7. Nichtinertialsysteme

$$\ddot{\underline{r}}' + \underline{b}_0 = \ddot{\underline{r}} \qquad (7.2)$$

KS' — Beschl. KS' relativ zu IS — IS

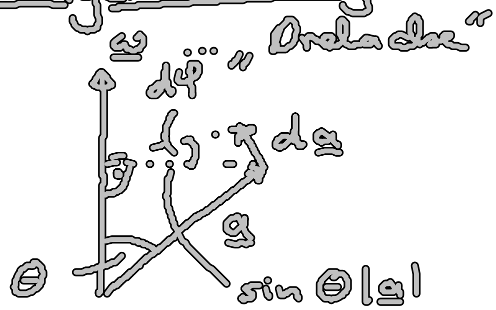
$$\longrightarrow \boxed{m\ddot{\underline{r}}' = \underline{F}' + \underline{F}_s' \,, \quad \underline{F}_s' = -m\underline{b}_0 \; \ldots \text{ Scheinkraft}} \qquad (7.3)$$

... Newton mit Scheinkraft

7.2 Rotierende BS

a) Winkelgeschwindigkeit

• Def:



$$\underline{\omega} = \frac{d\varphi}{dt}\,\underline{e}_\omega \,, \quad |\underline{e}_\omega| = 1 \qquad (7.4)$$

... Winkelgeschwind.

• bel. Vektor:

$$\left.\begin{aligned} &d\underline{a} \perp \underline{a}, \underline{\omega} \\ &|d\underline{a}| = \sin\theta \, |\underline{a}| \, d\varphi \end{aligned}\right\} \; d\underline{a} = \underbrace{d\varphi\, \underline{e}_\omega}_{\underline{\omega}\, dt} \times \underline{a} \xrightarrow{\frac{1}{dt}} \boxed{\frac{d\underline{a}}{dt} = \underline{\omega} \times \underline{a}} \qquad (7.5)$$

• Bsp: $\underline{a} = \underline{r} \longrightarrow \boxed{\underline{v} = \frac{d\underline{r}}{dt} = \underline{\omega} \times \underline{r}}$ (7.6)

• Bemerkungen:

(i) polare Vektoren $\underline{a}, \frac{d\underline{a}}{dt} \xrightarrow{(7.5)} \underline{\omega}$ ... Pseudovektor

[Punktspiegelung am Ursprung $\longrightarrow -\underline{a}, -\frac{d\underline{a}}{dt}$] $\longrightarrow \underline{\omega}$ ... bei Pkt.spiegelung!

(ii) $\underline{\omega}_1 + \underline{\omega}_2 = \underline{\omega}_2 + \underline{\omega}_1$! [s. (7.5)]

(iii) <u>Drehung</u>: $\underline{\underline{D}}(\underline{\varphi})$ $\qquad \underline{\varphi} = \varphi\, \underline{e}$

(1) $\underline{\underline{D}}(\underline{\varphi}_1)\,\underline{\underline{D}}(\underline{\varphi}_2) \neq \underline{\underline{D}}(\underline{\varphi}_2)\,\underline{\underline{D}}(\underline{\varphi}_1)$

(2) $\underline{\varphi} = d\underline{\varphi}$ mit $|d\underline{\varphi}| \ll 1$

$$\underline{\underline{D}}(d\underline{\varphi})\,\underline{a} = \underline{a} + d\underline{a} \underset{(7.5)}{=} (\underline{\underline{1}} + d\underline{\varphi}\times)\,\underline{a}$$

$$\longrightarrow \boxed{\underline{\underline{D}}(d\underline{\varphi}) \approx \underline{\underline{1}} + d\underline{\varphi}\times} \quad (7.7)$$

(3) $\underline{\underline{D}}(\underline{\varphi})$?

„n mal Rotation um $\underline{e}$ um $d\varphi = \frac{\varphi}{n}$"

$$\boxed{\begin{aligned}\underline{\underline{D}}(\underline{\varphi}) &= \left(\underline{\underline{1}} + \frac{\underline{\varphi}}{n}\times\right)^n \\ &:= e^{\underline{\varphi}\times} = \sum_{i=0}^{\infty} \frac{1}{i!}(\underline{\varphi}\times)^i\end{aligned}} \quad (7.8)$$

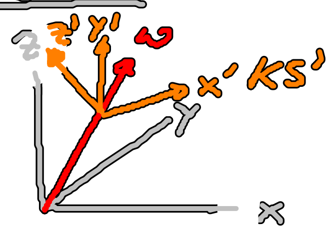
$\left(1+\frac{z}{n}\right)^n \underset{n\to\infty}{=} e^z$

… „Lie-Gruppen-Darstellung der Drehungen"

b) <u>rotierende BS</u>

• Geometrie

IS



• Vektor $\underline{a}(t)$: $\quad d\underline{a}_{IS} = d\underline{a}_{KS'} + \underline{\omega}\, dt \times \underline{a}$ (7.9)

$$\xrightarrow{(7.9)/dt} \boxed{\left(\frac{d\underline{a}}{dt}\right)_{IS} = \left(\frac{d\underline{a}}{dt}\right)_{KS'} + \underline{\omega}\times\underline{a}} \quad (7.10)$$

• „Zeitoperator": $\boxed{\left(\frac{d}{dt}\right)_{IS} = \left(\frac{d}{dt}\right)_{KS'} + \underline{\omega}\times}$ (7.11)

• gemeinsamer Ursprung von IS und KS': $\quad \underline{r}(t) = \underline{r}'(t)$

aber: $\quad \dot{\underline{r}} := \left(\frac{d\underline{r}}{dt}\right)_{IS} \neq \dot{\underline{r}}' := \left(\frac{d\underline{r}}{dt}\right)_{KS'}$ (7.12)

• $\boxed{\underline{\omega} = \text{const}}$ $\longrightarrow$ $\ddot{\underline{r}} = \left(\frac{d^2 \underline{r}}{dt^2}\right)_{IS} \overset{(6.11)}{=} \left[\left(\frac{d}{dt}\right)_{KS'} + \underline{\omega}\times\right]\left[\left(\frac{d}{dt}\right)_{KS'} + \underline{\omega}\times\right]\underline{r}$

$\underline{r} = \underline{r}'$

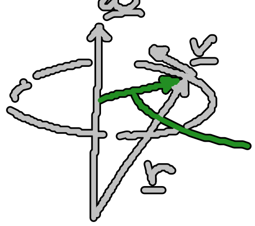
$$= \ddot{\underline{r}}' + \underbrace{2\underline{\omega}\times\dot{\underline{r}}' + \underline{\omega}\times(\underline{\omega}\times\underline{r})}_{\underline{b}_0} \qquad (7.13)$$

IS: $m\ddot{\underline{r}} = \underline{F}$

$\longrightarrow$ Bewgl. in KS':

$$\boxed{\begin{aligned} m\ddot{\underline{r}}' &= \underline{F}' + \underline{F}_S' \\ \text{mit } \underline{F}_S' &= \underbrace{-2m\,\underline{\omega}\times\dot{\underline{r}}'}_{\text{Corioliskraft}} \underbrace{-\, m\,\underline{\omega}\times(\underline{\omega}\times\underline{r})}_{\text{Zentrifugalkraft}} \end{aligned}} \qquad (7.14)$$

• Zentrifugalkraft:



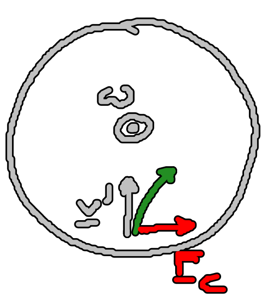
$\underline{r}_\perp = -r_\perp \underline{n}$
(vgl. (7.8))

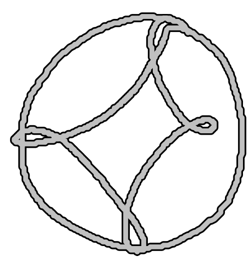
$$\longrightarrow -m\,\underline{\omega}\times(\underline{\omega}\times\underline{r}) = -m\,\underline{\omega}\times\underline{v}$$

$$= m\frac{v^2}{r_\perp}\underbrace{\frac{\underline{r}_\perp}{r_\perp}}_{-\underline{n}} \qquad \omega = \frac{v}{r_\perp}$$

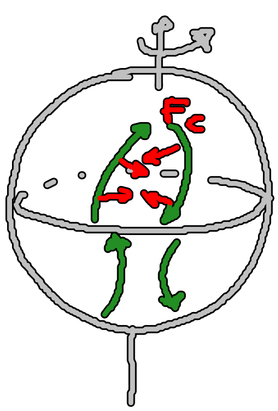
$$= m r_\perp \omega^2 \frac{\underline{r}_\perp}{r_\perp}$$

• Corioliskraft: falls $\dot{\underline{r}}' = \underline{v}' \neq 0$





Erde



Bewegung auf $\left.\begin{matrix}\text{Nord}\\ \text{Süd}\end{matrix}\right\}$ halbkugel: $\left.\begin{matrix}\text{„Rechts"}\\ \text{„Links"}\end{matrix}\right\}$ abweichung

Meteorologie: $\left.\begin{matrix}\text{NO}\\ \text{SO}\end{matrix}\right\}$ Passate

Unterspülung des $\left.\begin{matrix}\text{rechten}\\ \text{linken}\end{matrix}\right\}$ Ufers (in Flußrichtung)

II. Newtonsche Mechanik für Vielteilchen-Systeme

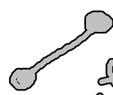
- System von N Massepktn
  Zustandsraum: Raum der 3N Ortskoordinaten
- **Bsp:** Zweikörperproblem (→ Kap. 6, 9)
  Stoßprozesse (Kap. 9)
  starrer Körper (→ Kap. 10)
  schwingende, gekoppelte Massepkte (→ Kap. 11)
- mechanische Freiheitsgrade f: beschreiben Lage der N Massepkte eindeutig

$$\boxed{f = 3N - Z}$$

Z: Zahl der Zwangsbed.

Bsp:

- $f = 6$



$f = 6 - 1 = 5$

8. Newtonsche Grundgleichungen & Folgerungen

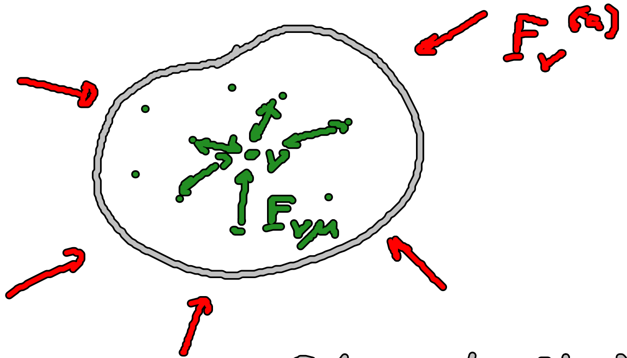
8.1 Grundgleichungen

„Newton" für Massepkte $\nu = 1 \dots N$:

$$\boxed{\begin{aligned} m_\nu \ddot{\underline{r}}_\nu &= \frac{d}{dt} \underline{p}_\nu = \underline{F}_\nu \\ \text{mit } \underline{F}_\nu &= \underline{F}_\nu^{(a)} + \underline{F}_\nu^{(i)} \\ &= \underbrace{\underline{F}_\nu^{(a)}}_{\text{äußere Kräfte}} + \underbrace{\sum_{\substack{\mu=1 \\ \mu \neq \nu}}^{N} \underline{F}_{\nu\mu}}_{\text{innere Kräfte}} \end{aligned}} \qquad (8.1)$$

3N DGl. für 3N Ortskoord.

innere Kräfte von allen anderen Massepktn.: Zweiteilchenkräfte!



- äußere Kräfte: Bsp: Schwerkraft der m_ν
  Kräfte aufgrund EM-Feld

  (i) äußere Gesamtkraft: $\underline{F}^{(a)} = \sum_{\nu=1}^{N} \underline{F}_\nu^{(a)}$ (8.2)

  (ii) „abgeschlossenes System": $\underline{F}^{(a)} = 0$ (8.3)

  im engeren Sinne: $\underline{F}_\nu^{(a)} = 0$

- innere Kräfte: Bsp: $\left.\begin{array}{l}\text{Coulomb}\\ \text{Gravitations}\end{array}\right\}$ Kräfte
  chem. Bindungskräfte (in Molekülen, etc...)

  (i) $\underline{F}_{\nu\mu} = \underbrace{\underline{F}_{\nu\mu}(\underline{r}_\nu - \underline{r}_\mu)}$ ... Kraft von m_μ auf m_ν

  Annahme: $\underline{F}_{\nu\mu} \parallel \underline{r}_\nu - \underline{r}_\mu$ ... Zentralkräfte

  (ii) actio = reactio: $\longrightarrow$ $\boxed{\underline{F}_{\nu\mu}(\underline{r}_\nu - \underline{r}_\mu) = -\underline{F}_{\mu\nu}(\underline{r}_\mu - \underline{r}_\nu)}$ (8.4)

  $\longrightarrow$ $\boxed{\underline{F}_{\nu\nu} = 0 \longrightarrow \sum_{\substack{\mu=1\\ \mu\neq\nu}}^{N} \underline{F}_{\nu\mu} = \sum_{\mu=1}^{N} \underline{F}_{\nu\mu}}$ (8.5)

  (iii) innere Gesamtkraft:

  $$\boxed{\underline{F}^{(i)} = \sum_{\nu=1}^{N} \underline{F}_\nu^{(i)} = \sum_{\nu\mu=1}^{N} \underline{F}_{\nu\mu} = 0} \quad (8.6)$$

8.2 Folgerungen

a) Impulssatz

- Def: Gesamtimpuls: $\boxed{\underline{P} = \sum_\nu \underline{p}_\nu = \sum_\nu m_\nu \dot{\underline{x}}_\nu}$ (8.7)