

7.3 Chimera states



Chimera on a red-figure plate 350-340 BC

Schimäre = Fabelwesen

In Greek mythology chimera is a fire-breathing monster, hybrid of a lion, a goat and a snake.

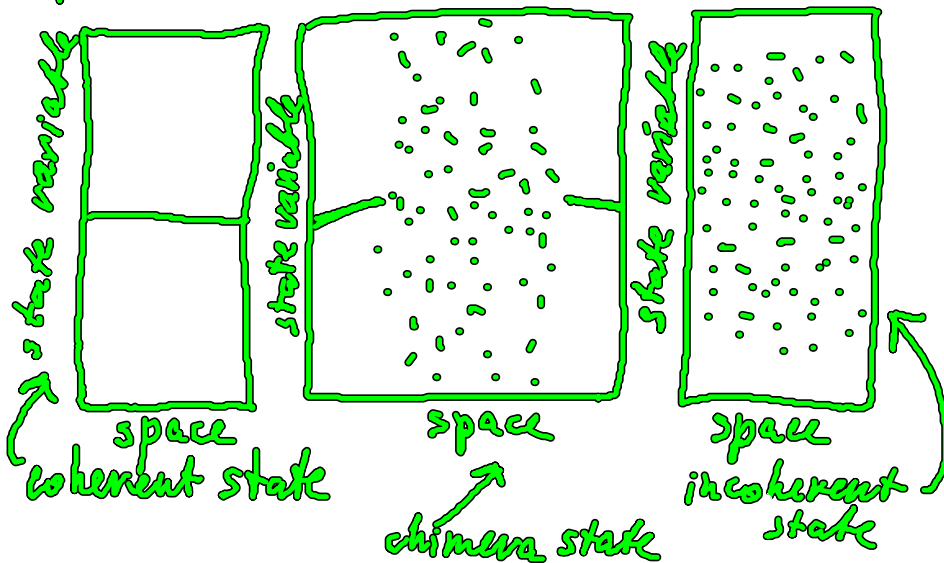
Nonlinear dynamics: chimera is a mathematical hybrid, a state of mixed synchronous (coherent) and asynchronous (incoherent) behaviour in a network of identically coupled oscillators.

Networks: power grids, neural network, social network

Chimera state is characterized by spatial coexistence of coherent and incoherent domains in a dynamical network.

Before the discovery of chimera states:

- it was believed that the dynamics of networks of identical elements was not interesting: identical oscillators were expected to either synchronize in phase or drift incoherently.



- coupled non-identical oscillators were known to exhibit complex phenomena: frequency locking, phase synchronization, partial synchronization and incoherence.

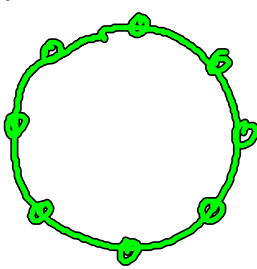
2002 Kuramoto and Battogtokh discovered chimera states

For a ring of identical and nonlocally coupled phase oscillators they found that for certain initial conditions, oscillators that were identically coupled to their neighbours and had identical natural frequencies could behave differently from each other; some of them could synchronize while others remained incoherent.

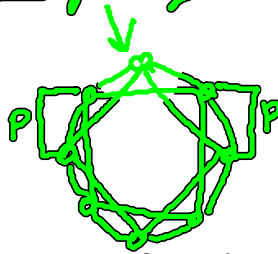
! This was not a transient state, but a stable persistent phenomenon.

2004 Abrams and Strogatz → named chimera states by Steve Strogatz

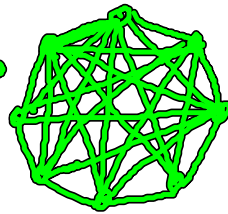
Non local coupling



local



nonlocal
P=2



global coupling

P - number of nearest neighbours in each direction on a ring

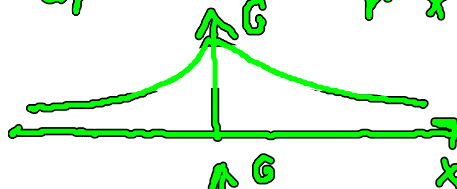
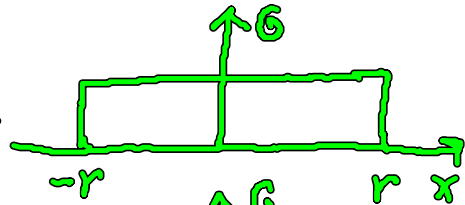
$$r = \frac{P}{N} - \text{coupling range}$$

Coupling function G

$$(i) G(x) = \begin{cases} \frac{1}{2r} & \text{if } |x| < r \\ 0 & \text{otherwise} \end{cases}$$

$$(ii) G_K(x) = \frac{K e^{-K|x|}}{2(1-e^{-K})}$$

$$(iii) G_A(x) = \frac{1 + A \cos \frac{x}{2}}{2\pi}$$



Phase oscillator network

$$\dot{\varphi}(x, t) = \omega - \int_{-\pi}^{\pi} G(x-x') \sin[\varphi(x', t) - \varphi(x, t) + \alpha] dx'$$

x: position on an interval [0; 2π];

φ(x, t): the phase of the oscillator at position x and time t.

Questions:

1) Are chimera states stable?
2) Do they exist in higher dimensional systems?

3) Are they robust to noise and heterogeneities?

↓ natural frequencies ↓ coupling topology

4) Are they robust enough to be observable in experiments?

5) Are more complex patterns possible?

- In many systems chimeras coexist with a stable fully synchronized state - this is why it was not found for a long time (it remained unknown).

x 6) What are the necessary conditions for a chimera state to exist?

- For certain systems, though they are stable as the number of oscillators $N \rightarrow \infty$, chimera states are very long lived transients for finite N .

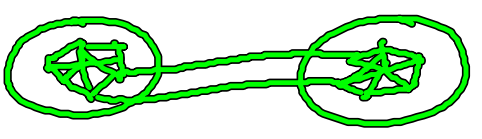
- Although the basins of attraction for chimera states are typically smaller than that of the fully synchronized state, chimeras are robust to many different types of perturbations.

2012 Experiments (first!)

Chimeras in a lab (Nature Physics):

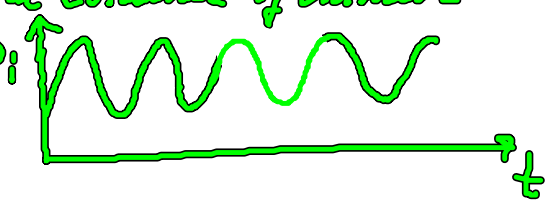
- Chimeras in a population of coupled chemical oscillators (K. Showalter, West Virginia University, USA)
- Chimeras in an optical experiment (R. Roy, University of Maryland, USA - in collaboration with F. Schöll)
- Chimeras in a mechanical oscillator system. TU Berlin (E. Martens, MPI-DS Göttingen, Germany, 2013)
- Chimeras in an electronic experiment (L. Larger, France, 2013)
- Chimeras in a photoelectrochemical experiment (K. Krischer, TU München, Germany, 2014)

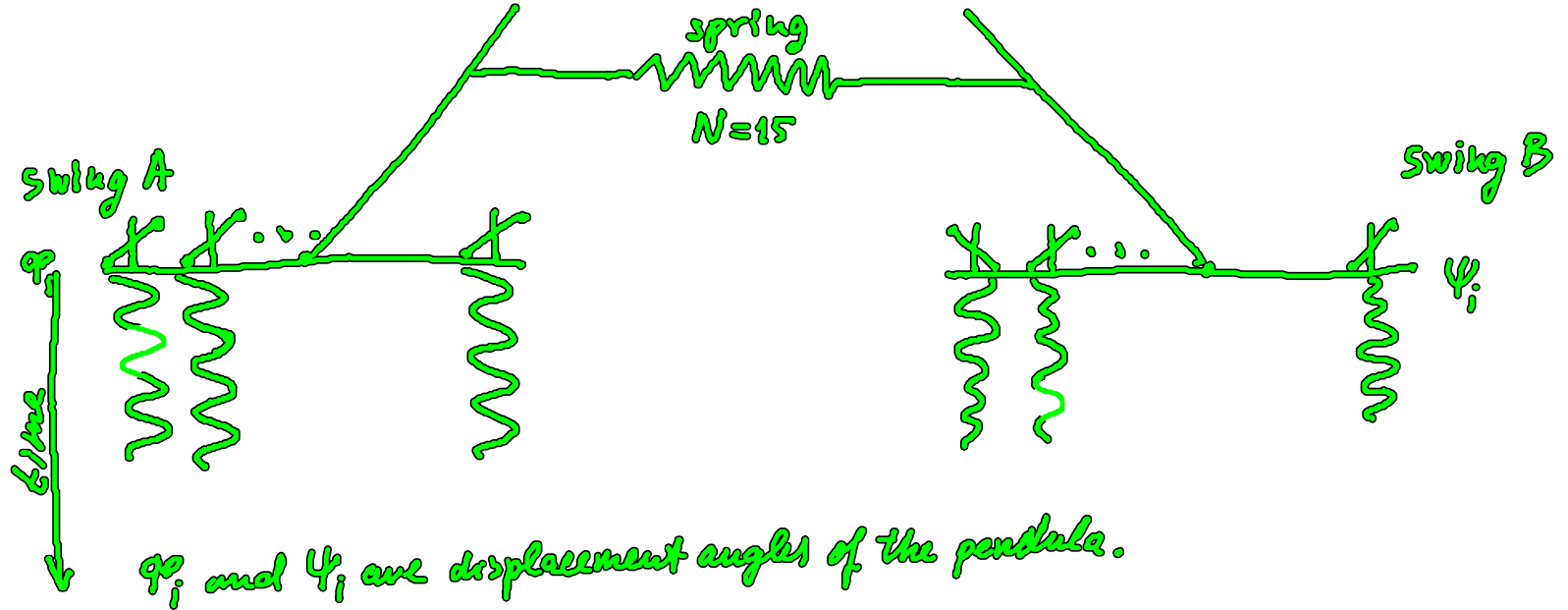
Two-population network



Physical experimental evidence of chimera states

Metronome





ϕ_i and ψ_i are displacement angles of the pendula.

If the spring connecting the swings was taut (high tension) the metronomes on both swings moved with in-phase synchrony. If the spring was loose (low tension), antiphase movement developed so that metronomes on one swing moved left as the others moved right, yet always in time. A chimera appeared when the spring's tension was in between.