1.2.07

$\langle \delta\sigma_i \, \delta\sigma_j \rangle$, $\quad \delta\sigma_i = \sigma_i - \langle \sigma_i \rangle$

$$\langle \delta\sigma_i \, \delta\sigma_j \rangle = (1-m_i^2)(1-m_j^2)\langle \varphi_i \varphi_j \rangle$$

$$\langle \varphi_i \varphi_j \rangle = \frac{\int \mathcal{D}[\varphi]\, \varphi_i \varphi_j \, e^{-S''[\varphi]}}{\int \mathcal{D}[\varphi]\, e^{-S''[\varphi]}}$$

Vergleich:

$$\langle x^2 \rangle = \frac{\int_{-\infty}^{\infty} dx\, x^2 p(x)}{\underbrace{\int_{-\infty}^{\infty} dx\, p(x)}_{1}}$$

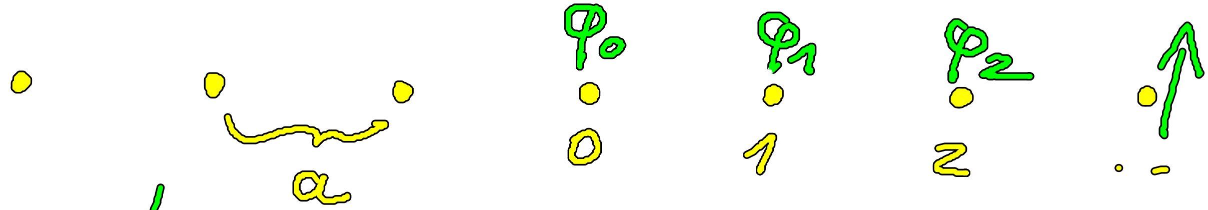
$$= \frac{\int_{-\infty}^{\infty} dx\, x^2 e^{-\frac{x^2}{2\sigma^2}}}{\int_{-\infty}^{\infty} dx\, e^{-x^2/2\sigma^2}} = \sigma^2$$

Hier ist $\quad S''[\varphi] = \frac{1}{2}\sum_{ij} \varphi_i A_{ij} \varphi_j$

$$A_{ij} = \frac{1}{\beta} \mathcal{J}^{-1}_{ij} - (1-m_i^2)\,\delta_{ij}$$

Fourierzerlegung auf Gittern

Beispiel: 1d Gitter mit Gitterkonstante a



$$\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a} dk\, e^{ikna} = \delta_{n,0}$$

$\Rightarrow$ Fourierzerlegung einer auf dem reziproken Gitter periodischen Funktion

$$\varphi(k) = \varphi\left(k + \frac{2\pi m}{a}\right)$$

im k-Raum

$$\tilde{\varphi}(k) = \sum_m \varphi_m\, e^{-ikma} \qquad \text{k-Raum}$$

$$\varphi_n = \frac{a}{2\pi}\int_{-\pi/a}^{\pi/a} dk\, e^{ikna}\, \tilde{\varphi}(k)$$

$$\Big(= \frac{a}{2\pi}\int_{-\pi/a}^{\pi/a} dk\, e^{ikna} \sum_m \varphi_m\, e^{-ikma} \hookrightarrow \delta_{m,n}$$

$$= \varphi_n \Big)$$

$$\varphi(k) = \sum_m \varphi_m e^{-ikma}$$

Basis-Fkt.

Entw. Koeffizienten

$$= \frac{a}{2\pi}\int_{-\pi/a}^{\pi/a} dk' \underbrace{\sum_m e^{-ikma}\, e^{+ik'ma}}_{\frac{2\pi}{a}\delta(k-k')} \varphi(k')$$

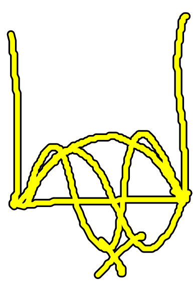
$$\sum_m e^{i(k'-k)ma} = \frac{2\pi}{a}\delta(k-k')$$

Notation einführen:

$$\frac{a}{2\pi}\int_{-\pi/a}^{\pi/a} dk \ldots \equiv \int_k$$

„Integral über die erste Brillouin-Zone"

Vergleich:



Teilchen im Kasten

$$\psi(x) = \sum_k c_k e^{ikx}$$

Ortsraum

k-Raum

Jetzt entsprechend für d-dimensionale Gitter $\{\underline{R}_i\}$, Funktionen $\varphi_i = \varphi(\underline{R}_i)$

$$\mathcal{J}_{ij} = \mathcal{J}(\underline{R}_i, \underline{R}_j)$$

Vektoren φ_i, $i = 1, 2, \dots$
bzw Matrizen $\mathcal{J}_{ij}$

$$\varphi_i = \int_{\underline{k}} e^{i\underline{k}\underline{R}_i}\, \varphi(\underline{k})$$

$\underline{k} = -\underline{k}'$

$$\Rightarrow \mathcal{J}_{ij} = \int_{kk'} e^{i\underline{k}\underline{R}_i}\, e^{i\underline{k}'\underline{R}_j}\, \mathcal{J}(\underline{k}, \underline{k}')$$

häufig $\quad \mathcal{J}_{ij} = \mathcal{J}(\underline{R}_i, \underline{R}_j) = \tilde{\mathcal{J}}(R_i - R_j)$

$$\Rightarrow \quad \mathcal{J}(\underline{k}, \underline{k}') = \underbrace{\frac{(2\pi)^d}{a^d}}_{\text{Volumen der 1. BZ}} \delta(\underline{k} + \underline{k}')\, \mathcal{J}(\underline{k})$$

Damit die Inverse berechnen:

$$\int_k e^{ik(R_i - R_l)} = \delta_{il} = \sum_j \mathcal{J}_{ij}\, \mathcal{J}^{-1}_{jl}$$

$$= \sum_j \int_{\underline{k}} \int_{\underline{k}'} \int_{\underline{k}''} \mathcal{J}(k)\, e^{i\underline{k}(\underline{R}_i - \underline{R}_j)}\, \mathcal{J}^{-1}(\underline{k}', \underline{k}'')\, e^{i\underline{k}'\underline{R}_j}\, e^{i\underline{k}''\underline{R}_l}$$

$$= \underbrace{\int_{\underline{k}}\int_{\underline{k}''} J(\underline{k})\, e^{i\underline{k}\underline{R}_i + i\underline{k}''\underline{R}_l}\, \overbrace{J^{-1}(\underline{k},\underline{k}'')}^{\frac{(2\pi)^d}{a^d}\delta(\underline{k}-\underline{k}')}}_{\left(\frac{2\pi}{a}\right)^d \delta(\underline{k}+\underline{k}'')\frac{1}{J(\underline{k})}}$$

$$\left\| \; J^{-1}(\underline{k},\underline{k}'') = \left(\frac{2\pi}{a}\right)^d \delta(\underline{k}+\underline{k}'')\,\frac{1}{J(\underline{k})} \; \right\|$$

$$S''[\varphi] = \frac{1}{2}\sum_{ij} \varphi_i A_{ij} \varphi_j =$$

$$= \frac{1}{2}\sum_{ij}\int_{kk'} e^{i\underline{k}\underline{R}_i + i\underline{k}'\underline{R}_j}\left[\frac{1}{\beta}J^{-1}_{ij} - (1-m_i^2)\delta_{ij}\right]\varphi(\underline{k})\varphi(\underline{k}')$$

$$= \frac{1}{2}\int_{kk'}\Bigg[\frac{1}{\beta}\frac{1}{J(-\underline{k})}\frac{(2\pi)^d}{a^d}\delta(\underline{k}+\underline{k}')\varphi(\underline{k})\varphi(\underline{k}')$$

$$\underbrace{- \sum_i (1-m_i^2)\, e^{i(\underline{k}+\underline{k}')\underline{R}_i}}_{\frac{(2\pi)^d}{a^d}\delta(\underline{k}+\underline{k}')}\, \varphi(\underline{k})\varphi(\underline{k}')\Bigg]$$

Annahme: $m_i = m$

$$= \frac{1}{2}\int_{\underline{k}}\left[\frac{1}{\beta J(-\underline{k})} - (1-m^2)\right]\varphi(\underline{k})\varphi(-\underline{k})$$

Wir wollen

$$\langle \varphi_i \varphi_j \rangle = \int_{k k'} e^{i \underline{k} \underline{R}_i} e^{i \underline{k}' \underline{R}_j} \underbrace{\langle \varphi(\underline{k}) \varphi(\underline{k}') \rangle}$$

Wegen Translationsinvarianz $\left(\frac{2\pi}{a}\right)^d \delta(\underline{k}+\underline{k}') \langle \varphi(\underline{k}) \varphi(-\underline{k}) \rangle$

$$\langle \varphi(\underline{k}) \varphi(-\underline{k}) \rangle = \frac{\int \mathcal{D}[\varphi]\, \varphi(\underline{k}) \varphi(-\underline{k})\, \exp[-S'']}{\int \mathcal{D}[\varphi]\, \exp[-S'']}$$

$$\left(S''[\varphi] = \frac{1}{2} \int_{\underline{k}} \varphi(\underline{k}) \left[\frac{1}{\beta J(-\underline{k})} - (1-m^2) \right] \varphi(-\underline{k}) \right)$$

$$= \frac{1}{\frac{1}{\beta J(-\underline{k})} - (1-m^2)}$$

$$\langle \delta\sigma_i \delta\sigma_j \rangle = (1-m_i^2)(1-m_j^2) \langle \varphi_i \varphi_j \rangle$$

$$\equiv G(R_i - R_j) = \int_k e^{i\underline{k}(\underline{R}_i - \underline{R}_j)} \tilde{G}(\underline{k})$$

$$\equiv (1-m_i^2)(1-m_j^2)\, G_\phi(R_i - R_j)$$

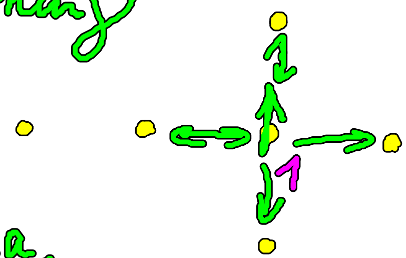
$$\tilde{G}_\phi(\underline{k}) \equiv \langle \varphi(k) \varphi(-k) \rangle = \left[\frac{1}{\beta J(-k)} - (1-m^2) \right]^{-1}$$

$$J(\underline{k}) = \sum_i J_{1i} \underbrace{e^{i \underline{k} \underline{R}_i}}$$

Jetzt immer

$$= \sum_i J_{1i} \left(1 + i\underline{k}\,\underline{R}_i - \frac{1}{2}(\underline{k}\,\underline{R}_i)^2 + \dots \right) m = m_i$$

Für „Nächste-Nachbar-Wechselwirkung"



$$J(\underline{k}) = J \left(e^{-i\underline{k}\,\underline{a}_1} + e^{i\underline{k}\,\underline{a}_1} + e^{-i\underline{k}\,\underline{a}_2} + e^{i\underline{k}\,\underline{a}_2} + \dots \right)$$

$$= 2J \sum_{\mu=1}^{d} \cos \underline{k}\,\underline{a}_\mu$$

$$= 2J \left(\sum_{\mu=1}^{d} 1 - \frac{1}{2} \sum_{\mu=1}^{d} (\underline{k}\,\underline{a}_\mu)^2 + \dots \right)$$

Damit

$$\tilde{G}_\varphi(\underline{k}) = \left[\frac{1}{\beta\left(2Jd - J\sum_{\mu=1}^{d}(\underline{k}\,\underline{a}_\mu)^2 + \dots\right)} - (1-m^2) \right]^{-1}$$

$$\underline{k} = (k_x, k_y, k_z)$$

$$\sum_\mu (\underline{k}\,\underline{a}_\mu)^2 = a^2 \cdot (k_x^2 + k_y^2 + \dots) = a^2 k^2$$

$$= \left[\frac{1}{\beta\left(2Jd - Jk^2a^2 + \dots\right)} - (1-m^2) \right]^{-1}$$

In MF: $T_c = \mathcal{J} = \sum J_{ij} = 2d J$

Damit $\tilde{G}_\varphi(k) = \dfrac{T_c}{\underbrace{T\left(1 - \frac{T_c}{T}(1-m^2)\right.}\left. + k^2 a^2/2d + O(k^4)\right)}$

denn $\left[\dfrac{1}{\beta(\underbrace{2Jd}_{T_c} - Jk^2a^2 \ldots)} - (1-m^2)\right]^{-1}$

$$= \frac{2d\, T_c / T a^2}{k^2 + K^2},$$

$$K^2 \equiv \frac{2d}{a^2}\left(1 - \frac{T_c}{T}(1-m^2)\right)$$

$$G_\varphi(R) \propto \frac{e^{-R/\xi}}{R} \quad \text{für } R \to \infty$$

$\xi = K^{-1}$: Korrelationslänge