

1.2.07

$$\langle \delta\sigma_i \delta\sigma_j \rangle, \quad \delta\sigma_i = \sigma_i - \langle \sigma_i \rangle$$

$$\langle \delta\sigma_i \delta\sigma_j \rangle = (1 - m_i^2)(1 - m_j^2) \langle \varphi_i \varphi_j \rangle$$

$$\langle \varphi_i \varphi_j \rangle = \frac{\int \mathcal{D}[\varphi] \varphi_i \varphi_j e^{-S''[\varphi]}}{\int \mathcal{D}[\varphi] e^{-S''[\varphi]}}$$

Vergleich:

$$\langle x^2 \rangle = \frac{\int_{-\infty}^{\infty} dx x^2 p(x)}{\int_{-\infty}^{\infty} dx p(x)}$$

$$= \frac{\int_{-\infty}^{\infty} dx x^2 e^{-\frac{x^2}{2\sigma^2}}}{\int_{-\infty}^{\infty} dx e^{-\frac{x^2}{2\sigma^2}}} = \sigma^2$$

Hier ist $S''[\varphi] = \frac{1}{2} \sum_{ij} \varphi_i A_{ij} \varphi_j$

$$A_{ij} = \frac{1}{\beta} \mathcal{F}_{ij}^{-1} - (1 - m_i^2) \delta_{ij}$$

Fourierzerlegung auf Gittern

Beispiel: 1d Gitter mit Gitterkonstante a

φ_0 φ_1 φ_2 \dots
 \bullet \bullet \bullet \bullet
 0 1 2 \dots

$\underbrace{\quad\quad\quad}_a$

$$\frac{a}{2\pi} \int_{-\pi/a}^{\pi/a} dk e^{ikna} = \delta_{n,0}$$

\Rightarrow Fourierreiheung linear auf dem reziproken Gitter periodischen Funktionen

$$\varphi(k) = \varphi\left(k + \frac{2\pi m}{a}\right)$$

im k -Raum

$$\overset{(\omega)}{\varphi}(k) = \sum_m \varphi_m e^{-ikma} \quad k\text{-Raum}$$

$$\varphi_n = \frac{a}{2\pi} \int_{-\pi/a}^{\pi/a} dk e^{ikna} \overset{(\omega)}{\varphi}(k)$$

$$\left(\begin{aligned} &= \frac{a}{2\pi} \int_{-\pi/a}^{\pi/a} dk e^{ikna} \sum_m \varphi_m e^{-ikma} \\ &= \varphi_n \end{aligned} \right) \quad \hookrightarrow \delta_{m,n}$$

$$\varphi(k) = \sum_m \underbrace{\varphi_m}_{\substack{\text{Entw. Koeffizienten} \\ \uparrow}} e^{-ikma} \quad \uparrow \text{ Basis-fkt.}$$

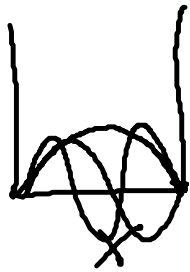
$$= \frac{a}{2\pi} \int_{-\pi/a}^{\pi/a} dk' \underbrace{\sum_m e^{-ikma} e^{ik'ma}}_{\dots\dots\dots} \varphi(k')$$

$$\sum_m e^{i(k'-k)ma} = \frac{2\pi}{a} \delta(k-k')$$

Notation einführen: $\frac{a}{2\pi} \int_{-\pi/a}^{\pi/a} dk \dots \equiv \int_{\text{BZ}}$

"Integral über die erste Brillouin-Zone"

Γ Vergleich:



Teilen mit Kästen

$$\psi(x) = \sum_k c_k e^{ikx}$$

↑ Ortsraum

↑ k-Raum

L

Jetzt entsprechend für d-dimensionalen Fall
 $\{R_i\}$, Funktionen $\varphi_i = \varphi(R_i)$

$$F_{ij} = F(\underline{R}_i, \underline{R}_j)$$

Vektoren \underline{R}_i
bzw Matrizen F_{ij} , $i = 1, 2, \dots$

$$\varphi_i = \int_{\underline{k}} e^{i\underline{k}\underline{R}_i} \varphi(\underline{k})$$

$$\Rightarrow F_{ij} = \int_{\underline{k}, \underline{k}'} e^{i\underline{k}\underline{R}_i} e^{i\underline{k}'\underline{R}_j} F(\underline{k}, \underline{k}')$$

häufig $F_{ij} = F(\underline{R}_i, \underline{R}_j) = \dots F(\underline{R}_i - \underline{R}_j)$

$$\Rightarrow F(\underline{k}, \underline{k}') = \underbrace{(2\pi)^d}_{a^d} \delta(\underline{k} + \underline{k}') F(\underline{k})$$

Volumen der 1. BZ

Damit die Inverse berechnen:

$$\int_{\underline{k}} e^{i\underline{k}(\underline{R}_i - \underline{R}_j)} = \delta_{ij} = \sum_j F_{ij} \underbrace{F^{-1}_{jl}}$$

$$= \sum_j \int_{\underline{k}} \int_{\underline{k}'} \int_{\underline{k}''} F(\underline{k}) e^{i\underline{k}(\underline{R}_i - \underline{R}_j)} \underbrace{F^{-1}(\underline{k}', \underline{k}'')}_{\uparrow} e^{i\underline{k}'\underline{R}_j} e^{i\underline{k}''\underline{R}_i}$$

$$= \int \int_{\underline{k} \underline{k}''} \mathcal{F}(\underline{k}) e^{i\underline{k} \underline{R}_i + i\underline{k}'' \underline{R}_e} \frac{(2\pi)^d}{a^d} \delta(\underline{k} - \underline{k}') \mathcal{F}^{-1}(\underline{k}, \underline{k}'')$$

$$\| \mathcal{F}^{-1}(\underline{k}, \underline{k}'') = \left(\frac{2\pi}{a} \right)^d \delta(\underline{k} + \underline{k}'') \frac{1}{\mathcal{F}(\underline{k})} \|$$

$$S''[\varphi] = \frac{1}{2} \sum_{ij} \varphi_i A_{ij} \varphi_j =$$

$$= \frac{1}{2} \sum_{ij} \int_{\underline{k} \underline{k}'} e^{i\underline{k} \underline{R}_i + i\underline{k}' \underline{R}_j} \left[\frac{1}{\beta} \mathcal{F}_{ij}^{-1} - (1 - m_i^2) \delta_{ij} \right] \varphi(\underline{k}) \varphi(\underline{k}')$$

$$= \frac{1}{2} \int_{\underline{k} \underline{k}'} \left[\frac{1}{\beta} \frac{1}{\mathcal{F}(-\underline{k})} \frac{(2\pi)^d}{a^d} \delta(\underline{k} + \underline{k}') \varphi(\underline{k}) \varphi(\underline{k}') - \sum_i (1 - m_i^2) e^{i(\underline{k} + \underline{k}') \underline{R}_i} \varphi(\underline{k}) \varphi(\underline{k}') \right]$$

↑ Annahme: $m_i = m$

$$\frac{(2\pi)^d}{a^d} \delta(\underline{k} + \underline{k}')$$

$$= \frac{1}{2} \int_{\underline{k}} \left[\frac{1}{\beta \mathcal{F}(-\underline{k})} - (1 - m^2) \right] \varphi(\underline{k}) \varphi(-\underline{k})$$

Wir wollen

$$\langle \varphi_i \varphi_j \rangle = \int_{kk'} e^{ikR_i} e^{ik'R_j} \underbrace{\langle \varphi(k) \varphi(k') \rangle}$$

Wegen Translationsinvarianz

$$\left(\frac{2\pi}{a}\right)^d \delta(k+k')$$

$$\langle \varphi(k) \varphi(-k) \rangle = \frac{\int \mathcal{D}[\varphi] \varphi(k) \varphi(-k) \exp[-S'']}{\int \mathcal{D}[\varphi] \exp[-S'']}$$

$$S''[\varphi] = \frac{1}{2} \int_{\underline{k}} \varphi(\underline{k}) \left[\frac{1}{\beta \tilde{\epsilon}(\underline{k})} - (1-m^2) \right] \varphi(-\underline{k})$$

$$= \frac{1}{\beta \tilde{\epsilon}(\underline{k}) - (1-m^2)}$$

$$\langle \delta\sigma_i \delta\sigma_j \rangle = (1-m_i^2)(1-m_j^2) \langle \varphi_i \varphi_j \rangle$$

$$\equiv G(R_i - R_j) = \int_{\underline{k}} e^{ik(R_i - R_j)} \tilde{G}(\underline{k})$$

$$\equiv (1-m_i^2)(1-m_j^2) G_\phi(R_i - R_j)$$

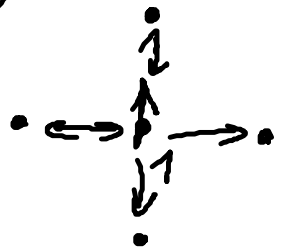
$$\tilde{G}_\phi(\underline{k}) \equiv \langle \varphi(\underline{k}) \varphi(-\underline{k}) \rangle = \left[\frac{1}{\beta \tilde{\epsilon}(\underline{k})} - (1-m^2) \right]^{-1}$$

$$\tilde{\epsilon}(\underline{k}) \equiv \sum_i \frac{1}{\epsilon_i} e^{ikR_i}$$

Faltet immer

$$\equiv \sum_i \mathcal{J}_{1i} \left(1 + i k \underline{r}_i - \frac{1}{2} (k \underline{r}_i)^2 + \dots \right)^{m = m_i}$$

Für "Nächste-Nachbar-Wechselwirkung"



$$\begin{aligned} \mathcal{J}(\underline{k}) &= \mathcal{J} \left(e^{-i \underline{k} \underline{a}_1} + e^{i \underline{k} \underline{a}_1} \right. \\ &\quad + e^{-i \underline{k} \underline{a}_2} + e^{i \underline{k} \underline{a}_2} \\ &\quad + \dots + e^{-i \underline{k} \underline{a}_d} + e^{i \underline{k} \underline{a}_d} \left. \right) \\ &= 2\mathcal{J} \sum_{\mu=1}^d \cos \underline{k} \underline{a}_\mu \\ &= 2\mathcal{J} \left(1 - \frac{1}{2} \sum_{\mu=1}^d (k \underline{a}_\mu)^2 + \dots \right) \end{aligned}$$

Damit

$$\tilde{G}_\varphi(\underline{k}) = \left[\frac{1}{\beta \left(2\mathcal{J}d - \mathcal{J} \sum_{\mu=1}^d (k \underline{a}_\mu)^2 + \dots \right) - (1 - m^2)} \right]^{-1}$$

$$\underline{k} = (k_x, k_y, k_z)$$

$$\sum_{\mu} (k \underline{a}_\mu)^2 = a^2 \cdot (k_x^2 + k_y^2 + \dots)$$

$$= \left[\frac{1}{\beta (2\mathcal{J}d - \mathcal{J} k^2 a^2 + \dots) - (1 - m^2)} \right]^{-1} = a^2 k^2$$

$$\text{Im MF: } T_c = \mathbb{F} = \sum_i \mathbb{F}_{i_j} = 2d\mathbb{F}$$

Damit $\tilde{G}_\varphi(k) \Rightarrow \frac{T_c}{T \left(1 - \frac{T_c}{T} (1 - m^2) + k^2 a^2 / 2d + O(k^4) \right)}$

denm $\left[\frac{1}{\beta \left(\frac{2d}{T_c} - \partial k^2 a^2 \dots - (1 - m^2) \right)} \right]^{-1}$

$$= \frac{2d T_c / T a^2}{k^2 + K^2}$$

$$K^2 = \frac{2d}{a^2} \left(1 - \frac{T_c}{T} (1 - m^2) \right)$$

$$G_\varphi(R) \propto \frac{e^{-R/\xi}}{R} \quad \text{für } R \rightarrow \infty$$

$\xi = K^{-1}$: Korrelationslänge