem Ordnungsparameter dar.

Literatur

[1] A. T. Winfree: *The Geometry of Biological Time* (Springer, New York, 1980).

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[3] H. Sakaguchi and Y. Kuramoto: *A soluble active rotater model showing phase transitions via mutual entertainment*, Progress of Theoretical Physics **76**, 576 (1986).

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Projekt 10: *Relay synchronization of chimeras in multiplex networks with various toplogies*

Betreuer: Jakub Sawicki

Complex networks consisting of several interacting layers allow for remote synchronization of distant layers via an intermediate relay layer. The notion of relay synchronization can be extend to chimera states, where one can study the scenarios of relay synchronization in a three-layer network of FitzHugh-Nagumo oscillators, where each layer has a nonlocal coupling topology. Varying the coupling strength and time delay in the inter-layer connections, one can observe relay synchronization between chimera states, i.e., complex spatio-temporal patterns of coexisting coherent and incoherent domains [1], in the outer network layers. Special regimes where only the coherent domains of chimeras are synchronized, and the incoherent domains remain desynchronized, as well as transitions between different synchronization regimes can be analyzed. These results can be useful for secure communication and modeling of brain dynamics.
Within this project networks with three layers of FitzHugh-Nagumo (FHN) oscillators will be considered:

$$\dot{\boldsymbol{x}}_i^m(t) = \boldsymbol{F}(\boldsymbol{x}_i^m(t)) + \frac{\sigma_m}{2R_m} \sum_{j=i-R_m}^{i+R_m} \boldsymbol{H}[\boldsymbol{x}_j^m(t) - \boldsymbol{x}_i^m(t)] + \sum_{l=1}^{3} \sigma_{ml} \boldsymbol{H}[\boldsymbol{x}_i^l(t) - \boldsymbol{x}_i^m(t)] \qquad (1)$$

with $i \in \{1, ..., N\}$, $m \in \{1, ..., 3\}$. The coupling radius in layer m is denoted by R_m. The dynamics of each individual oscillator is governed by

$$\boldsymbol{F}(\boldsymbol{x}) = \begin{pmatrix} \epsilon^{-1}(u - \frac{u^3}{3} - v) \\ u + a \end{pmatrix}, \qquad (2)$$

where $\epsilon > 0$ is a small parameter characterizing the time scale separation. Depending on the threshold parameter a the FHN element exhibits either oscillatory ($|a| < 1$) or excitable ($|a| > 1$) behavior. The parameter σ_m denotes the coupling strength inside the layer (intra-layer coupling), and σ_{ml} is the inter-layer coupling. The interaction is realized through diffusive coupling with coupling matrix $\boldsymbol{H}$ and coupling phase $\phi = \frac{\pi}{2} - 0.1$ [2]. The aim of this project is to understand what kind of synchronization scenarios can be observed in the three-layer network.

- Provide a literature study on chimera states [1][2].
- Reproduce the results from [3].
- Construct a three-layer network of FHN oscillators with non local couplings within the layer. First, consider identical layers. System parameters can be chosen as in [3] in order to obtain chimera states in isolated layers.
- Check how the mismatch of the coupling radius R_m between the layers ($R_1 \neq R_2 \neq R_3$) influences the synchronization scenarios. Can chimera states still be observed in each layer? Are they synchronized? Which qualitative changes can be observed with a mismatch $R_{2/3} << R_1$?
- Contact the group working on synchronization in multiplex networks and compare your results.

Literatur

[1] Panaggio, M. J. and Abrams, D. M., *Chimera states: Coexistence of coherence and incoherence in networks of coupled oscillators, Nonlinearity* **28**, R67 (2015).
[2] Omelchenko, I., Omel'chenko, O. E., Hövel, P. and Schöll, E., *When nonlocal coupling between oscillators becomes stronger: patched synchrony or multichimera states*, Phys. Rev. Lett. **110**, 224101 (2013).
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Projekt 11: *Robustness of synchronization scenarios of chimeras in multiplex networks*

Betreuer: Jakub Sawicki

Complex networks consisting of several interacting layers allow for remote synchronization of distant layers via an intermediate relay layer. The notion of relay synchronization can be extend to chimera states, where one can study the scenarios of relay synchronization in a three-layer network of FitzHugh-Nagumo oscillators, where each layer has a nonlocal coupling topology. Varying the coupling strength and time delay in the inter-layer connections, one can observe relay synchronization between chimera states, i.e., complex spatio-temporal patterns of coexisting coherent and incoherent domains [1], in the outer network layers. Special regimes where only the coherent domains of chimeras are synchronized, and the incoherent domains remain desynchronized, as well as transitions between different synchronization regimes can be analyzed. These results can be useful for secure communication and modeling of brain dynamics.
Within this project networks with three layers of FitzHugh-Nagumo (FHN) oscillators will be considered:

$$\dot{\boldsymbol{x}}_i^m(t) = \boldsymbol{F}(\boldsymbol{x}_i^m(t)) + \frac{\sigma_m}{2R_m} \sum_{j=i-R_m}^{i+R_m} \boldsymbol{H}[\boldsymbol{x}_j^m(t) - \boldsymbol{x}_i^m(t)] + \sum_{l=1}^{3} \sigma_{ml} \boldsymbol{H}[\boldsymbol{x}_i^l(t) - \boldsymbol{x}_i^m(t)] \qquad (1)$$

with $i \in \{1, ..., N\}$, $m \in \{1, ..., 3\}$. The coupling radius in layer m is denoted by R_m. The dynamics of each individual oscillator is governed by

$$\boldsymbol{F}(\boldsymbol{x}) = \begin{pmatrix} \epsilon^{-1}(u - \frac{u^3}{3} - v) \\ u + a_m \end{pmatrix}, \qquad (2)$$

where $\epsilon > 0$ is a small parameter characterizing the time scale separation. Depending on the threshold parameter a_m the FHN element exhibits either oscillatory ($|a_m| < 1$) or excitable ($|a_m| > 1$) behavior. The parameter σ_m denotes the coupling strength inside the layer (intra-layer coupling), and σ_{ml} is the inter-layer coupling. The interaction is realized through diffusive coupling with coupling matrix $\boldsymbol{H}$ and coupling phase $\phi = \frac{\pi}{2} - 0.1$ [2]. The aim of this project is to understand what kind of synchronization scenarios can be observed in the three-layer network.

- Provide a literature study on chimera states [1][2].
- Reproduce the results from [3].
- Construct a three-layer network of FHN oscillators with non local couplings within the layer. First, consider identical layers. System parameters can be chosen as in [3] in order to obtain chimera states in isolated layers.
- Check how the mismatch of the threshold parameter a_m between the layers ($a_1 \neq a_2 \neq a_3$) influences the synchronization scenarios. Can chimera states still be observed in each layer? Are they synchronized? Which qualitative changes can be observed with a mismatch $|a_{1/2}| < 1$, $|a_2| > 1$?
- Contact the group working on synchronization in multiplex networks and compare your results.

Literatur

[1] Panaggio, M. J. and Abrams, D. M., *Chimera states: Coexistence of coherence and incoherence in networks of coupled oscillators, Nonlinearity* **28**, R67 (2015).
[2] Omelchenko, I., Omel'chenko, O. E., Hövel, P. and Schöll, E., *When nonlocal coupling between oscillators becomes stronger: patched synchrony or multichimera states*, Phys. Rev. Lett. **110**, 224101 (2013).
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