Cumulantenentwicklung

3. Ordnung

$$tr\left(c^{(+)} c^{(+)} c^{(+)} \rho_B\right) = 0$$

4. Ordnung

$$e^{-\mathcal{F}_1(t) - \mathcal{F}_2(t)}\Big|_{4.\,\text{Ordnung}} = -\mathcal{F}_2(t) + \frac{\mathcal{F}_1(t)\,\mathcal{F}_1(t)}{2!} \overset{!}{=}$$

$$= \frac{1}{\hbar^4} \sum_{l'} g_{l'} \sum_{l''} g_{l''} \sum_{l'''} g_{l'''} \sum_{l''''} g_{l''''} \Big($$

$$\int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \int_{t_0}^{t''} dt''' \int_{t_0}^{t'''} dt'''' \, e^{-i(\omega_{l'} t' + \omega_{l''} t'' - \omega_{l'''} t''' - \omega_{l''''} t'''')} \, tr\left(c^+_{l'} c^+_{l''} c_{l'''} c_{l''''} \rho_B\right) \quad (1)$$

$$+ \quad '' \quad e^{-i(\omega_{l'} t' - \omega_{l''} t'' + \omega_{l'''} t''' - \omega_{l''''} t'''')} \, tr\left(c^+_{l'} c_{l''} c^+_{l'''} c_{l''''} \rho_B\right) \quad (2)$$

$$+ \quad '' \quad e^{-i(\omega_{l'} t' - \omega_{l''} t'' - \omega_{l'''} t''' + \omega_{l''''} t'''')} \, tr\left(c^+_{l'} c_{l''} c_{l'''} c^+_{l''''} \rho_B\right) \quad (3)$$

$$+ \quad '' \quad e^{-i(-\omega_{l'} t' + \omega_{l''} t'' + \omega_{l'''} t''' - \omega_{l''''} t'''')} \, tr\left(c_{l'} c^+_{l''} c^+_{l'''} c_{l''''} \rho_B\right) \quad (4)$$

$$+ \quad '' \quad e^{-i(-\omega_{l'} t' + \omega_{l''} t'' - \omega_{l'''} t''' + \omega_{l''''} t'''')} \, tr\left(c_{l'} c^+_{l''} c_{l'''} c^+_{l''''} \rho_B\right) \quad (5)$$

$$+ \quad '' \quad e^{-i(-\omega_{l'} t' - \omega_{l''} t'' + \omega_{l'''} t''' + \omega_{l''''} t'''')}$$

$tr(c_{\zeta'} c_{\zeta''} c^+_{\zeta'''} c^+_{\zeta''''} \rho_B)$ (6)

$$tr(c_1^+ c_2^+ c_3 c_4 \rho_B) = tr(c_1^+ c_4 \rho_B)\, tr(c_2^+ c_3 \rho_B) + tr(c_1^+ c_3 \rho_B)\, tr(c_2^+ c_4 \rho_B)$$

$$tr(c_1^+ c_2 \rho_B) = n_1 \delta_{12}$$

(1) $n_{\zeta'} n_{\zeta''} \delta_{\zeta'\zeta'''} \delta_{\zeta''\zeta''''} + n_{\zeta'} n_{\zeta''} \delta_{\zeta'\zeta''''} \delta_{\zeta''\zeta'''}$ ⟸ Auswerten

(2) $n_{\zeta'}(n_{\zeta''}+1) \delta_{\zeta'\zeta''''} \delta_{\zeta''\zeta'''} + n_{\zeta'} n_{\zeta''} \delta_{\zeta'\zeta''} \delta_{\zeta'''\zeta''''}$

(3) $(n_{\zeta'''}+1) n_{\zeta'} \delta_{\zeta'''\zeta''''} \delta_{\zeta''\zeta'} + (n_{\zeta''}+1) n_{\zeta'} \delta_{\zeta''''\zeta''} \delta_{\zeta'\zeta'''}$

(4) $(n_{\zeta'}+1) n_{\zeta'''} \delta_{\zeta'\zeta''} \delta_{\zeta'''\zeta''''} + (n_{\zeta'}+1) n_{\zeta''} \delta_{\zeta'\zeta'''} \delta_{\zeta''\zeta''''}$

(5) $(n_{\zeta'}+1)(n_{\zeta'''}+1) \delta_{\zeta'\zeta''} \delta_{\zeta'''\zeta''''} + (1+n_{\zeta'}) n_{\zeta''} \delta_{\zeta'\zeta''''} \delta_{\zeta''\zeta'''}$

(6) $(n_{\zeta'''}+1)(n_{\zeta''}+1) \delta_{\zeta'''\zeta'} \delta_{\zeta''\zeta''''} + (n_{\zeta''}+1)(n_{\zeta'}+1) \delta_{\zeta'''\zeta''} \delta_{\zeta''''\zeta'}$

$$\sum_{\zeta}\sum_{\tilde\zeta} \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \int_{t_0}^{t''} dt''' \int_{t_0}^{t'''} dt'''' \Big[n_\zeta n_{\tilde\zeta} \Big(e^{-i\omega_{\tilde\zeta}(t'-t''')} e^{-i\omega_\zeta(t''-t''')} + e^{-i\omega_\zeta(t'-t''')} e^{-i\omega_{\tilde\zeta}(t''-t''')} + e^{-i(\omega_\zeta t' - \omega_\zeta t')} e^{-i\omega_{\tilde\zeta}(t''-t''')}$$

$$= \underbrace{\sum_\zeta \sum_{\tilde\zeta} n_\zeta n_{\tilde\zeta}}\Big(\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} d\tilde t_1 \int_{t_0}^{\tilde t_1} dt_2 \int_{t_0}^{t_2} d\tilde t_2\, \underbrace{e^{-i\omega_\zeta(t_1-t_2)} e^{-i\omega_{\tilde\zeta}(\tilde t_1-\tilde t_2)}}_{I(t_1,t_2,\tilde t_1,\tilde t_2)}$$

$$+ \int_{t_0}^{t} dt_1 \int_{t_0}^{t} d\tilde t_1 \int_{t_0}^{\tilde t_1} d\tilde t_2 \int_{t_0}^{\tilde t_2} dt_2\, e^{-i\omega_\zeta(t_1-t_2)} e^{-i\omega_{\tilde\zeta}(\tilde t_1-\tilde t_2)}$$

$$+ \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_1}^{t_2} d\tilde t_1 \int_{t_0}^{\tilde t_1} d\tilde t_2\, e^{-i\omega_\zeta(t_1-t_2)} e^{-i\omega_{\tilde\zeta}(\tilde t_1-\tilde t_2)}$$

$$= \frac{1}{2}\Big[\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} d\tilde t_1 \int_{t_0}^{\tilde t_1} dt_2 \int_{t_0}^{t_2} d\tilde t_2\, I(\ldots) \;(1) + \int_{t_0}^{t} d\tilde t_1 \int_{t_0}^{\tilde t_1} dt_1 \int_{t_0}^{t_1} d\tilde t_2 \int_{t_0}^{\tilde t_2} dt_2\, I(\ldots) \;(2)$$

$$+ \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} d\tilde t_1 \int_{t_0}^{\tilde t_1} d\tilde t_2 \int_{t_0}^{\tilde t_2} dt_2\, I(\ldots) \;(3) + \int_{t_0}^{t} d\tilde t_1 \int_{t_0}^{\tilde t_1} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_2} d\tilde t_2\, I(\ldots) \;(4)$$

$$+ \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_2} d\tilde t_1 \int_{t_0}^{\tilde t_1} d\tilde t_2\, I(\ldots) \;(5) + \int_{t_0}^{t} d\tilde t_1 \int_{t_0}^{\tilde t_1} d\tilde t_2 \int_{t_0}^{\tilde t_2} dt_1 \int_{t_0}^{t_1} dt_2\, I(\ldots)\Big] \;(6)$$

(1) $t_0 < \tilde t_2 < t_2 < \tilde t_1 < t_1 < t$ (2) $t_0 < t_2 < \tilde t_2 < t_1 < \tilde t_1 < t$

(3) $t_0 < t_2 < \tilde t_2 < \tilde t_1 < t_1 < t$ (4) $t_0 < \tilde t_2 < t_2 < t_1 < \tilde t_1 < t$

(5) $t_0 < \tilde t_2 < \tilde t_1 < t_2 < t_1 < t$ (6) $t_0 < t_2 < t_1 < \tilde t_2 < \tilde t_1 < t$

(1)+(5) $t_0 < \tilde t_2 < \tilde t_1 < t_1 < t$ $\wedge$ $\tilde t_2 < t_2 < t_1$

(2)+(6) $t_0 < t_2 < t_1 < \tilde t_1 < t$ $\wedge$ $t_2 < \tilde t_2 < \tilde t_1 < t$

(1)+(5)+(3) $t_0 < \tilde t_2 < \tilde t_1 < t_1 < t$

$t_0 < t_2 < t_1 < t$

$\underbrace{\tilde t_2 < \tilde t_1,\; t_2 < t_1 \quad \tilde t_1 < t_1}$

(2)+(6)+(4) $t_0 < \tilde t_2 < \tilde t_1 < t$

$t_0 < t_2 < t_1 < \tilde t_1 < t$

$\underbrace{\tilde t_2 < \tilde t_1,\; t_2 < t_1 \quad t_1 < \tilde t_1}$

$1 + \ldots + 6$

$$\begin{aligned} t_0 < \tilde t_2 < \tilde t_1 < t \\ t_0 < t_2 < t_1 < t \end{aligned} \Rightarrow \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t} d\tilde t_1 \int_{t_0}^{\tilde t_1} d\tilde t_2\, I(\ldots)$$

$\Downarrow$

$$= \frac{1}{2}\left(\sum_{n_1} \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2\, e^{-i\omega_{n_1}(t_1 - t_2)}\right)^2$$

Also ~~Ex~~ Rest analog

$F_2 = 0$. Damit 4. Ordnung = 0

$$\circledast = \frac{1}{2}\left(-\frac{1}{\hbar^2}\sum_l g_l^2 \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \left(e^{-i\omega_l(t'-t'')} n_l + e^{i\omega_l(t'-t'')}(n_l+1)\right)\right)^2$$

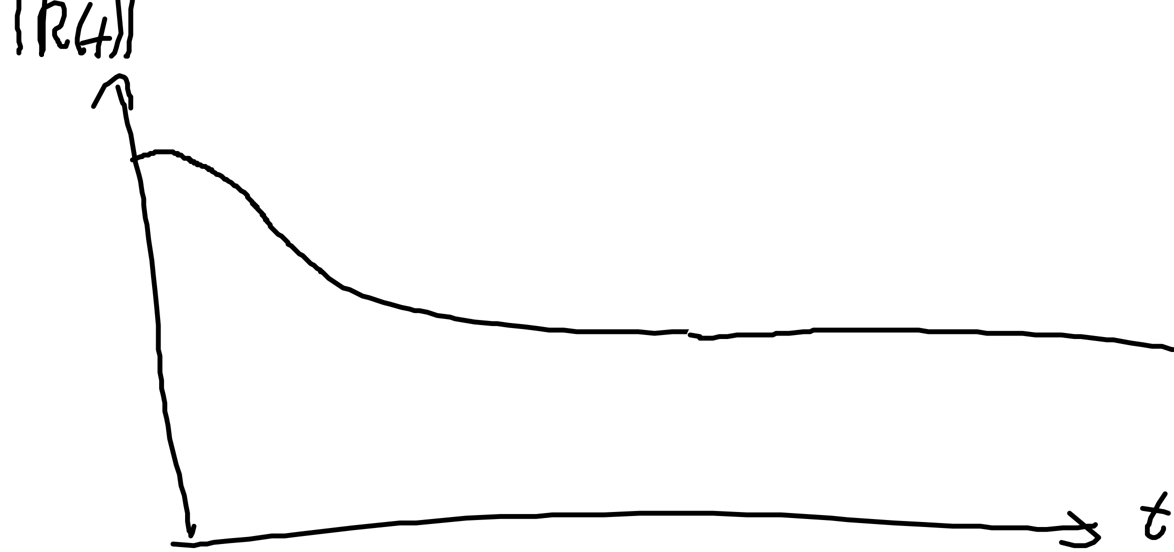
$$= \frac{1}{2} F_1(t)^2 \qquad \stackrel{!}{=} -F_2(t) + \frac{F_1(t)\,F_1(t)}{2!}$$

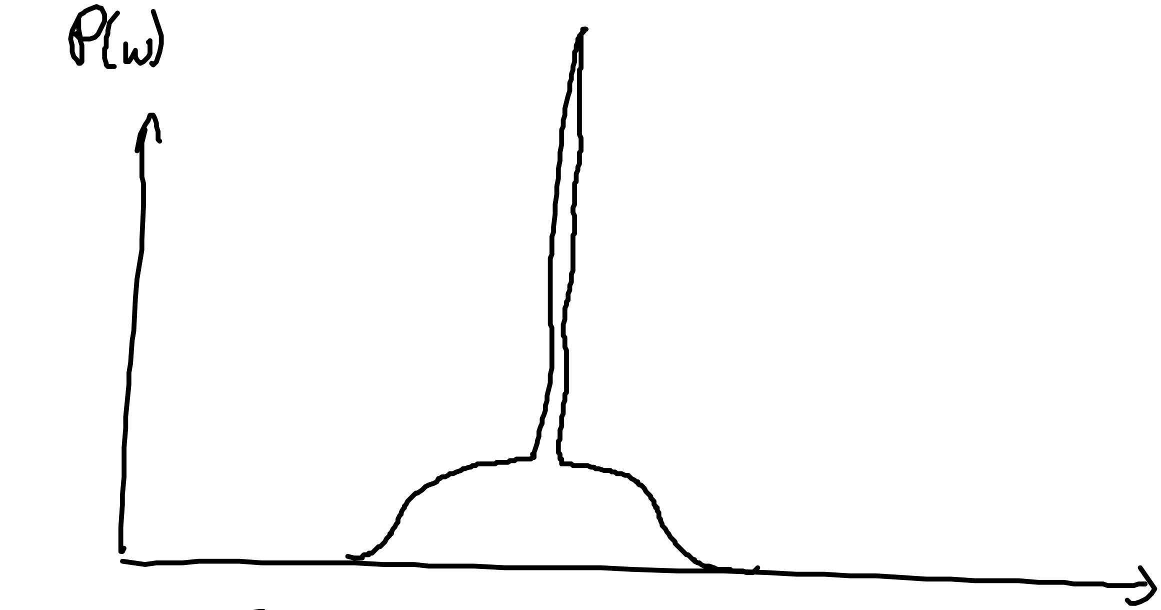
$$\Rightarrow F_2(t) = 0$$

Alles nur wegen Wicks Theorem.

<u>Also</u>

$$R(t) = \exp\left(-\frac{1}{\hbar^2}\sum_l g_l^2 \left(\frac{(1-e^{-i\omega_l(t-t_0)})}{\omega_l^2} n_l + \frac{(1-e^{i\omega_l(t-t_0)})}{\omega_l^2}(n_l+1) + \frac{i}{\omega_l}(t-t_0)\right)\right)$$





Beispiel für komplexe Anregung

$$R(t, t_0, \tilde{t}, \tilde{t}_0) = tr_B\Big(\underbrace{T_{\leftarrow} \exp\Big(-\frac{i}{\hbar}\int_{t_0}^{t} dt' \sum_l g_l \big(c_l^{\dagger} e^{-i\omega_l (t'-t)} + c_l e^{i\omega_l (t'-t)}\big)\Big)}_{= A}$$

$$\underbrace{T_{\rightarrow} \exp\Big(-\frac{i}{\hbar}\int_{\tilde{t}_0}^{\tilde{t}} dt' \sum_l g_l \big(c_l^{\dagger} e^{-i\omega_l (t'-t)} + c_l e^{i\omega_l (t'-t)}\big)\Big)}_{B =}\Big)$$