

English Summary:

Stabilization of unstable fixed point

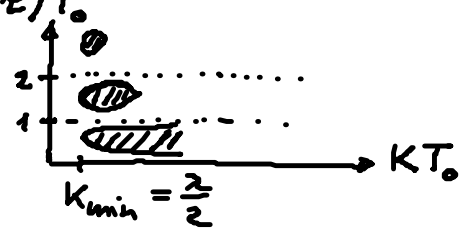
Stab. boundaries: $\operatorname{Re} \lambda = 0 \rightarrow z/T$

multiple feedback (ETDAS):

$$K \sum_{n=0}^{\infty} R^n [x(t-n\tau) - x(t-(n+1)\tau)]$$

$$(0 \leq R < 1)$$

$$\Rightarrow \lambda + K \frac{1 - e^{-\lambda\tau}}{1 - R e^{-\lambda\tau}} = \lambda + i\omega$$



Stabilisierung instab. period. orbits:

subkrit. Hopf-Bif.: $\dot{z} = (\lambda + i\omega + (1+i\gamma)|z|^2)z + b(z(t-\tau) - z(t))$

$$\lambda < 0, \omega = 1, \gamma < 0, b = b_0 e^{i\beta} \in \mathbb{C}$$

nichtinvasive Kontrolle:

$$\text{Wähle } \tau \stackrel{!}{=} nT = \frac{2\pi n}{1 - \gamma\lambda} \quad (T = \text{Periode des UPO})$$

(Pythagoras-Kurve in der (τ, λ) -Ebene)

Hopf-Kurve: lin. Stab. des Fixp. $z(t) \sim e^{z\tau}$

$$z + b(1 - e^{-z\tau}) = \lambda + i \quad \text{vgl. 3.2.1}$$

$$\text{Hopf-Bif.: } z = i\omega : 0 = \lambda + b_0 [\cos(\beta - \omega\tau) - \cos\beta] \quad (1)$$

$$\omega - 1 = b_0 [\sin(\beta - \omega\tau) - \sin\beta] \quad (2)$$

$$(1) \Rightarrow \omega\tau = \pm \arccos\left(\frac{b_0 \cos\beta - \lambda}{b_0}\right) + \beta + 2\pi n$$

$$\left(\frac{b_0 \cos\beta - \lambda}{b_0}\right)^2 = \cos^2(\beta - \omega\tau)$$

$$\left(\frac{\omega - 1 + b_0 \sin\beta}{b_0}\right)^2 = \sin^2(\beta - \omega\tau)$$

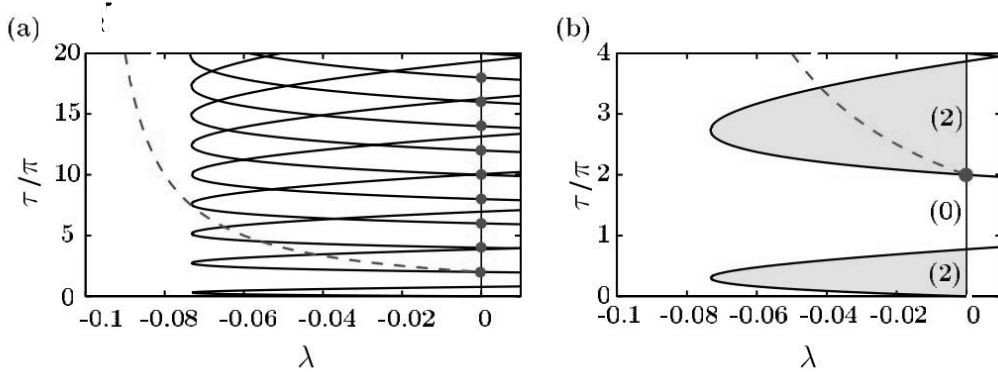
$$\left. \begin{array}{l} \cos^2 + \sin^2 = 1 \end{array} \right\} (3)$$

$$\stackrel{(3)}{\Rightarrow} \omega = g(\lambda, b_0, \beta) \stackrel{\text{elim.}}{\Rightarrow} \tau = h(\lambda, b_0, \beta)$$

Hopf-Kurven in (λ, τ) -Ebene

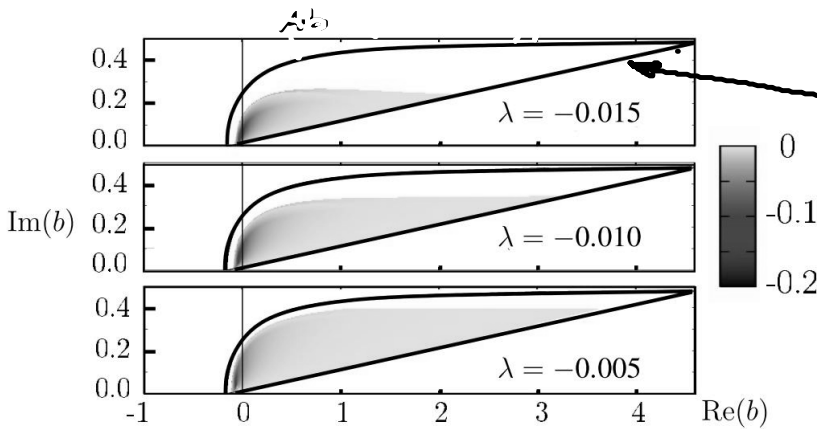
ohne Kontrolle ($b=0$): \sim subkrit. Hopf

mit Kontrolle $b = b_0 e^{i\beta}$ \sim superkrit. Hopf entlang der Pyragaskurve



Fiedler et al.
PRL 98, 114101
(2007)

FIG. 1 (color online). Pyragas (red dashed) and Hopf (black solid) curves in the (λ, τ) plane: (a) Hopf bifurcation curves $n = 0, \dots, 10$, (b) Hopf bifurcation curves $n = 0, 1$ in an enlarged scale. Yellow (light gray) shading marks the domains of unstable $z = 0$ and numbers in parentheses denote the dimension of the unstable manifold of $z = 0$ ($\gamma = -10$, $b_0 = 0.3$, and $\beta = \pi/4$).



analyt. Bed.
 $\tau_{\text{Hopf}}(\lambda) < \tau_{\text{Pyragas}}(\lambda)$

Koppl. phase $\beta \neq 0$!

FIG. 3 (color online). Domain of control in the plane of the complex feedback gain $b = b_0 e^{i\beta}$ for three different values of the bifurcation parameter λ . The black solid curves indicate the boundary of stability in the limit $\lambda \nearrow 0$; see (18) and (19). The color-shading shows the magnitude of the largest (negative) real part of the Floquet exponents of the periodic orbit ($\gamma = -10$, $\tau = \frac{2\pi}{1-\gamma\lambda}$).

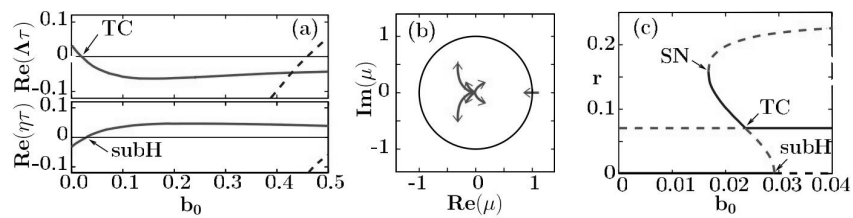


FIG. 2 (color online). (a) Top: Real part of Floquet exponents Λ of the periodic orbit vs feedback amplitude b_0 . Bottom: Real part of eigenvalue η of the steady state vs feedback amplitude b_0 . (b) Floquet multipliers $\mu = \exp(\Lambda\tau)$ in the complex plane with the feedback amplitude $b_0 \in [0, 0.3]$ as a parameter. (c) Radii of periodic orbits. Solid (dashed) lines correspond to stable (unstable) orbits. ($\lambda = -0.005$, $\gamma = -10$, $\tau = \frac{2\pi}{1-\gamma\lambda}$, $\beta = \pi/4$).

korrektes Odd-Number-Theorem: E.W. Hooton, A. Amann, PRL 103, 154101 (2012)

Analytical Limitations for time-delayed feedback control in Autonomous Systems

A. Amann, E.W. Hooton: Phil. Trans. R. Soc. A (2013)

Exp. Verification durch Laserexperiment (Schikora, Wünsche, Henneberger 2011)

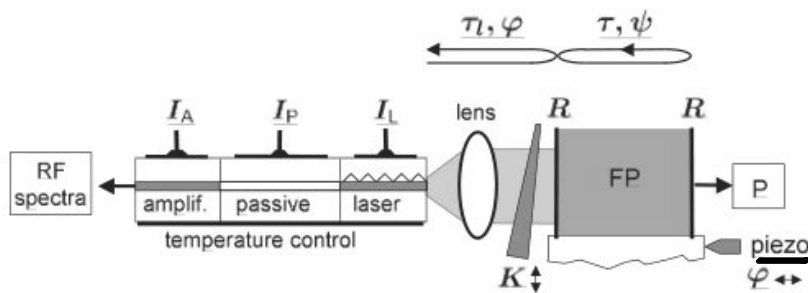


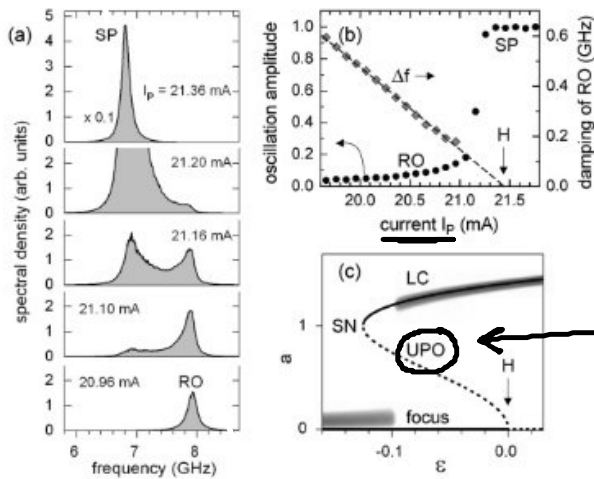
FIG. 1. All-optical control setup: multisection laser with optical feedback from a planar FP glass etalon. rf spectra: 50-GHz photo diode and electrical spectrum analyzer. P: power measurement by infrared photodiode. R : reflectivity of the FP mirrors. τ and ψ : round-trip time and corresponding optical phase shift in the FP, respectively. τ_l and φ : round-trip time and phase shift between laser and FP, respectively. K : control gain comprising all losses on the round trip between laser and FP (including the factor \sqrt{R} for a single reflection). K and φ are adjusted by a variable neutral density filter and a piezo positioning with high resolution (20 nm), respectively. Laser details: see text.

Multisektion Laser

(auch Stabilisierung des Exp. - cw möglich:

Schikora et al, PRL (2006))

ohne Kontrolle



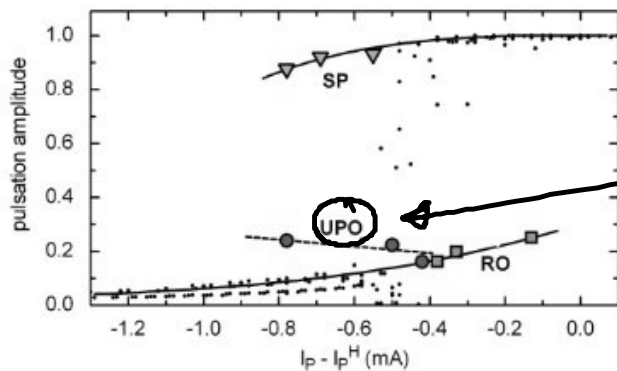
Ziel: Stabilisierung des UPO (Selbstpuls. mit kleiner Amplitude)

$\lambda \approx$

FIG. 2. (Color online) Subcritical Hopf bifurcation in the experimental device. (a) Power spectra of ROs and SPs for selected currents I_p . (b) Dots: amplitudes of ROs and SPs normalized to maximum versus I_p . Diamonds: damping of ROs; the linear extrapolation to zero defines the position of the Hopf bifurcation H . (I_L, I_A) = (87.1, 8.7) mA. (c) Corresponding scenario of a Duffing-van der Pol oscillator [31]. Lines: normalized amplitudes of deterministic oscillations and focus ($a = 0$) versus bifurcation parameter ϵ . UPO: unstable periodic orbit. SN: saddle-node bifurcation of orbits. LC: stable limit cycle. Gray (red) scale coded: normalized amplitude probability distribution (3) for noise level $D = 10^{-3}$. Probabilities below 1% are not shown.

[31] Sakharov et al., PRE 81, 011006 (2010)

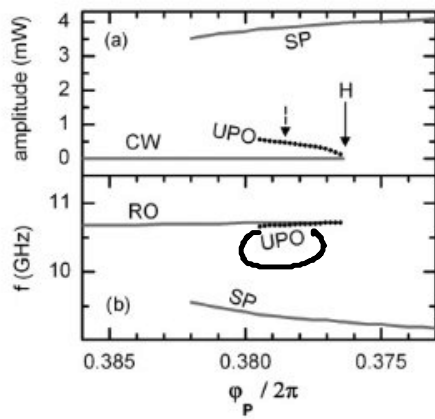
mit Kontrolle



$\approx \lambda$

FIG. 3. (Color online) Control experiments at subcritical Hopf bifurcation: normalized SP amplitude versus distance to bifurcation. Cyan triangles: SPs stabilized with $\tau = 145$ ps, Green squares: CW emission stabilized with $\tau = 59$ ps. In both cases $R = 0.5, K = 0.02$. Red circles: UPO stabilized with $\tau = 130$ ps, $R = 0.3, K = 0.06$. The uncertainty of K is always about 15%. Black dots: Data of the free-running device, which are collecting measurements for all operation points used in the control demonstrations. Lines: guide to the eye.

Stabilisierung des Odd-number orbits

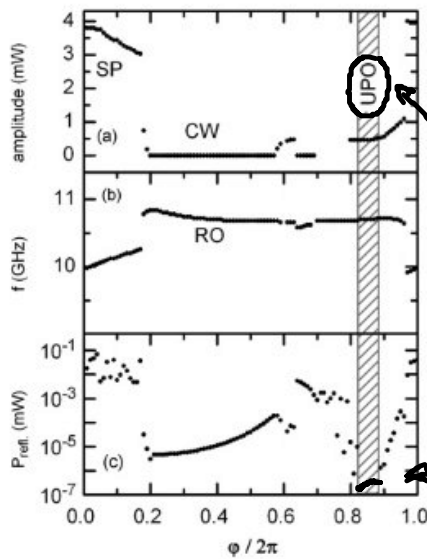


Kontroll-Szenario :

geeignete Wahl des Phasenstroms der passiven Sektion
 \rightarrow interne Phase φ_p

FIG. 5. Simulations of the control scenario, (a) Pulsation amplitude and (b) frequency versus internal phase shift in the AFL. Solid gray: stable states of the free-running laser [RO in (b): damped relaxation oscillations toward CW], black dotted: UPO stabilized with $\varphi/2\pi = 0.84, R = 0.3, K = 0.06, \tau = 0$.

ODD-NUMBER THEOREM: OPTICAL FEEDBACK CONTROL ...



Stabilisierung nur möglich für endl. Kontrollphase
 $\beta \equiv \varphi$

nicht-invasive Kontrolle des Odd-number UPO

$\cong \beta$

FIG. 6. Simulations of phase tuning, (a) Pulsation amplitude and (b) frequency versus control phase φ at $\varphi_p/2\pi = 0.3785$ for otherwise identical parameters as in Fig. 5. (c) Feedback power reinjected into the laser when stopping iterations. Shaded area: range for noninvasive control of UPO. Feedback power is here below 10⁻⁵ of laser output.

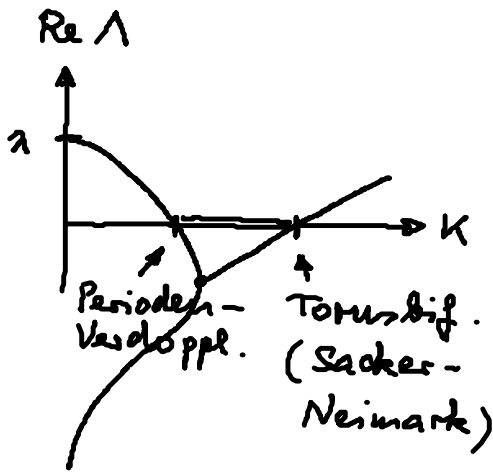
3.2.3 Chaos-Kontrolle durch zeitverzögerte Rückkopplung

Pyrnagas, Phys. Lett. A 170, 421 (1992)

$$\dot{x} = f(x) + KA [x(t-\tau) - x(t)] \quad x \in \mathbb{R}^n, \quad n \times n \text{ Kopplungsmatrix } A$$

$x = f(x)$: chaot. Attraktor mit unendl. vielen UPOs

→ s. Kap. 2



unkontroll. Floquet-Problem

$$\delta x = e^{\Lambda t} u(t) = u(t+T)$$

$$(\lambda + i\omega)u + \dot{u} = Df u$$

kontroll. (diagonale Kontrolle)

$$\Lambda u + \dot{u} = Df u + K (e^{-\Lambda T} - 1) u$$

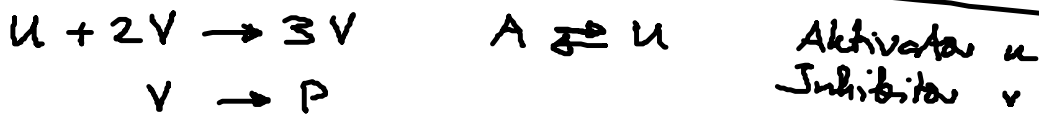
führt $\Rightarrow \Lambda + K(1 - e^{-\Lambda T}) = \lambda + i\omega$

Schöll u. Schuster (eds.): Handbook of Chaos Control (2008)

3.2.4 Kontrolle räum-zeitlicher Systeme

Kuznetsov, Ryzhov, Hogan, Schöll: Chaos 13, 043126 (2009)

Gray-Scott-Modell (Reaktions-Diff.-System)



$$\begin{pmatrix} \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} -uv^2 + a(1-u) & + D_u \nabla_u^2 \\ uv^2 - (a+b)v & + D_v \nabla_v^2 \end{pmatrix} + KA \begin{pmatrix} u(t-\tau) - u(t) \\ v(t-\tau) - v(t) \end{pmatrix}$$

bis zu 3 räumlich-homogene Fixpunkte. $E_0 = (1, 0)$ immer stabil
 $E_{1,2}$ nichttrivial

Aktivator-Kontrolle $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Inhibitor-Kontrolle $A = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$