

# 6.1 Erzeugungs- und Vernichtungsoperatoren

Orthogonalität

$$\langle n_1 n_2 \dots | n'_1 n'_2 \dots \rangle = \dots \langle \psi_{n_1}(1) \psi_{n_2}(2) \dots | \psi_{n'_1}(1) \psi_{n'_2}(2) \dots \rangle$$

$$= \langle \psi_{n_1}(1) | \psi_{n'_1}(1) \rangle \langle \psi_{n_2}(2) | \psi_{n'_2}(2) \rangle \dots \langle \psi_{n_N}(N) | \psi_{n'_N}(N) \rangle$$

Normierung:  $N=2$

$$|n_1 n_2 \dots \rangle = \frac{1}{\sqrt{2}} [\psi_{n_1}(1) \psi_{n_2}(2) - \psi_{n_2}(1) \psi_{n_1}(2)]$$

$$\langle n_1 n_2 \dots | n_1 n_2 \dots \rangle = \frac{1}{2} [\langle \psi_{n_1}(1) | \psi_{n_1}(1) \rangle \langle \psi_{n_2}(2) | \psi_{n_2}(2) \rangle - \langle \psi_{n_2}(1) | \psi_{n_1}(1) \rangle \langle \psi_{n_1}(2) | \psi_{n_2}(2) \rangle - 0 + 1] = 1$$

$$A(\hat{z}) |n_1 n_2 \dots \rangle = \dots \sum_{PES} (\pm 1)^P T_P \{ \psi_{n_1}(1) \dots A(\hat{z}) \psi_{n_2}(2) \dots \psi_{n_N}(N) \}$$

$$A(\hat{z}) \psi_{n_2}(z) = \sum_{\lambda=1}^{\infty} \psi_{\lambda}(z) A_{\lambda n_2} \text{ mit } A_{\lambda n_2} = \langle \psi_{\lambda}(z) | A(\hat{z}) | \psi_{n_2}(z) \rangle$$

1)  $\lambda = n_2 \rightarrow A_{n_2 n_2} \leftrightarrow |n_1 n_2 \dots \rangle$

2)  $\lambda \neq n_2 \rightarrow A_{\lambda n_2} \leftrightarrow |n_1 n_2 \dots n_{\lambda+1} \dots n_{n_2-1} \dots \rangle$

Def:  $a_{\lambda} |n_1 n_2 \dots n_{\lambda} \dots \rangle = \sqrt{n_{\lambda}} |n_1 n_2 \dots n_{\lambda-1} \dots \rangle$

$$a_{\lambda}^{\dagger} |n_1 n_2 \dots n_{\lambda} \dots \rangle = \sqrt{n_{\lambda}+1} |n_1 n_2 \dots n_{\lambda+1} \dots \rangle$$

$$\dots A_{\lambda n_2} a_{\lambda}^{\dagger} a_{n_2} |n_1 n_2 \dots n_{\lambda} \dots n_{n_2} \dots \rangle$$

$$a_\lambda a_\lambda^\dagger |n_1, n_2, \dots\rangle = a_\lambda \sqrt{n_\lambda + 1} |n_1, n_2, \dots, n_\lambda + 1, \dots\rangle$$

$$= (n_\lambda + 1) |n_1, n_2, \dots\rangle$$

$$\Rightarrow [a_\lambda, a_\mu^\dagger] = \delta_{\lambda\mu} 1$$

6.2 Feldoperatoren

$$\hat{\psi}(x) = \sum_\nu \varphi_\nu(x) a_\nu$$

$$\hat{\psi}^\dagger(x) = \sum_\mu \varphi_\mu^*(x) a_\mu^\dagger$$

Kommutator:

$$[\psi_\nu(x), \psi_\mu^\dagger(x')] = \sum_{\nu, \mu} \varphi_\nu(x) \varphi_\mu^*(x') [a_\nu, a_\mu^\dagger] = \delta_{\nu\mu} 1$$

$$= \sum_\nu \varphi_\nu(x) \varphi_\nu^*(x') 1 = \delta(x-x') 1$$

$$\psi(x) = \sum_\nu \varphi_\nu(x) c_\nu \quad \text{mit } c_\nu = \int \varphi_\nu^*(x') \psi(x') dx'$$

$$= \int \underbrace{\left( \sum_\nu \varphi_\nu(x) \varphi_\nu^*(x') \right)}_{=\delta(x-x')} \psi(x') dx' = \int \delta(x-x') \psi(x') dx' = \psi(x)$$