Cumulantenentwicklung

3. Ordnung

$$tr\left(c^{(+)} c^{(+)} c^{(+)} \rho_B \right) = 0$$

4. Ordnung

$$e^{-\tilde{F}_1(t) - \tilde{F}_2(t)}\Big|_{4.\,\text{Ordnung}} = -\tilde{F}_2(t) + \frac{\tilde{F}_1(t)\,\tilde{F}_1(t)}{2!} \overset{!}{=}$$

$$= \frac{1}{\hbar^4} \sum_{j'} g_{j'} \sum_{j''} g_{j''} \sum_{j'''} g_{j'''} \sum_{j''''} g_{j''''} \Big(\int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \int_{t_0}^{t''} dt''' \int_{t_0}^{t'''} dt'''' \, e^{-i(\omega_{j'} t' + \omega_{j''} t'' - \omega_{j'''} t''' - \omega_{j''''} t'''')} \, tr\left(c^+_{j'} c^+_{j''} c_{j'''} c_{j''''} \rho_B \right) \quad 1$$

$$+ \quad '' \quad e^{-i(\omega_{j'} t' - \omega_{j''} t'' + \omega_{j'''} t''' - \omega_{j''''} t'''')} \, tr\left(c^+_{j'} c_{j''} c^+_{j'''} c_{j''''} \rho_B \right) \quad 2$$

$$+ \quad '' \quad e^{-i(\omega_{j'} t' - \omega_{j''} t'' - \omega_{j'''} t''' + \omega_{j''''} t'''')} \, tr\left(c^+_{j'} c_{j''} c_{j'''} c^+_{j''''} \rho_B \right) \quad 3$$

$$+ \quad '' \quad e^{-i(-\omega_{j'} t' + \omega_{j''} t'' + \omega_{j'''} t''' - \omega_{j''''} t'''')} \, tr\left(c_{j'} c^+_{j''} c^+_{j'''} c_{j''''} \rho_B \right) \quad 4$$

$$+ \quad '' \quad e^{-i(-\omega_{j'} t' + \omega_{j''} t'' - \omega_{j'''} t''' + \omega_{j''''} t'''')} \, tr\left(c_{j'} c^+_{j''} c_{j'''} c^+_{j''''} \rho_B \right) \quad 5$$

$$+ \quad '' \quad e^{-i(-\omega_{j'} t' - \omega_{j''} t'' + \omega_{j'''} t''' + \omega_{j''''} t'''')}$$

$$\mathrm{tr}(c_{1'}\, c_{1''}\, c^+_{1'''}\, c^+_{1''''}\, \rho_B)\Big)$$ ⑥

$$\mathrm{tr}(c^+_1 c^+_2 c_3 c_4 \rho_B) = \mathrm{tr}(c^+_1 c_4 \rho_B)\,\mathrm{tr}(c^+_2 c_3 \rho_B)$$

$$+\,\mathrm{tr}(c^+_1 c_3 \rho_B)\,\mathrm{tr}(c^+_2 c_4 \rho_B)$$

$$\mathrm{tr}(c^+_1 c_2 \rho_B) = n_1 \delta_{12}$$

(1) $n_{\varsigma'} n_{\varsigma''} \delta_{\varsigma'\varsigma'''} \delta_{\varsigma''\varsigma''''} + n_{\varsigma'} n_{\varsigma''} \delta_{\varsigma'\varsigma''''} \delta_{\varsigma''\varsigma'''}$ ← Ansatz

(2) $n_{\varsigma'} (n_{\varsigma''}+1) \delta_{\varsigma'\varsigma''''} \delta_{\varsigma''\varsigma'''} + n_{\varsigma'} n_{\varsigma''} \delta_{\varsigma'\varsigma''} \delta_{\varsigma'''\varsigma''''}$

(3) $(n_{\varsigma'''}+1) n_{\varsigma'} \delta_{\varsigma''\varsigma''''} \delta_{\varsigma'''\varsigma'} + (n_{\varsigma''}+1) n_{\varsigma'} \delta_{\varsigma''''\varsigma''} \delta_{\varsigma'\varsigma'''}$

(4) $(n_{\varsigma'}+1) n_{\varsigma'''} \delta_{\varsigma'\varsigma''} \delta_{\varsigma'''\varsigma''''} + (n_{\varsigma'}+1) n_{\varsigma''} \delta_{\varsigma'\varsigma''''} \delta_{\varsigma''\varsigma''''}$

(5) $(n_{\varsigma'}+1)(n_{\varsigma''''}+1) \delta_{\varsigma'\varsigma''} \delta_{\varsigma'''\varsigma''''} + (1+n_{\varsigma'}) n_{\varsigma''} \delta_{\varsigma'\varsigma''''} \delta_{\varsigma''\varsigma'''}$

(6) $(n_{\varsigma'''}+1)(n_{\varsigma''''}+1) \delta_{\varsigma''''\varsigma'} \delta_{\varsigma''\varsigma'''} + (n_{\varsigma''}+1)(n_{\varsigma'}+1) \delta_{\varsigma''''\varsigma''} \delta_{\varsigma'''\varsigma'}$

$$\sum_{1}\sum_{\varsigma} \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \int_{t_0}^{t''} dt''' \int_{t_0}^{t'''} dt'''' \Big[n_\varsigma n_\varsigma \Big(e^{-i\omega_\varsigma (t'-t''')}\, e^{-i\omega_\varsigma (t''-t'''')}$$

$$+\, e^{-i\omega_\varsigma (t'-t'''')}\, e^{-i\omega_\varsigma (t''-t''')}$$

$$+\, e^{-i(\omega_\varsigma t' - \omega_\varsigma t'')}\, e^{-i\omega_\varsigma (t''' - t'''')}$$

$$= \underline{\sum_{1}\sum_{\varsigma} n_1 n_\varsigma} \Big(\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} d\tilde{t}_1 \int_{t_0}^{\tilde{t}_1} dt_2 \int_{t_0}^{t_2} d\tilde{t}_2\, \underbrace{e^{-i\omega_1 (t_1 - t_2)}\, e^{-i\omega_\varsigma (\tilde{t}_1 - \tilde{t}_2)}}_{I(t_1, t_2, \tilde{t}_1, \tilde{t}_2)}$$

$$+ \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} d\tilde{t}_1 \int_{t_0}^{\tilde{t}_1} d\tilde{t}_2 \int_{t_0}^{\tilde{t}_2} dt_2\, e^{-i\omega_1 (t_1 - t_2)}\, e^{-i\omega_\varsigma (\tilde{t}_1 - \tilde{t}_2)}$$

$$+ \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_2} d\tilde{t}_1 \int_{t_0}^{\tilde{t}_1} d\tilde{t}_2\, e^{-i\omega_1 (t_1 - t_2)}\, e^{-i\omega_\varsigma (\tilde{t}_1 - \tilde{t}_2)}$$

$$= \frac{1}{2} \Big(\int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} d\tilde{t}_1 \int_{t_0}^{\tilde{t}_1} dt_2 \int_{t_0}^{t_2} d\tilde{t}_2 \, I(\ldots) + \int_{t_0}^{t} d\tilde{t}_1 \int_{t_0}^{\tilde{t}_1} dt_1 \int_{t_0}^{t_1} d\tilde{t}_2 \int_{t_0}^{\tilde{t}_2} dt_2 \, I(\ldots)$$

$$+ \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} d\tilde{t}_1 \int_{t_0}^{\tilde{t}_1} d\tilde{t}_2 \int_{t_0}^{\tilde{t}_2} dt_2 \, I(\ldots) + \int_{t_0}^{t} d\tilde{t}_1 \int_{t_0}^{\tilde{t}_1} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_2} d\tilde{t}_2 \, I(\ldots)$$

$$+ \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_2} d\tilde{t}_1 \int_{t_0}^{\tilde{t}_1} d\tilde{t}_2 \, I(\ldots) + \int_{t_0}^{t} d\tilde{t}_1 \int_{t_0}^{\tilde{t}_1} d\tilde{t}_2 \int_{t_0}^{\tilde{t}_2} dt_1 \int_{t_0}^{t_1} dt_2 \, I(\ldots) \Big)$$

1 $t_0 < \tilde{t}_2 < t_2 < \tilde{t}_1 < t_1 < t$

2 $t_0 < t_2 < \tilde{t}_2 < t_1 < \tilde{t}_1 < t$

3 $t_0 < t_2 < \tilde{t}_2 < \tilde{t}_1 < t_1 < t$

4 $t_0 < \tilde{t}_2 < t_2 < t_1 < \tilde{t}_1 < t$

5 $t_0 < \tilde{t}_2 < \tilde{t}_1 < t_2 < t_1 < t$

6 $t_0 < t_2 < t_1 < \tilde{t}_2 < \tilde{t}_1 < t$

1+5 $t_0 < \tilde{t}_2 < \tilde{t}_1 < t_1 < t$
$\wedge$ $\tilde{t}_2 < t_2 < t_1$

2+6 $t_0 < t_2 < t_1 < \tilde{t}_1 < t$
$\wedge$ $t_2 < \tilde{t}_2 < \tilde{t}_1 < t$

1+5+3 $t_0 < \tilde{t}_2 < \tilde{t}_1 < t_1 < t$
$t_0 < t_2 < t_1 < t$

$\tilde{t}_2 < \tilde{t}_1$, $\tilde{t}_1 < t_1$, $t_2 < t_1$

2+6+4 $t_0 < \tilde{t}_2 < \tilde{t}_1 < t$
$t_0 < t_2 < t_1 < \tilde{t}_1 < t$

$\tilde{t}_2 < \tilde{t}_1$, $t_2 < t_1$, $t_1 < \tilde{t}_1$

1+...+6

$t_0 < \tilde{t}_2 < \tilde{t}_1 < t$
$t_0 < t_2 < t_1 < t$
$$\Rightarrow \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t} d\tilde{t}_1 \int_{t_0}^{\tilde{t}_1} d\tilde{t}_2 \, I(\ldots)$$

$$\Downarrow$$

$$= \frac{1}{2} \Big(\sum_{l} \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \, e^{-i\omega_l (t_1 - t_2)} h_l \Big)^2$$

Aber ~~Rest analog~~ $\mathcal{F}_2 = 0$. Damit 4. Ordnung $= 0$

$$\Theta = \frac{1}{2}\left(-\frac{1}{\hbar^2}\sum_l g_l^2 \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \left(e^{-i\omega_l(t'-t'')} n_l + e^{i\omega_l(t'-t'')}(n_l+1)\right)\right)^2$$

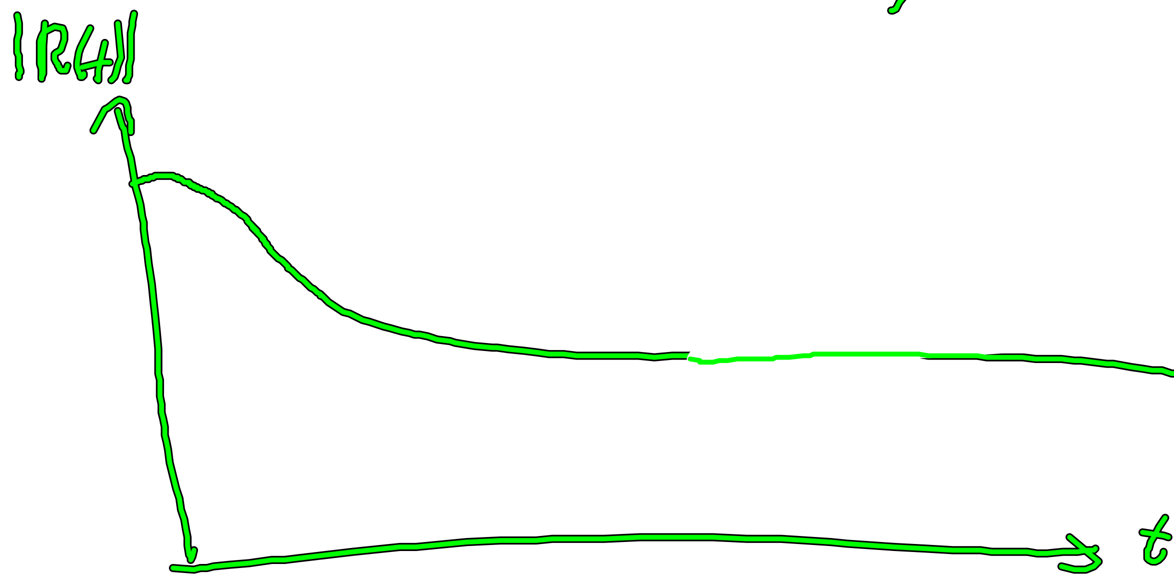
$$= \frac{1}{2}\mathcal{F}_1(t)^2 \overset{!}{=} -\mathcal{F}_2(t) + \frac{\mathcal{F}_1(t)\,\mathcal{F}_1(t)}{2!}$$

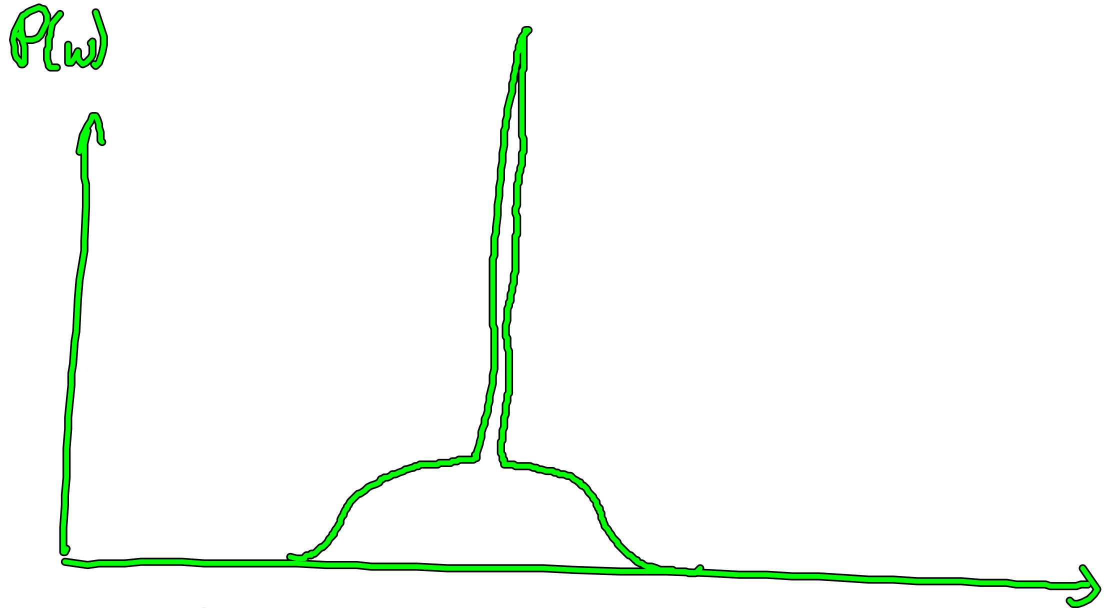
$$\Rightarrow \mathcal{F}_2(t) = 0$$

Alles nur wegen Wicks Theorem.

<u>Aber</u>

$$R(t) = \exp\left(-\frac{1}{\hbar^2}\sum_l g_l^2\left(\frac{(1-e^{-i\omega_l(t-t_0)})}{\omega_l^2} n_l + \frac{(1-e^{i\omega_l(t-t_0)})}{\omega_l^2}(n_l+1) + \frac{i}{\omega_l}(t-t_0)\right)\right)$$





Beispiel für komplexe Anordnung

A

$$R(t,t_0,\tilde t,\tilde t_0) = \mathrm{tr}_B\Big(T_{\leftarrow} \exp\Big(-\frac{i}{\hbar}\int_{t_0}^{t} dt' \sum_l g_l \big(c_l^{\dagger} e^{-i\omega_l(t'-t)} + c_l e^{i\omega_l(t'-t)}\big)\Big)$$

$$T_{\rightarrow} \exp\Big(-\frac{i}{\hbar}\int_{\tilde t_0}^{\tilde t} dt' \sum_l g_l \big(c_l^{\dagger} e^{-i\omega_l(t'-t)} + c_l e^{i\omega_l(t'-t)}\big)\Big)\Big)$$

B