

Chapter 9 Superconductivity

So far our materials were "mix" of electrons and ions:

- Born-Oppenheimer: electrons and ions separable (elec-phonon coupling requires different treatment)
- independent electron approximation for many-electron ground state (always exists (DFT))
- independent quasiparticle picture as simple extension (e.g. photoemission, Fermi liquid theory)

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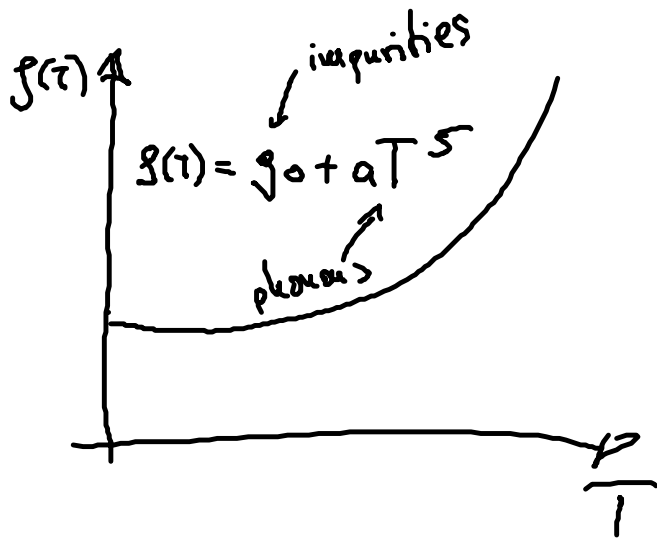
but: independent electron/quasiparticle picture

can break down:

e.g. most magnetic ordering phenomena are

Many-body in nature

even worse:



normal conductor



Superconductor

Kamerling Onnes 1911

- But perfect conductivity "should" not exist at $T \approx 0$
- It's also not an isolated phenomenon
- Typical transition temperatures:

Hg (α) 4.15 K

Sn 3.72 K

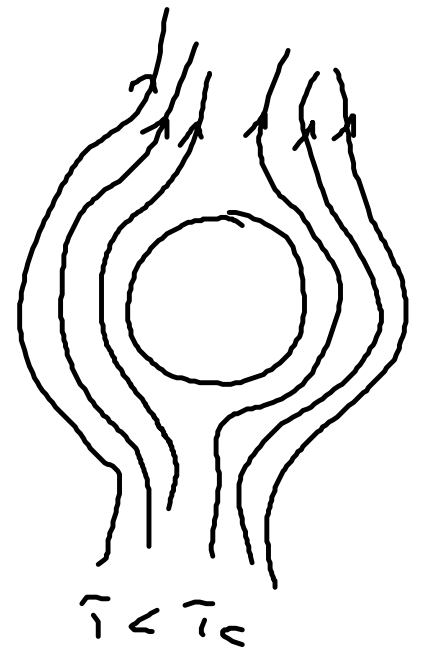
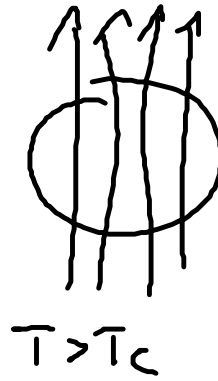
Pb 7.19 K

NB

9.26 K

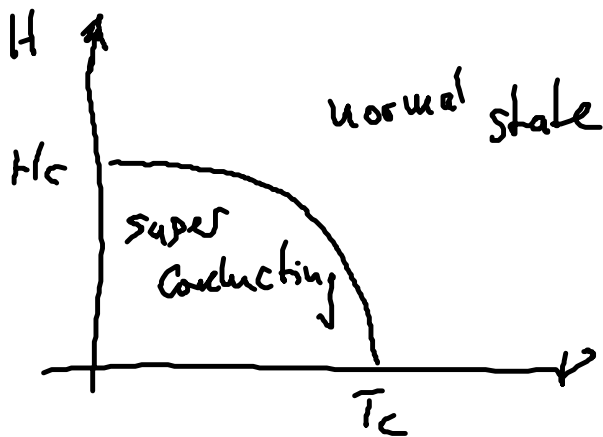
1933 Meissner and Ochsenfeld

- external field can't enter SC
- any original field in SC material is expelled in SC state



$$0 = \underline{B} = \mu_0 \mu \underline{H} = \mu_0 (1 + \chi) \underline{H}$$

$\Rightarrow \chi = -1$ very large diamagnetism



this odd, so let's consider a perfect conductor

n_s is fraction of superconducting electrons
(rest scatters normally)

\underline{E} applied elec field, accelerates electrons:

$$m \frac{d\underline{v}_s}{dt} = -e \underline{E}$$

current density: $\underline{j} = -e \underline{v}_s n_s$

$$\Rightarrow \frac{d\vec{j}}{dt} = \frac{n_s e}{m} \vec{E}$$

Substituting this into Faraday's law of induction:

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t} \left(\nabla \times \vec{j} + \frac{n_s e^2}{mc} \vec{B} \right) = 0$$

together with Maxwell's equation

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{j}$$

this determines \vec{B} and \vec{j} in superconductor.

- in particular: any change in \vec{B} field is immediately screened ✓

- however: there are static \vec{j} and \vec{B} that satisfy equations

↳ inconsistent with expulsion of magnetic fields

→ perfect conductivity not sufficient to explain Meissner effect.

F. and H. London proposed to restrict \vec{j} and \vec{B} of SC to those that satisfy:

$$\nabla \times \vec{j} + \frac{n_s e^2}{mc} \vec{B} = 0 \quad (\text{London equation})$$

↳ Maxwell's equation:

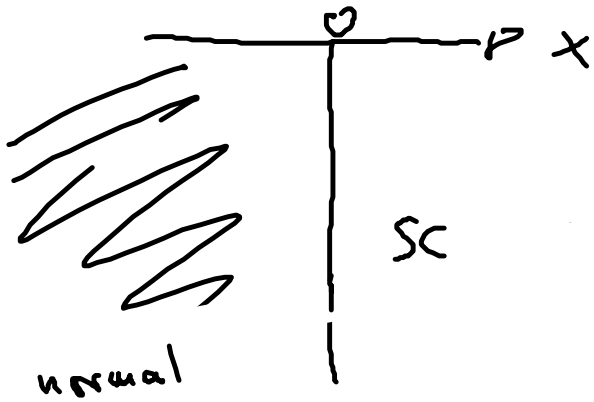
$$\nabla \times (\nabla \times \vec{B}) + \frac{4\pi n_s e^2}{mc^2} \vec{B} = 0$$

$$\nabla \cdot (\nabla \cdot \underline{B}) - \nabla^2 \underline{B}$$

Gauss law = 0

$$\nabla^2 \underline{B} = \frac{4\pi n_s e^2}{mc^2} \underline{B}$$

consider half infinite SC

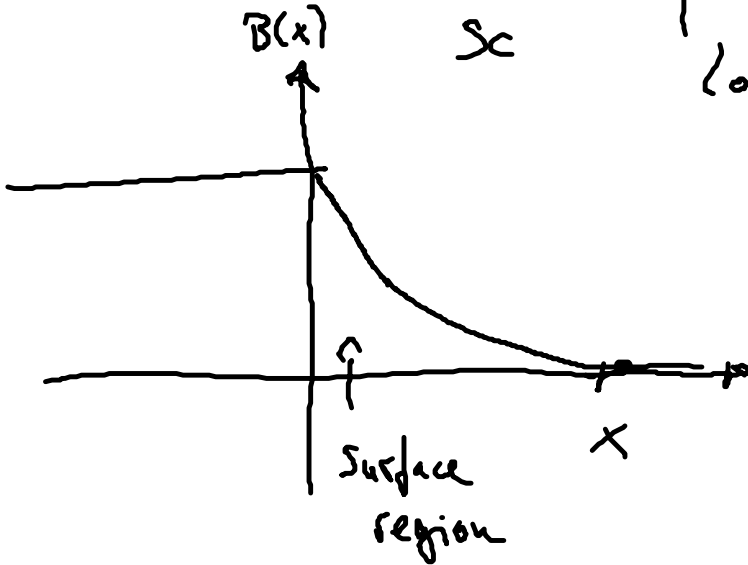


$$B(x) = B(0) e^{-\frac{x}{\lambda_L}}$$

$$\lambda_L = \sqrt{\frac{mc^2}{4\pi n_s e^2}} = 41.9 \left(\frac{r_s}{a_0}\right)^{3/2} \left(\frac{h}{m_s}\right)^{1/2} \text{ \AA}$$

$\approx 10^2 - 10^3 \text{ \AA}$ typically

↑ London penetration depth



- magnetic field only penetrates SC surface
- eddy currents at surface expel the field

Questions:

What creates phase transition of electronic structure to Superconducting state?

How can we create a microscopic theory (e.g. of London's equation) beyond standard electrodynamics?

9.2. London theory

- still phenomenological
- but quantum mechanical analog of London equation

Current density

$$\mathbf{j}(\underline{r}) = \frac{ie}{2m} (\psi^* \nabla \psi - \psi \nabla \psi^*) - \frac{e^2}{m} \underline{A} \psi^* \psi$$

Sum over all
electron states below
Fermi energy

with vector potential $\underline{A}(\underline{r})$ such that
 $\nabla \times \underline{A} = \underline{B}$

- for electrons this yields weak London diamagnetism
(see Chapter on magnetism)

\Rightarrow cannot explain Meissner effect

so let's assume bosons with charge e^* and mass m^*

- they satisfy the Schrödinger equation:

$$-\frac{1}{2m^*} \left(\nabla + ie^* \underline{A} \right)^2 \phi(\underline{r}) = E \phi(\underline{r})$$

- assume Maxwell gauge $\nabla \cdot \underline{A} = 0$ and expand $()^2$

$$\Rightarrow \left(-\frac{1}{2m^*} \nabla^2 - \frac{ie^*}{m^*} \underline{A} \cdot \nabla + \frac{e^{*2} A^2}{2m^*} \right) \phi(\underline{r}) = E \phi(\underline{r})$$

now we assume \underline{A} to be small: apply perturbation theory

$$\phi(\underline{r}) = \phi_0(\underline{r}) + \phi_e(\underline{r}) \quad \text{for ground state}$$

ground state
without field

comes from weak
perturbation A

↓ Bose condensate at low temperature

$$\phi_0(\underline{r}) = \frac{1}{\sqrt{V}} \quad , \quad \underline{k} = 0, \quad E_0 = 0$$

↪ insert ϕ into SE and keep terms to first order in A and ϕ_e

$$\underbrace{-\frac{1}{2m^*} \nabla^2 \phi_0}_0 - \frac{1}{2m^*} \nabla^2 \phi_e - \underbrace{\frac{ie^*}{m^*} \underline{A} \cdot \nabla \phi_0}_0 = \underbrace{E_0}_{0} \phi_e(\underline{r}) + \underbrace{E_1}_{0} \phi_0(\underline{r})$$

$E_1 \sim \nabla \cdot \underline{A}$ (because it has to be first order in A and a scalar)

⇒ $E_1 = 0$ vanishes in Maxwell gauge

⇒ $\phi_e(\underline{r}) = 0$ first order change vanishes (to first order in A)

$$\underline{j}(\underline{r}) = \frac{ie^*}{2m^*} \left(\psi^\dagger \nabla \psi - \psi \nabla \psi^\dagger \right) - \frac{e^*}{2m^*} \underline{A} \psi \psi^\dagger$$

$\psi = \phi_0$ " 0 # of SC carriers (from sum over states)

$$\underline{j}(\underline{r}) = -\frac{e^* e}{2m^*} \underbrace{\left(\frac{N_s}{V} \right)}_{n_s} \cdot \underline{A}$$

with $\nabla \times \underline{A} = \underline{B}$ and Maxwell's eq. : $\nabla \times \underline{B} = \frac{4\pi}{c} \underline{j}$

$$\Rightarrow \underline{B} + \lambda_L^2 \nabla \times \nabla \times \underline{B} = 0 \quad \text{London equation}$$

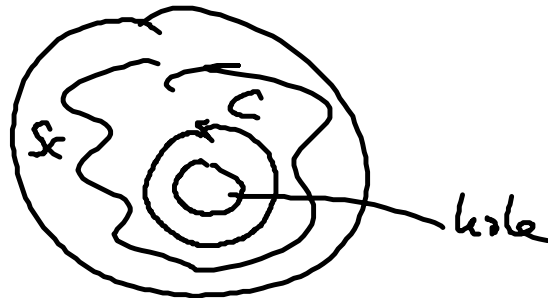
but this time derived quantum mechanically

and we know: Bosons are the carriers in
Superconductivity $\begin{matrix} \uparrow \uparrow \\ 0 \ 0 \end{matrix}$

Flux quantization

$$\phi(\underline{r}) = \sqrt{n_s} e^{i\varphi(\underline{r})}$$

↖ phase factor



Can we understand phase?

magnetic flux through hole:

$$\Phi_B = \int_{\text{surf}} d\underline{s} \cdot \underline{B} = \oint_C d\underline{\ell} \cdot \underline{A}(\underline{r})$$

$$\underline{B} = \nabla \times \underline{A} + \text{Stoke theorem}$$