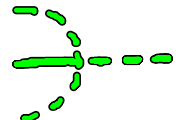
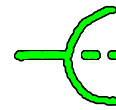
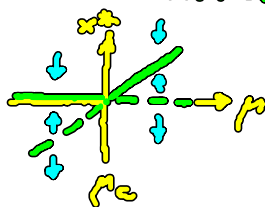
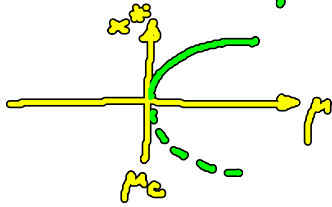


English Summary:

1.3 Bifurcations

(A) Zero-eigenvalue bifurcation: $\lambda < 0 \rightarrow \lambda > 0$
 $\det A > 0$ $\det A < 0$
 saddle-node bifurcation transcrit. bif. pitchfork bif.



supercrit.

subcrit.

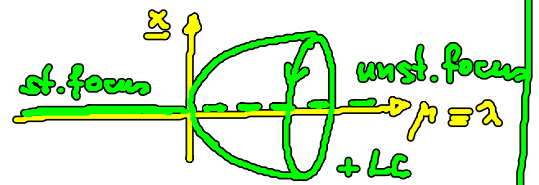
(B) Hopf bifurcation: $\lambda_{1,2} = \lambda_0 \pm i\omega$, $\lambda_0 < 0 \rightarrow \lambda_0 > 0$

Hopf normal form
 (Stuart-Landau eq.) $z = r e^{i\varphi} \in \mathbb{C}$

$$\dot{z} = (\lambda + i\omega \mp (1 + i\gamma)|z|^2)z$$

$$\dot{r} = (\lambda \mp r^2)r \quad \text{super/subcrit.}$$

$$\dot{\varphi} = (\omega \mp \gamma r^2)$$



Supercrit.

amplitude $r \rightarrow 0$
 frequency $\omega \neq 0$

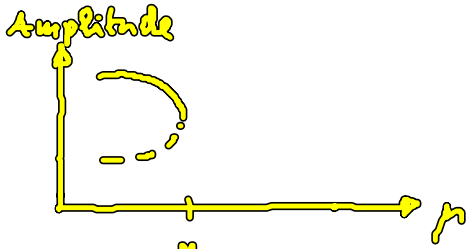
linear stab.:
 Floquet modes
 $Sz = e^{\lambda t} u(t)$
 Floquet multiplier $\rho = e^{\lambda T}$ ($u(t+T)$)

(C) Lokale Bifurkation von Grenzzyklen

Stabilit. getestet: Grenzzyklus, nicht Fixpt.
 keine einfache lin. Stabilitätsanalyse

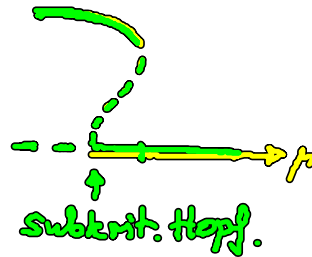
(C1) Sattel-Knoten-Bifurkation eines Grenzzyklus

(Kondensation von Pfaden, fold bifurcation of limit cycles LC)

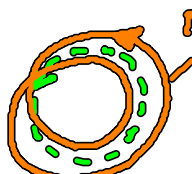


Amplitude $\neq 0$
Frequenz $\neq 0$

Bistabilität zwischen Fixpt. und LC, häufig:

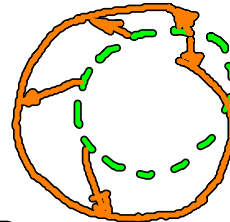


(C2) Perioden-Verdopplung (flip-Bifurkation, subharmon. Bif.)



Möbiusband

mind. 3D Phasentraum



phase flip π
nach 1 Umlauf

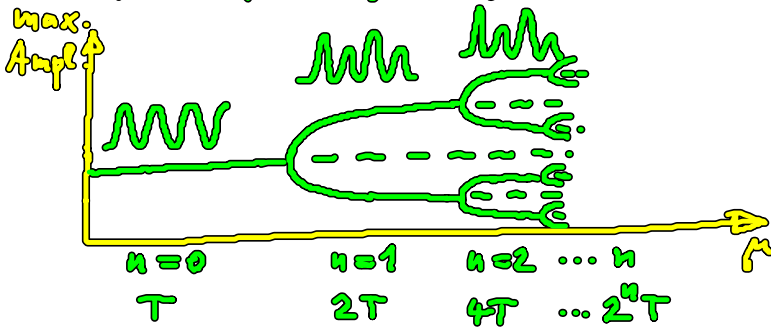
Torsion benachbarter Traj. !

Floquet-Exp. $\Lambda = \lambda + i\omega$

Bif. : $\lambda = 0, \omega T = \pi$

⇒ Floquet-Multiplikation

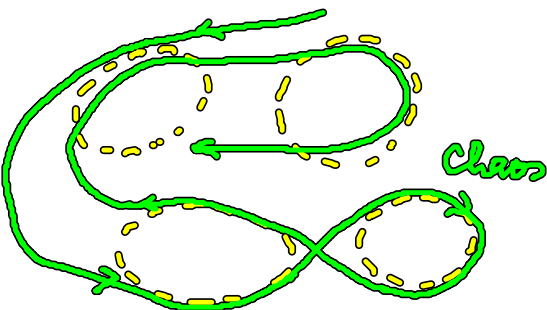
$\mu = e^{\Lambda T} = e^{i\pi} = -1$



häufig Periodenverdopplungskaskade

→ Chaos (Feigenbaum-Szenario)

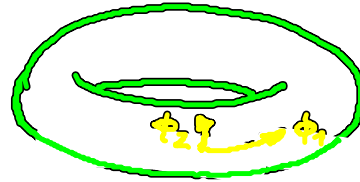
→ unendlich viele instabile periodische Orbits der Periode $2^n T$ ($n=0, 1, 2, \dots$)



(C3) Sacker-Neimark-Bifurkation
 (sekundäre Hopf-Bif. eines LC)



- 1. Hopf-Bif. des Fixpt. : $\phi_1 = \omega_1 t$
- 2. Hopf-Bif. des LC : $\phi_2 = \omega_2 t$



LC \longrightarrow 2-Torus

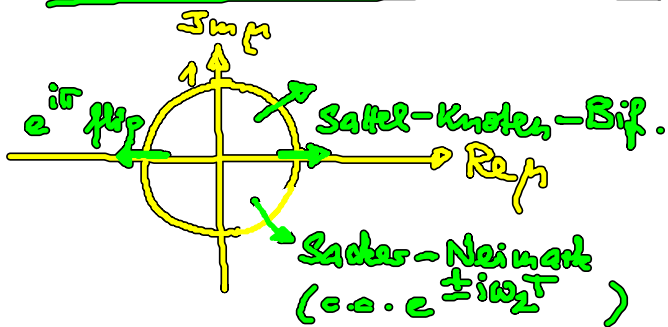
mind. 3D Phasenraum

inkommensurable Frequenzen
 (quasi-periodisch) $\frac{\omega_1}{\omega_2} \notin \mathbb{Q}$
 irrational

\rightarrow Trajektorie schließt sich nicht, liegt dicht auf dem Torus

falls $\frac{\omega_1}{\omega_2} \in \mathbb{Q} \Rightarrow$ geschlossener Orbit = LC
 (frequency locking)
 Modenkopplung

• Stabilität von Grenzzyklen : Floquet-Multiplikator



$\mu = e^{\lambda T}$
 Stab.grenze $|\mu| = 1$

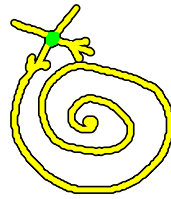
oder transkrit. Bif. eines LC

(D) Globale Bifurkation von Grenzzyklen

globale qualitative Änderung des Phasenporträts,
 z.B.



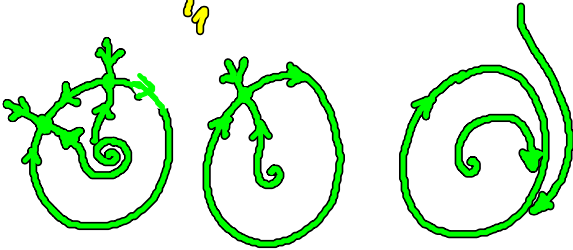
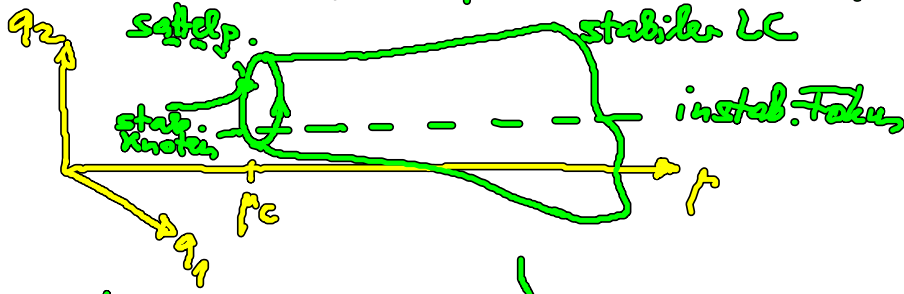
invar. Mannigfaltigk. des Sattelpnts.



(D1) Sattel-Knoten-Bif. auf einem Grenzzyklus (Omega explosion)

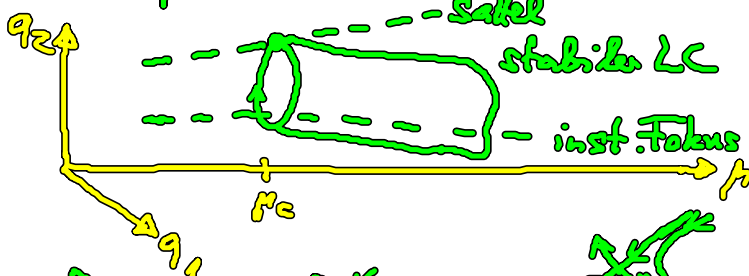
saddle-node on invariant cycle (SNIC)
 saddle-node infinite period (SNIPER)

Amplitude $\neq 0$
 Freq. $\rightarrow 0$

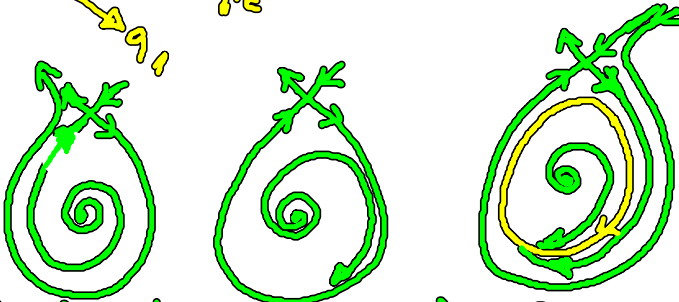


(D2) Homokline Bif. (blue-sky catastrophe)

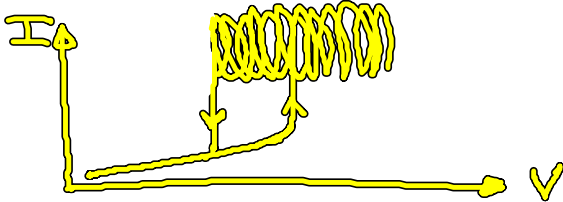
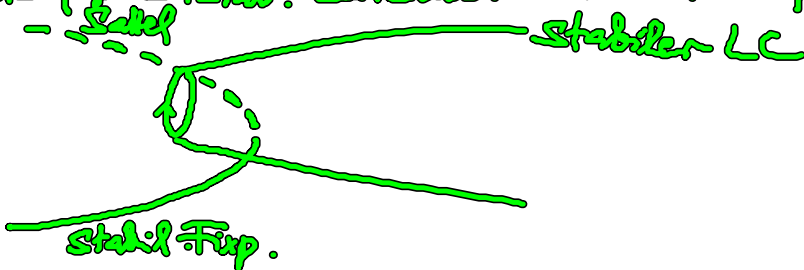
Sattelpkt. kollidiert mit LC: homokliner Orbit (saddle-to-saddle loop)



Ampl. $\rightarrow 0$
 Freq. $\rightarrow 0$



häufige Bistab. zwischen Osz. u. Fixpkt., z.B.:



z.B. nichtlinearer Halbleiter: Hypothese von Osz. u. stationärem Strom