Dielektrische Verschiebung

$$\vec{D} = \underline{\varepsilon_0 \varepsilon} \cdot \vec{E} + \vec{P}^{NL} \quad \text{mit } \vec{P}^{NL} = \varepsilon_0 \underline{\chi}^{(2)} : \vec{E}\vec{E}$$

$$\varepsilon_0 \vec{E} + \varepsilon_0 \underline{\chi}^{e} \cdot \vec{E} \quad , \quad \underline{\varepsilon} = 1 + \underline{\chi}^{e}$$

$$\Delta \vec{E}_3 - \nabla \underbrace{\nabla \cdot \vec{E}_3}_{=0 \text{ für } E_\perp} + \frac{\omega_3^2}{c^2} \underline{\varepsilon} \cdot \vec{E}_3 = -\frac{\omega_3^2}{c^2} \underline{\chi}^{(2)} : \vec{E}_1 \vec{E}_2$$

$$\exp\{-i\omega_3 t\} \longleftrightarrow \exp\{-i(\omega_1 + \omega_2)t\}$$

$$\left(\frac{\partial^2}{\partial z^2} + k^2\right) E_\perp = -\frac{\omega_3^2}{c^2} \chi_\perp^{(2)} E_{10} E_{20} \exp\{i(k_1 + k_2)z\}$$

$$\frac{\partial^2}{\partial z^2} E_\perp = \frac{\partial^2 F}{\partial z^2} \exp\{ikz\} - k^2 F \exp\{ikz\} + 2ik \frac{\partial F}{\partial z} \exp\{ikz\}$$

$$\frac{\partial}{\partial z}\left[\frac{\partial F}{\partial z} + 2ikF\right] \exp\{ikz\}, \quad \text{Ann.} \left|\frac{\partial F}{\partial z}\right| \ll k|F| = \frac{2\pi}{\lambda}|F|$$

$$2ik \frac{\partial F}{\partial z} \exp\{ikz\} = -\frac{\omega_3^2}{c^2} \chi_\perp^{(2)} E_{10} E_{20} \exp\{i(k_1 + k_2)z\}$$

$$\frac{\partial F}{\partial z} = -\frac{1}{2ik} \frac{\omega_3^2}{c^2} \chi_\perp^{(2)} E_{10} E_{20} \exp\{iKz\} \quad ; K = k_1 + k_2 - k$$

z

$\int_0 dz$ mit $F(z=a\omega_3)=0$

$$F(z,\omega_3) = \cdots \frac{\exp\{izkz\}}{izk}\Big|_0^a = \cdots \frac{\exp\{izkz\}-1}{izk}$$

$$= \cdots \exp\left\{i\frac{kz}{2}\right\} \frac{2i\sin\left\{\frac{kz}{2}\right\}}{izk} \qquad \upsilon k=\omega_3$$

Intensität

$$I(z) = \upsilon\varepsilon\langle\vec{E}^2\rangle = \upsilon\frac{c^2}{\upsilon^2}\varepsilon_0\cdot\frac{1}{2}\left(\frac{\omega_3^2}{2kc^2}\right)^2\left(\chi_\perp^{(2)}E_{10}E_{20}\right)^2\left[\frac{\sin\left\{\frac{kz}{2}\right\}}{\frac{kz}{2}}\right]^2$$

$$= \frac{\varepsilon_0}{8c^2}\frac{\omega_3^2}{k}\left(\chi_\perp^{(2)}E_{10}E_{20}\right)^2 z^2\left[\frac{\sin\left\{\frac{kz}{2}\right\}}{\frac{kz}{2}}\right]^2$$

Optische Fasern