$$\mathcal{L}(\psi, \psi^*, \partial_\mu \psi, \partial_\mu \psi^*) = \frac{\hbar^2}{2m}\left((\partial_0 \psi^*)(\partial_0 \psi) - (\partial_x \psi^*)(\partial_x \psi) - \lambda^{-2} \psi^* \psi\right)$$

$$0 = \frac{\partial \mathcal{L}}{\partial \psi^*} - \overset{\text{"}\sum\text{"}}{\partial_\mu} \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \psi^*)} \right)$$

$$= \frac{\partial \mathcal{L}}{\partial \psi^*} - \partial_0 \left(\frac{\partial \mathcal{L}}{\partial(\partial_0 \psi^*)} \right) - \partial_x \left(\frac{\partial \mathcal{L}}{\partial(\partial_x \psi^*)} \right) - \left[\partial_y \underbrace{\left(\frac{\partial \mathcal{L}}{\partial(\partial_y \psi^*)} \right)}_{=0} - \partial_z \underbrace{\left(\frac{\partial \mathcal{L}}{\partial(\partial_z \psi^*)} \right)}_{=0} \right]$$

$$\frac{\partial \mathcal{L}}{\partial \psi^*} = -\frac{\hbar^2}{2m} \lambda^{-2} \psi$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_0 \psi^*)} = \frac{\hbar^2}{2m} \partial_0 \psi$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_x \psi^*)} = -\frac{\hbar^2}{2m} \partial_x \psi$$

$$0 = \left(-\frac{\hbar^2}{2m} \lambda^{-2} - \frac{\hbar^2}{2m} \partial_0^2 + \frac{\hbar^2}{2m} \partial_x^2\right) \psi$$

$$\Rightarrow 0 = \left(\partial_x^2 - \frac{1}{c^2} \partial_t^2 - \lambda^{-2}\right) \psi$$

Das ist die Klein-Gordon-Gl. (1d), sie beschreibt Spin-0 Teilchen.

$\boxed{A2}$ Pauli-Gleichung

$$H = \left(H_0 - \frac{q}{2m} \underbrace{(\vec{r} \times \vec{p})}_{\vec{l}} \cdot \vec{B}\right) \mathbb{1} - \frac{q}{m} \hat{\vec{S}} \cdot \vec{B}$$

$$\vec{B} = B \vec{e}_z \quad \Rightarrow \quad \vec{l} \cdot \vec{B} = B\, l_z$$

$$\& \quad \vec{S} \cdot \vec{B} = B\, S_z$$

$$H|n,m_l,m_s\rangle = \left[\left(H_0 - \frac{q}{2m}BL_z\right)\mathbb{1} - \frac{q}{m}BS_z\right]|n,m_l,m_s\rangle$$

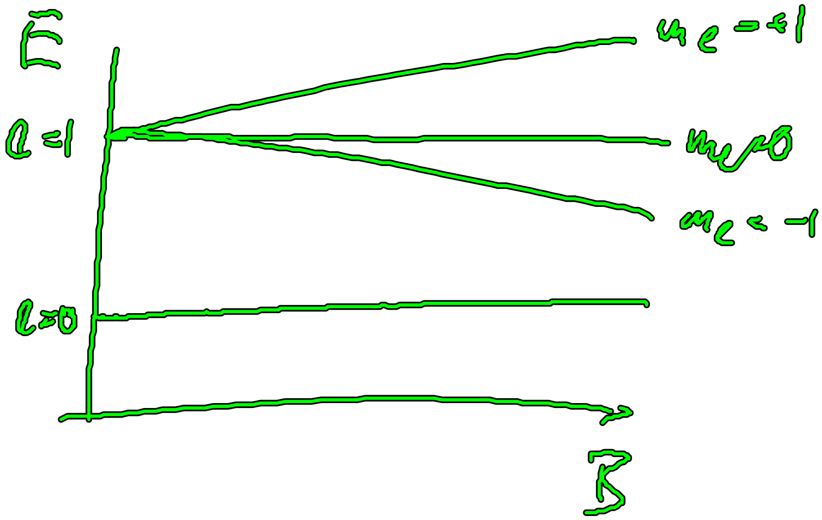
$$= \left[-\frac{E_0}{n^2} - \frac{q}{2m}B\hbar m_l - \frac{q}{m}B\hbar m_s\right]|n,m_l,m_s\rangle$$

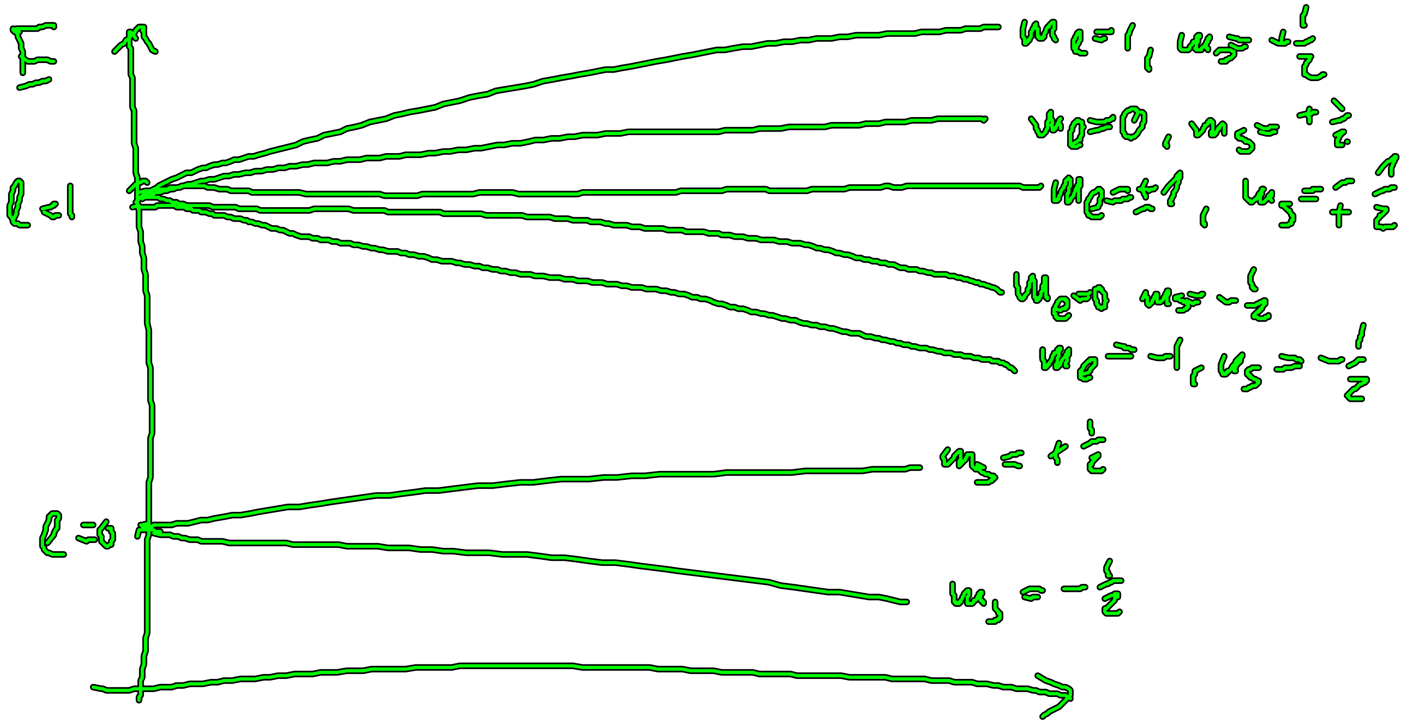
$$\Rightarrow E = -\frac{E_0}{n^2} - \frac{qB\hbar}{2m}(m_l + 2m_s)$$

$$= -\frac{E_0}{n^2} + \frac{eB\hbar}{2m}(m_l + 2m_s)$$

$q = -e$

$S = 0$, ohne Spin





→ Energieaufspaltung nach Spin & Bahndrehimpuls ($\vec{B}$)

$\boxed{A3}$ gestörter harmonischer Oszillator

$$b = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,x + i\,\frac{1}{\sqrt{m\hbar\omega}}\,p_x\right)$$

$$\Rightarrow x = \sqrt{\frac{\hbar}{2m\omega}}\,(b^+ + b)$$

(i) $\Rightarrow V = \frac{\hbar\omega}{\sqrt{2}}(b^+ + b)$

$$\langle m|V|n\rangle = \frac{\hbar\omega}{\sqrt{2}}\left(\sqrt{n+1}\,\delta_{m,n+1} + \sqrt{n}\,\delta_{m,n-1}\right)$$

(ii)
$$|\psi_n\rangle \approx |\varphi_n\rangle^0 + |\varphi_n\rangle^{(1)}$$
$$= |\varphi_n\rangle^0 + \sum_{m\neq n} \frac{\langle m|\lambda V|n\rangle}{E_n^0 - E_m^0}\,|m\rangle^0$$
$$= |\varphi_n\rangle^0 - \lambda\sqrt{\frac{n+1}{2}}\,|n+1\rangle + \lambda\sqrt{\frac{n}{2}}\,|n-1\rangle$$

(iii)
$$E_n \approx E_n^0 + E_n^1 + E_n^{(2)}$$
$$= \hbar\omega\left(n+\tfrac{1}{2}\right) + \underbrace{\langle n|\lambda V|n\rangle}_{=0} + \sum_{m\neq n}\frac{|\langle m|\lambda V|n\rangle|^2}{E_n^0 - E_m^0}$$
$$= \hbar\omega\left(n + \frac{1}{2} - \frac{\lambda^2}{2}\right)$$

$\boxed{A4}$ Zwei-Niveau-System

$$H = \sum_i \hbar\nu_i\, a_i^+ a_i + \sum_{ij} \hbar\Omega_{ij}\, a_i^+ a_j$$

$$= \hbar\nu_1 a_1^+ a_1 + \hbar\nu_2 a_2^+ a_2 + \hbar\Omega_{12}\, a_1^+ a_2 + \hbar\Omega_{21}\, a_2^+ a_1$$

(i) $[a_i, a_j^+]_+ = \delta_{ij}$

$\Big\ulcorner\; a_i a_j^+ + a_j^+ a_i = \delta_{ij} \Leftrightarrow a_i a_j^+ = \delta_{ij} - a_j^+ a_i$

$[a_i^{(+)}, a_j^{(+)}]_+ = 0 \Leftrightarrow a_i^{(+)} a_j^{(+)} = - a_j^{(+)} a_i^{(+)} \;\Big\lrcorner$

(ii) $[a_1^+ a_2, H]_- = \hbar\nu_1 [a_1^+ a_2, a_1^+ a_1] + \hbar\nu_2 [a_1^+ a_2, a_2^+ a_2] + \hbar\Omega_{12} [a_1^+ a_2, a_1^+ a_2] + \hbar\Omega_{21} [a_1^+ a_2, a_2^+ a_1]$

$-\hbar\nu_1 a_1^+ a_2 \qquad +\hbar\nu_2 a_1^+ a_2 \qquad 0 \qquad \hbar\Omega_{21}(a_2^+ a_2 - a_1^+ a_1)$

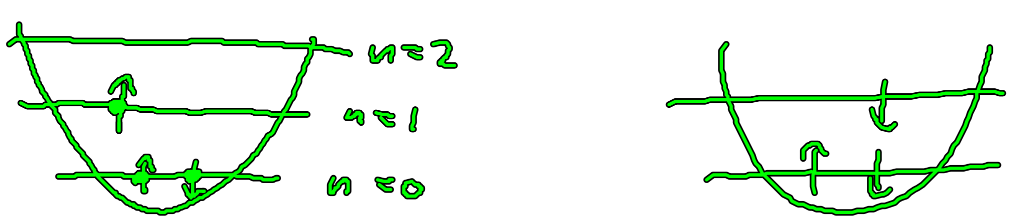
allg. $[a_1^+ a_2, a_i^+ a_j]_- = a_1^+ a_2 a_i^+ a_j - a_i^+ a_j a_1^+ a_2$

$$= \delta_{2i} a_1^+ a_j - a_1^+ a_i^+ a_2 a_j - \delta_{j1} a_i^+ a_2 + \underbrace{a_i^+ a_1^+ a_j a_2}_{a_1^+ a_i^+ a_2 a_j}$$

$$\langle [a_1^+ a_2, H] \rangle = \hbar(\nu_2 - \nu_1)\underbrace{\langle a_1^+ a_2 \rangle}_{p} + \hbar\Omega_{21}(\underbrace{\langle a_1^+ a_1 \rangle}_{1-f} - \underbrace{\langle a_2^+ a_2 \rangle}_{f})$$

$$\dot p = -\frac{i}{\hbar}\langle [a_1^+ a_2, H] \rangle = -i(\nu_2 - \nu_1)p - i\Omega(1-2f)$$

$\boxed{A5}$ Drei-Teilchen-Zustand



(i) $E_g = 2E_0 + E_1 = \frac{5}{2}\hbar\omega$

(ii) 2-fach, da $n=1$ entweder Spin $\uparrow$ o. $\downarrow$

(iii)

$$\Psi_g = \frac{1}{\sqrt{3!}} \begin{vmatrix} \varphi_{\alpha_1}(r_1) & \varphi_{\alpha_2}(r_1) & \varphi_{\alpha_3}(r_1) \\ \varphi_{\alpha_1}(r_2) & \varphi_{\alpha_2}(r_2) & \varphi_{\alpha_3}(r_2) \\ \varphi_{\alpha_1}(r_3) & \varphi_{\alpha_2}(r_3) & \varphi_{\alpha_3}(r_3) \end{vmatrix}$$

$$= \frac{1}{\sqrt{6}} \Big(\varphi_{\alpha_1}(r_1)\varphi_{\alpha_2}(r_2)\varphi_{\alpha_3}(r_3)$$
$$+ \varphi_{\alpha_2}(r_1)\varphi_{\alpha_3}(r_2)\varphi_{\alpha_1}(r_3)$$
$$+ \varphi_{\alpha_3}(r_1)\varphi_{\alpha_1}(r_2)\varphi_{\alpha_2}(r_3)$$
$$- \varphi_{\alpha_3}(r_1)\varphi_{\alpha_2}(r_2)\varphi_{\alpha_1}(r_3)$$
$$- \varphi_{\alpha_2}(r_1)\varphi_{\alpha_1}(r_2)\varphi_{\alpha_3}(r_3)$$
$$- \varphi_{\alpha_1}(r_1)\varphi_{\alpha_3}(r_2)\varphi_{\alpha_2}(r_3) \Big)$$

dabei $(\alpha_1, \alpha_2, \alpha_3) = (0\uparrow, 0\downarrow, 1\uparrow)$

oder $(\alpha_1, \alpha_2, \alpha_3) = (0\uparrow, 0\downarrow, 1\downarrow)$